

② $f(x) = \frac{x^2}{x+1}$

① $\text{Dom } f = \{x \in \mathbb{R} \mid x \neq -1\}$

② P. corte ejes

Eje X ($y=0$) $\Rightarrow \frac{x^2}{x+1} = 0 \Rightarrow x^2 = 0 \Rightarrow x=0 \Rightarrow C(0,0)$

Eje Y ($x=0$) $\Rightarrow \frac{0^2}{0+1} = \frac{0}{1} = 0 \xrightarrow{\text{Asimptota}} (0,0)$

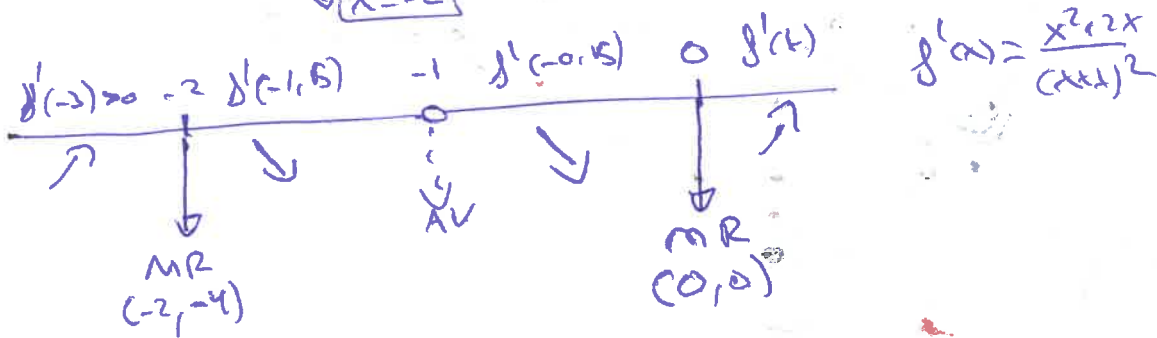
③ A. verticales

$\lim_{x \rightarrow -1} \frac{x^2}{x+1} = \frac{1}{0} \Rightarrow \text{AV en } x = -1$

④ Monotonía ($f'(x) = 0$)

$f'(x) = \frac{2x(x+1) - x^2 \cdot 1}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} \parallel \frac{x^2 + 2x}{(x+1)^2} = 0 \Rightarrow x^2 + 2x = 0$

$\Rightarrow x(x+2) = 0 \Rightarrow \begin{cases} x=0 \\ x=-2 \end{cases}$



⑤ Concavidad ($f''(x) = 0$)

SUPRIMIBLE

$f''(x) = \frac{(2x+2) \cdot (x+1)^2 - (x^2+2x) \cdot 2(x+1) \cdot 1}{[(x+1)^2]^2} = \frac{(x+1) [(2x+2)(x+1) - 2(x^2+2x)]}{(x+1)^4} = \frac{2x^2+2x+2x+2-2x^2-4x}{(x+1)^3}$

$= \frac{2}{(x+1)^3} \parallel \frac{2}{(x+1)^3} = 0 \Rightarrow 2=0 \nexists \text{ solución}$



