1 SJ $f(x) = \dfrac{x^2-4}{x-1}$

① $\text{Dom} f = \mathbb{R} - \{1\}$

② <u>P. corte ejes</u>

Eje X $(y=0)$ $\Rightarrow \dfrac{x^2-4}{x-1}=0 \Rightarrow x^2-4=0 \Rightarrow x^2=4 \Rightarrow x=\pm\sqrt{4} \Rightarrow \boxed{x=\pm 2}$

$\Rightarrow (-2,0)\,,\,(2,0)$

Eje Y $(x=0)$ $\Rightarrow f(0) = \dfrac{-4}{-1} = 4 \Rightarrow (0,4)$

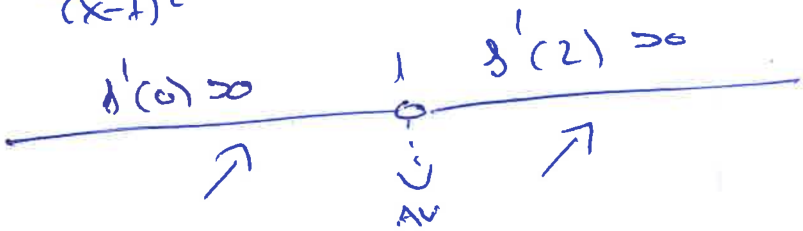
③ <u>A. verticales</u>

$\lim\limits_{x\to 1} \dfrac{x^2-4}{x-1} = \dfrac{-3}{0} \Rightarrow$ AV en $x=1$

④ <u>Monotonía</u> $(f'(x)=0)$

$$f'(x) = \frac{2x\cdot(x-1)-(x^2-4)\cdot 1}{(x-1)^2} = \frac{2x^2-2x-x^2+4}{(x-1)^2} = \frac{x^2-2x+4}{(x-1)^2} \,//$$

$\dfrac{x^2-2x+4}{(x-1)^2}=0 \Rightarrow x^2-2x+4=0$ $\nexists$ solución

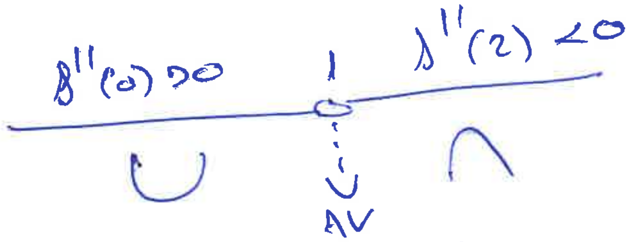


$f'(x) = \dfrac{x^2-2x+4}{(x-1)^2}$

⑤ <u>Curvatura</u> $(f''(x)=0)$

$$f''(x) = \frac{(2x-2)\cdot(x-1)^2-(x^2-2x+4)\,2(x-1)\,1}{[(x-1)^2]^2} = \frac{\cancel{(x-1)}\,[(2x-2)(x-1)-(x^2-2x+4)2]}{(x-1)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{2x^2}-2x-2x+2-\cancel{2x^2}+\cancel{4x}-8}{(x-1)^3} = \frac{-6}{(x-1)^3}\,// \qquad \frac{-6}{(x-1)^3}=0 \Rightarrow -6=0 \;\nexists \text{ solución}$$



$f''(x) = \dfrac{-6}{(x-1)^3}$

⑥ <u>R. infinitas</u> $f(x) = \dfrac{x^2-4}{x-1}$ $\begin{matrix}\to G:2 \\ \to G:1\end{matrix}\Big\} \Rightarrow$ Asíntota oblicua: $y=mx+n$

