

|    |                   |                      |
|----|-------------------|----------------------|
| 7  | $w \wedge q$      | $E \rightarrow 2, 6$ |
| 8  | $w$               | $E \wedge 7$         |
| 9  | $p$               | $E \rightarrow 3, 8$ |
| 10 | $q$               | $E \rightarrow 4, 9$ |
| 11 | $s \rightarrow q$ | $I \rightarrow 5-10$ |

4)

|    |                                                   |                                                          |
|----|---------------------------------------------------|----------------------------------------------------------|
| —1 | $t \rightarrow (p \rightarrow (s \rightarrow r))$ | $\vdash p \rightarrow (s \rightarrow (t \rightarrow r))$ |
| 2  | $p$                                               |                                                          |
| 3  | $s$                                               |                                                          |
| 4  | $t$                                               |                                                          |
| 5  | $p \rightarrow (s \rightarrow r)$                 | $E \rightarrow 1, 4$                                     |
| 6  | $s \rightarrow r$                                 | $E \rightarrow 5, 2$                                     |
| 7  | $r$                                               | $E \rightarrow 6, 3$                                     |
| 8  | $t \rightarrow r$                                 | $I \rightarrow 4-7$                                      |
| 9  | $s \rightarrow (t \rightarrow r)$                 | $I \rightarrow 3-8$                                      |
| 10 | $p \rightarrow (s \rightarrow (t \rightarrow r))$ | $I \rightarrow 2-9$                                      |

5)

|    |                                                                     |                                                              |
|----|---------------------------------------------------------------------|--------------------------------------------------------------|
| —1 | $t \rightarrow (p \rightarrow s) \wedge (s \rightarrow \neg r)$     |                                                              |
| —2 | $(\neg s \vee r) \wedge (\neg \neg t \wedge \neg(p \rightarrow q))$ | $\vdash \neg(p \rightarrow q) \wedge (s \rightarrow \neg r)$ |
| 3  | $\neg \neg t \wedge \neg(p \rightarrow q)$                          | $E \wedge 2$                                                 |
| 4  | $\neg \neg t$                                                       | $E \wedge 3$                                                 |
| 5  | $t$                                                                 | $E \neg 4$                                                   |
| 6  | $(p \rightarrow s) \wedge (s \rightarrow \neg r)$                   | $E \rightarrow 1, 5$                                         |
| 7  | $s \rightarrow \neg r$                                              | $E \wedge 6$                                                 |
| 8  | $\neg(p \rightarrow q)$                                             | $E \wedge 3$                                                 |
| 9  | $\neg(p \rightarrow q) \wedge (s \rightarrow \neg r)$               | $I \wedge 8, 7$                                              |

6)

|    |                                                                    |                          |
|----|--------------------------------------------------------------------|--------------------------|
| —1 | $r \vee \neg s \rightarrow (s \vee p) \wedge (\neg q \vee \neg t)$ |                          |
| —2 | $s \rightarrow q$                                                  | $\vdash r \rightarrow q$ |
| —3 | $p \rightarrow q$                                                  |                          |
| 4  | $r$                                                                |                          |
| 5  | $r \vee \neg s$                                                    | $I \vee 4$               |
| 6  | $(s \vee p) \wedge (\neg q \vee \neg t)$                           | $E \rightarrow 1, 5$     |
| 7  | $s \vee p$                                                         | $E \wedge 6$             |

|    |                   |                        |
|----|-------------------|------------------------|
| 8  | $s$               |                        |
| 9  | $q$               | $E \rightarrow 2, 8$   |
| 10 | $p$               |                        |
| 11 | $q$               | $E \rightarrow 3, 10$  |
| 12 | $q$               | $E \vee 7, 8-9, 10-11$ |
| 13 | $r \rightarrow q$ | $I \rightarrow 4-12$   |

