3

a) $\cos(30^\circ+\alpha)=\operatorname{sen}\alpha \Rightarrow \cos 30^\circ \cos\alpha - \operatorname{sen}30^\circ \operatorname{sen}\alpha = \operatorname{sen}\alpha \Rightarrow$

$\frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\operatorname{sen}\alpha - \operatorname{sen}\alpha = 0 \Rightarrow \frac{\sqrt{3}}{2}\cos\alpha - \frac{3}{2}\operatorname{sen}\alpha = 0 \xRightarrow{\cdot 2} \sqrt{3}\cos\alpha - 3\operatorname{sen}\alpha = 0$

$\Rightarrow \sqrt{3}\cos\alpha = 3\operatorname{sen}\alpha \xRightarrow[\boxed{\cos\alpha \neq 0}]{} \frac{\sqrt{3}}{3} = \tan\alpha \Rightarrow \tan\alpha = \frac{\sqrt{3}}{3} \Rightarrow \alpha = \operatorname{arctan}\frac{\sqrt{3}}{3} = 30^\circ$

$180^\circ + 30^\circ = 210^\circ$

$$\Rightarrow \begin{cases} \alpha_1 = 30^\circ + K360^\circ \\ \alpha_2 = 210^\circ + K360^\circ \end{cases} \quad K \in \mathbb{Z}$$

b) $\operatorname{sen}(2\alpha) = \tan\alpha \Rightarrow 2\operatorname{sen}\alpha\cos\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \Rightarrow 2\operatorname{sen}\alpha\cos^2\alpha = \operatorname{sen}\alpha \Rightarrow$

$2\operatorname{sen}\alpha\cos^2\alpha - \operatorname{sen}\alpha = 0 \Rightarrow \operatorname{sen}\alpha(2\cos^2\alpha - 1) = 0 \rightarrow$

$\rightarrow \begin{cases} \operatorname{sen}\alpha = 0 \quad (1) \\ 2\cos^2\alpha - 1 = 0 \Rightarrow 2\cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{1}{2} \Rightarrow \cos\alpha = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} \end{cases}$

$= \pm\frac{\sqrt{2}}{2} \begin{cases} \cos\alpha = \frac{\sqrt{2}}{2} \quad (2) \\ \cos\alpha = -\frac{\sqrt{2}}{2} \quad (3) \end{cases}$

<u>CASO 1</u>

$\operatorname{sen}\alpha = 0 \Rightarrow \alpha = \operatorname{arc}\operatorname{sen} 0 = 0^\circ \Rightarrow \begin{cases} \alpha_1 = 0^\circ + K360^\circ \\ \alpha_2 = 180^\circ + K360^\circ \end{cases}$

$180^\circ - 0^\circ = 180^\circ$

<u>CASO 2</u>

$\cos\alpha = \frac{\sqrt{2}}{2} \Rightarrow \alpha = \operatorname{arc}\cos\frac{\sqrt{2}}{2} = 45^\circ \Rightarrow \begin{cases} \alpha_1 = 45^\circ + K360^\circ \\ \alpha_2 = 315^\circ + K360^\circ \end{cases}$

$360^\circ - 45^\circ = 315^\circ$

<u>CASO 3</u>

$\cos\alpha = -\frac{\sqrt{2}}{2} \Rightarrow \alpha = \operatorname{arc}\cos\left(-\frac{\sqrt{2}}{2}\right) = 135^\circ \Rightarrow \begin{cases} \alpha_1 = 135^\circ + K360^\circ \\ \alpha_2 = 225^\circ + K360^\circ \end{cases}$

$360^\circ - 135^\circ = 225^\circ$

<u>SOLUCIONES</u>

$\alpha_1 = 0^\circ + K360^\circ$

$\alpha_2 = 45^\circ + K360^\circ$

$\alpha_3 = 135^\circ + K360^\circ$

$\alpha_4 = 180^\circ + K360^\circ$

$\alpha_5 = 225^\circ + K360^\circ$

$\alpha_6 = 315^\circ + K360^\circ$

$K \in \mathbb{Z}$

d) $\cos(2x) = 1 + 4\,\mathrm{sen}\,x \rightarrow \cos^2 x - \mathrm{sen}^2 x = 1 + 4\,\mathrm{sen}\,x \rightarrow$

$1 - \mathrm{sen}^2 x - \mathrm{sen}^2 x = 1 + 4\,\mathrm{sen}\,x \rightarrow -2\,\mathrm{sen}^2 x + 1 = 1 + 4\,\mathrm{sen}\,x$

$\rightarrow -2\,\mathrm{sen}^2 x + 1 - 1 - 4\,\mathrm{sen}\,x = 0 \rightarrow -2\,\mathrm{sen}^2 x - 4\,\mathrm{sen}\,x = 0 \xrightarrow{:(-2)}$

$\mathrm{sen}^2 x + 2\,\mathrm{sen}\,x = 0 \rightarrow \mathrm{sen}\,x(\mathrm{sen}\,x + 2) = 0$

- $\nearrow \mathrm{sen}\,x = 0$ (A)
- $\searrow \mathrm{sen}\,x + 2 = 0 \rightarrow \mathrm{sen}\,x = -2$ X

Imposible pues $-1 \leq \mathrm{sen}\,x \leq 1$

CASO A

$\mathrm{sen}\,x = 0 \Rightarrow x = \arcsin 0 = 0^\circ \Rightarrow \begin{cases} \alpha_1 = 0^\circ + k360^\circ \\ \alpha_2 = 180^\circ + k360^\circ \end{cases}$ $k \in \mathbb{Z}$

$180^\circ - 0^\circ = 180^\circ$