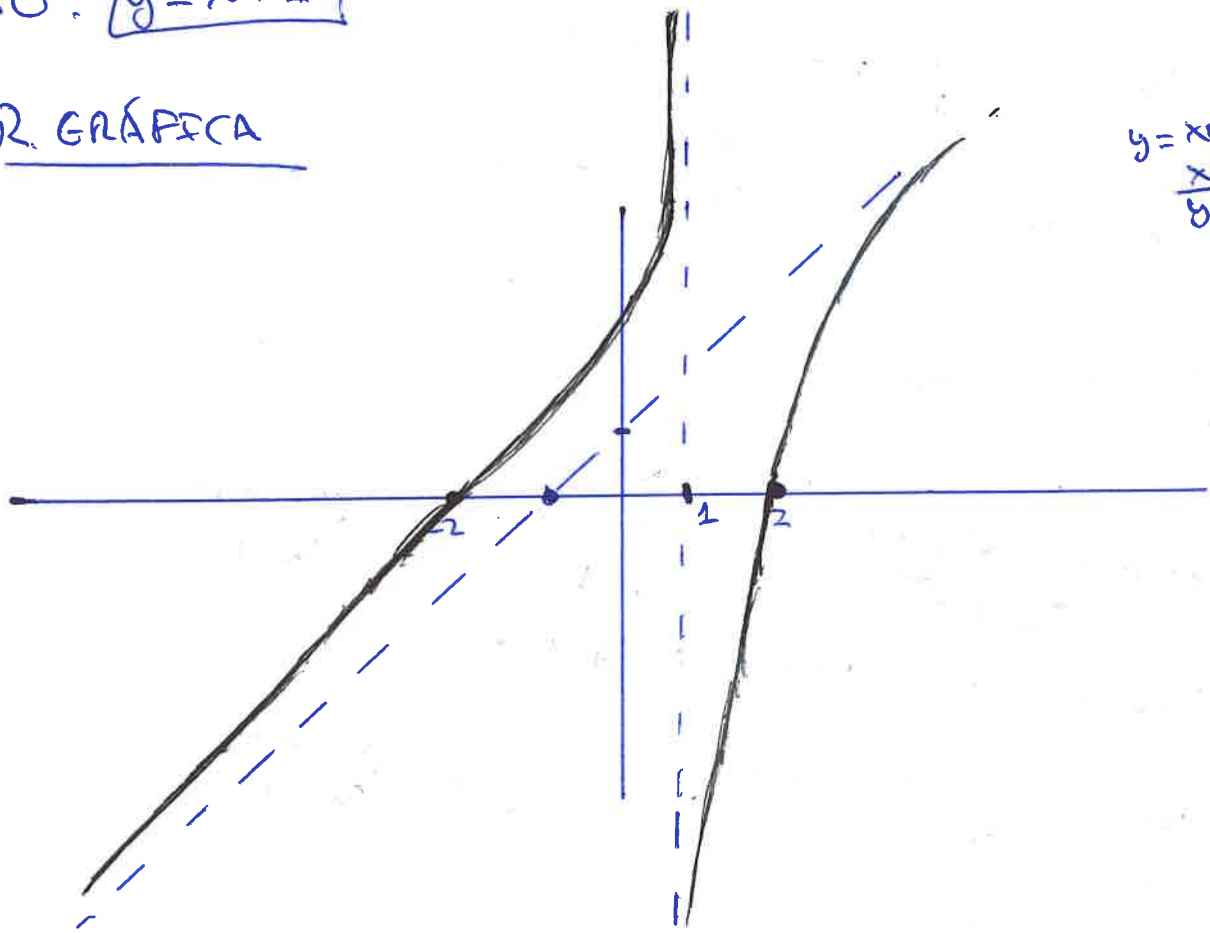
$$m = \lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{\frac{x^2-4}{x-1}}{x} = \lim_{x\to\infty}\frac{x^2-4}{x(x-1)} = \lim_{x\to\infty}\frac{x^2-4}{x^2-x} = \lim_{x\to\infty}\frac{x^2}{x^2} = 1$$

$$n = \lim_{x \to \infty} f(x) - mx = \lim_{x \to \infty} \frac{x^2-4}{x-1} - x = \lim_{x \to \infty} \frac{x^2-4}{x-1} + \frac{-x^2+x}{x-1} = \lim_{x \to \infty} \frac{x-4}{x-1} =$$

$$= \lim_{x \to \infty} \frac{x}{x} = 1$$

AO: $\boxed{y = x + 1}$

R. GRÁFICA



$y = x+1$

x	0	-1
y	1	0