7)

|    |                                            |                         |
|----|--------------------------------------------|-------------------------|
| —1 | $p \rightarrow t \vee r$                   |                         |
| —2 | $t \rightarrow s \wedge m$                 |                         |
| —3 | $r \rightarrow \neg s$                     | $\vdash \neg p$         |
| —4 | $m \vee \neg s \rightarrow \neg(t \vee r)$ |                         |
| 5  | $p$                                        |                         |
| 6  | $t \vee r$                                 | $E \rightarrow 1, 5$    |
| 7  | $t$                                        |                         |
| 8  | $s \wedge m$                               | $E \rightarrow 2, 7$    |
| 9  | $m$                                        | $E \wedge 8$            |
| 10 | $m \vee \neg s$                            | $I \vee 9$              |
| 11 | $\neg(t \vee r)$                           | $E \rightarrow 4, 10$   |
| 12 | $(t \vee r) \wedge \neg(t \vee r)$         | $I \wedge 6, 11$        |
| 13 | $r$                                        |                         |
| 14 | $\neg s$                                   | $E \rightarrow 3, 13$   |
| 15 | $m \vee \neg s$                            | $I \vee 14$             |
| 16 | $\neg(t \vee r)$                           | $E \rightarrow 4, 15$   |
| 17 | $(t \vee r) \wedge \neg(t \vee r)$         | $I \wedge 6, 16$        |
| 18 | $(t \vee r) \wedge \neg(t \vee r)$         | $E \vee 6, 7-12, 13-17$ |
| 19 | $\neg p$                                   | $I \neg 5-18$           |

## BLOQUE II

(NOTA: A continuación se presenta una posible solución a cada uno de los ejercicios propuestos, pero evidentemente caben otras múltiples soluciones alternativas.)

1)

|    |                   |            |
|----|-------------------|------------|
| —1 | $t \rightarrow p$ |            |
| —2 | $s \rightarrow t$ | $\vdash p$ |
| —3 | $s$               |            |

- 4 t             $E \rightarrow 2, 3$   
 5 p             $E \rightarrow 1, 4$
- 2)  
 $\neg 1 r \vee q \rightarrow t$   
 $\neg 2 t \rightarrow p$              $\vdash p$   
 $\neg 3 r \vee q$   
 4 t             $E \rightarrow 1, 3$   
 5 p             $E \rightarrow 2, 4$
- 3)  
 $\neg 1 (s \rightarrow t) \rightarrow r \vee s$   
 $\neg 2 r \vee s \rightarrow w$              $\vdash w$   
 $\neg 3 s \rightarrow t$   
 4 r  $\vee$  s             $E \rightarrow 1, 3$   
 5 w             $E \rightarrow 2, 4$
- 4)  
 $\neg 1 r \vee s \rightarrow t \vee w$   
 $\neg 2 t \vee w \rightarrow r$              $\vdash p \wedge q$   
 $\neg 3 r \rightarrow p \wedge q$   
 $\neg 4 r \vee s$   
 5 t  $\vee$  w             $E \rightarrow 1, 4$   
 6 r             $E \rightarrow 2, 5$   
 7 p  $\wedge$  q             $E \rightarrow 3, 6$
- 5)  
 $\neg 1 \neg p$   
 $\neg 2 \neg r \vee s \rightarrow (\neg p \rightarrow \neg q)$              $\vdash \neg q$   
 $\neg 3 \neg r \vee s$   
 4  $\neg p \rightarrow \neg q$              $E \rightarrow 2, 3$   
 5  $\neg q$              $E \rightarrow 4, 1$
- 6)  
 $\neg 1 s \wedge t$   
 $\neg 2 p \rightarrow q \wedge \neg r$              $\vdash w$   
 $\neg 3 (p \rightarrow q \wedge \neg r) \rightarrow (s \wedge t \rightarrow w)$   
 4 s  $\wedge$  t  $\rightarrow$  w             $E \rightarrow 3, 2$   
 5 w             $E \rightarrow 4, 1$

