① Sabiendo que α está en el 3º cuadrante y que $\cos\alpha = -\frac{\sqrt{3}}{2}$, se pide calcular sin calculadora:

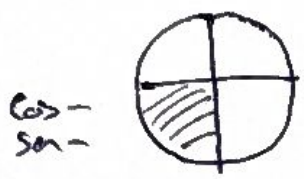
a) $\cos(\alpha+45°) = \cos\alpha\cos 45° - \operatorname{sen}\alpha \operatorname{sen} 45° = \left(-\frac{\sqrt{3}}{2}\right)\cdot\frac{\sqrt{2}}{2} - \left(-\frac{1}{2}\right)\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{-\sqrt{6}+\sqrt{2}}{4}$

CÁLCULO DE $\operatorname{sen}\alpha$

$\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \to \operatorname{sen}^2\alpha + \left(-\frac{\sqrt{3}}{2}\right)^2 = 1 \to \operatorname{sen}^2\alpha + \frac{3}{4} = 1 \to \operatorname{sen}^2\alpha = 1 - \frac{3}{4}$

$\to \operatorname{sen}^2\alpha = \frac{1}{4} \to \operatorname{sen}\alpha = \pm\sqrt{\frac{1}{4}} = -\sqrt{\frac{1}{4}} = -\frac{1}{\sqrt{4}} = -\frac{1}{2}$



b) $\operatorname{sen}(2\alpha) = 2\operatorname{sen}\alpha\cos\alpha = 2\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{\sqrt{3}}{2}\right) = \frac{2\sqrt{3}}{4\cdot 2} = \frac{\sqrt{3}}{2}$

c) $\cos(2\alpha) = \cos^2\alpha - \operatorname{sen}^2\alpha = \left(-\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

d) $\operatorname{sen}(\alpha-30°) = \operatorname{sen}\alpha\cos 30° - \cos\alpha\operatorname{sen}30° = \left(-\frac{1}{2}\right)\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\frac{1}{2} = -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 0$

e) $\tan(45°+\alpha) = \frac{\tan 45° + \tan\alpha}{1 - \tan 45°\cdot\tan\alpha} = \frac{1+\frac{\sqrt{3}}{3}}{1-1\cdot\frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} = *$

$\tan\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{(\sqrt{3})^2} = \frac{\sqrt{3}}{3}$

$* = \frac{(3+\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{9+3\sqrt{3}+3\sqrt{3}+(\sqrt{3})^2}{3^2-(\sqrt{3})^2} = \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6} = \frac{6(2+\sqrt{3})}{6}$

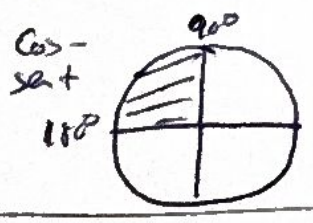
$= 2+\sqrt{3}$

d) $\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} =$

$= -\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

SIGNO DE $\cos\frac{\alpha}{2}$

$180° < \alpha < 270° \rightarrow \frac{180°}{2} < \frac{\alpha}{2} < \frac{270°}{2} \rightarrow 90° < \frac{\alpha}{2} < 135°$



Como $\frac{\alpha}{2}$ está en el 2º cuadrante, $\cos\frac{\alpha}{2}$ es negativo