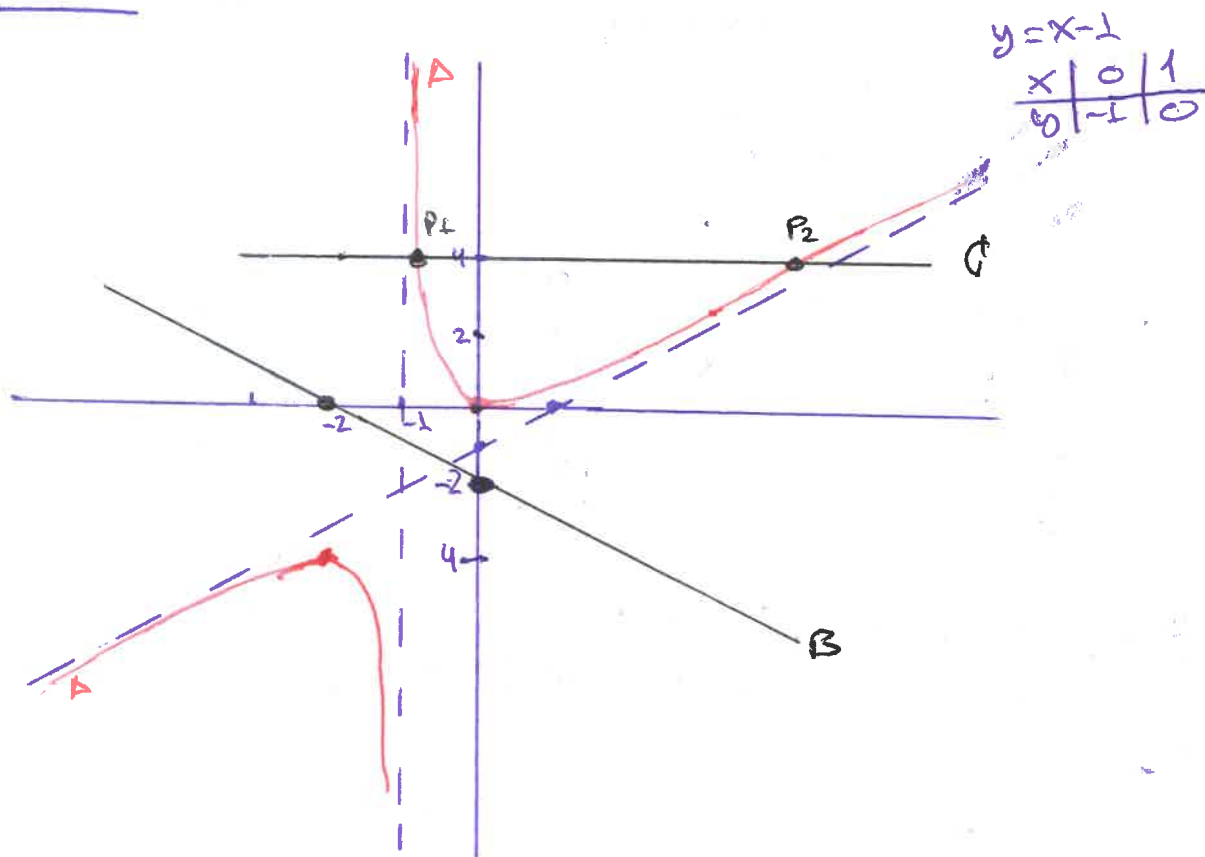
⑥ R. infinitas $f(x) = \frac{x^2}{x+1} \rightarrow G:2 \mid \Rightarrow \text{Asintota oblicua } y = mx + n$

$$m = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x+1}}{x} = \lim_{x \rightarrow \infty} \frac{x^2}{x(x+1)} = \lim_{x \rightarrow \infty} \frac{x^2}{x^2+x} = \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = 1$$

$$n = \lim_{x \rightarrow \infty} f(x) - mx = \lim_{x \rightarrow \infty} \frac{x^2}{x+1} - x = \lim_{x \rightarrow \infty} \frac{x^2}{x+1} + \frac{-x^2-x}{x+1} = \lim_{x \rightarrow \infty} \frac{x^2-x^2-x}{x+1} = \lim_{x \rightarrow \infty} \frac{-x}{x+1} = -1$$

$$AO: \boxed{y = x - 1}$$

R. GRÁFICA



b) $B \rightarrow y = -x - 2$

x	0	-2
y	-2	0

claramente no se cortan.

c) $G \rightarrow y = 4$

claramente las curvas A y G se cortan en 2 puntos

P_1 y P_2 .

CÁLCULO DE LOS PUNTOS DE CORTE

$$f(x) = 4 \Rightarrow \frac{x^2}{x+1} = 4 \Rightarrow x^2 = 4(x+1) \Rightarrow x^2 = 4x + 4 \Rightarrow x^2 - 4x - 4 = 0$$

$$\Rightarrow x^2 - 4x - 4 = 0 \quad x = \frac{4 \pm \sqrt{16 + 16}}{2} = \frac{4 \pm \sqrt{32}}{2} = \begin{cases} x_1 = \frac{4 + \sqrt{32}}{2} \approx 4,83 \\ x_2 = \frac{4 - \sqrt{32}}{2} \approx -0,83 \end{cases}$$

Los puntos pedidos son:

$$P_1 = (-0,83, 4) \quad P_2 = (4,83, 4)$$