- 7)  
 $\neg 1 s \rightarrow p$   
 $\neg 2 p \rightarrow q$              $\vdash s \rightarrow m$   
 $\neg 3 q \rightarrow r$   
 $\neg 4 r \rightarrow m$   
 $\neg 5 s$   
 6 p             $E \rightarrow 1, 5$   
 7 q             $E \rightarrow 2, 6$   
 8 r             $E \rightarrow 3, 7$   
 9 m             $E \rightarrow 4, 8$   
 10 s  $\rightarrow$  m             $I \rightarrow 5-9$
- 8)  
 $\neg 1 r \vee t \rightarrow p \wedge q$   
 $\neg 2 p \rightarrow (w \leftrightarrow s)$              $\vdash (r \vee t) \rightarrow (w \leftrightarrow s)$   
 $\neg 3 p \wedge q \rightarrow (r \vee s \rightarrow t \wedge m)$   
 $\neg 4 (s \vee r \rightarrow t \wedge w) \rightarrow p$   
 $\neg 5 r \vee t$   
 6 p  $\wedge$  q             $E \rightarrow 1, 5$   
 7 p             $E \wedge 6$   
 8 w  $\leftrightarrow$  s             $E \rightarrow 2, 7$   
 9 (r  $\vee$  t)  $\rightarrow$  (w  $\leftrightarrow$  s)             $I \rightarrow 5-8$
- 9)  
 $\neg 1 r \rightarrow w \wedge p \wedge q$   
 $\neg 2 (p \rightarrow q) \rightarrow (r \rightarrow q)$              $\vdash (s \vee t \rightarrow s) \rightarrow (r \rightarrow q)$   
 $\neg 3 (s \vee t \rightarrow s) \rightarrow r$   
 $\neg 4 w \wedge p \wedge q \rightarrow (p \rightarrow q)$   
 $\neg 5 s \vee t \rightarrow s$   
 6 r             $E \rightarrow 3, 5$   
 7 w  $\wedge$  p  $\wedge$  q             $E \rightarrow 1, 6$   
 8 p  $\rightarrow$  q             $E \rightarrow 4, 7$   
 9 r  $\rightarrow$  q             $E \rightarrow 2, 8$   
 10 (s  $\vee$  t  $\rightarrow$  s)  $\rightarrow$  (r  $\rightarrow$  q)             $I \rightarrow 5-9$

10)

|   |                                                      |                      |                                                             |
|---|------------------------------------------------------|----------------------|-------------------------------------------------------------|
| 1 | $\neg s \vee t \rightarrow (q \rightarrow p \vee r)$ |                      | $\vdash q \rightarrow (\neg s \vee t \rightarrow p \vee r)$ |
| 2 | $q$                                                  |                      |                                                             |
| 3 | $\neg s \vee t$                                      |                      |                                                             |
| 4 | $q \rightarrow p \vee r$                             | $E \rightarrow 1, 3$ |                                                             |
| 5 | $p \vee r$                                           | $E \rightarrow 4, 2$ |                                                             |
| 6 | $\neg s \vee t \rightarrow p \vee r$                 | $I \rightarrow 3-5$  |                                                             |
| 7 | $q \rightarrow (\neg s \vee t \rightarrow p \vee r)$ | $I \rightarrow 2-6$  |                                                             |

11)

|    |                                                   |                      |                                                          |
|----|---------------------------------------------------|----------------------|----------------------------------------------------------|
| 1  | $q \rightarrow (s \rightarrow (t \rightarrow w))$ |                      | $\vdash t \rightarrow (q \rightarrow (s \rightarrow w))$ |
| 2  | $t$                                               |                      |                                                          |
| 3  | $q$                                               |                      |                                                          |
| 4  | $s$                                               |                      |                                                          |
| 5  | $s \rightarrow (t \rightarrow w)$                 | $E \rightarrow 1, 3$ |                                                          |
| 6  | $t \rightarrow w$                                 | $E \rightarrow 5, 4$ |                                                          |
| 7  | $w$                                               | $E \rightarrow 6, 2$ |                                                          |
| 8  | $s \rightarrow w$                                 | $I \rightarrow 4-7$  |                                                          |
| 9  | $q \rightarrow (s \rightarrow w)$                 | $I \rightarrow 3-8$  |                                                          |
| 10 | $t \rightarrow (q \rightarrow (s \rightarrow w))$ | $I \rightarrow 2-9$  |                                                          |

12)

|   |                            |                      |            |
|---|----------------------------|----------------------|------------|
| 1 | $r \rightarrow s$          |                      |            |
| 2 | $s \rightarrow p \wedge q$ |                      | $\vdash q$ |
| 3 | $t \wedge r$               |                      |            |
| 4 | $r$                        | $E \wedge 3$         |            |
| 5 | $s$                        | $E \rightarrow 1, 4$ |            |
| 6 | $p \wedge q$               | $E \rightarrow 2, 5$ |            |
| 7 | $q$                        | $E \wedge 6$         |            |

13)

|   |                            |                      |                          |
|---|----------------------------|----------------------|--------------------------|
| 1 | $s \rightarrow p \wedge t$ |                      |                          |
| 2 | $t \rightarrow w \wedge q$ |                      | $\vdash s \rightarrow q$ |
| 3 | $s$                        |                      |                          |
| 4 | $p \wedge t$               | $E \rightarrow 1, 3$ |                          |
| 5 | $t$                        | $E \wedge 4$         |                          |
| 6 | $w \wedge q$               | $E \rightarrow 2, 5$ |                          |
| 7 | $q$                        | $E \wedge 6$         |                          |

8  $s \rightarrow q$        $I \rightarrow 3-7$ 

14)

|   |                                     |                      |            |
|---|-------------------------------------|----------------------|------------|
| 1 | $w \rightarrow p \wedge t$          |                      |            |
| 2 | $t \wedge r \wedge s \rightarrow q$ |                      | $\vdash q$ |
| 3 | $w \wedge r \wedge s$               |                      |            |
| 4 | $w$                                 | $E \wedge 3$         |            |
| 5 | $p \wedge t$                        | $E \rightarrow 1, 4$ |            |
| 6 | $r \wedge s$                        | $E \wedge 3$         |            |
| 7 | $t$                                 | $E \wedge 5$         |            |
| 8 | $t \wedge r \wedge s$               | $I \wedge 7, 6$      |            |
| 9 | $q$                                 | $E \rightarrow 2, 8$ |            |

15)

|    |                                                     |                      |                     |
|----|-----------------------------------------------------|----------------------|---------------------|
| 1  | $(q \rightarrow s \wedge t) \rightarrow r \wedge t$ |                      |                     |
| 2  | $q \rightarrow s \wedge t$                          |                      | $\vdash p \wedge t$ |
| 3  | $r \rightarrow (s \wedge t \rightarrow p)$          |                      |                     |
| 4  | $q \wedge w$                                        |                      |                     |
| 5  | $r \wedge t$                                        | $E \rightarrow 1, 2$ |                     |
| 6  | $r$                                                 | $E \wedge 5$         |                     |
| 7  | $s \wedge t \rightarrow p$                          | $E \rightarrow 3, 6$ |                     |
| 8  | $q$                                                 | $E \wedge 4$         |                     |
| 9  | $s \wedge t$                                        | $E \rightarrow 2, 8$ |                     |
| 10 | $p$                                                 | $E \rightarrow 7, 9$ |                     |
| 11 | $t$                                                 | $E \wedge 5$         |                     |
| 12 | $p \wedge t$                                        | $I \wedge 10, 11$    |                     |

16)

|   |                                                   |                      |                     |
|---|---------------------------------------------------|----------------------|---------------------|
| 1 | $r \rightarrow p \wedge q$                        |                      |                     |
| 2 | $p \wedge q \rightarrow (s \vee t \rightarrow w)$ |                      | $\vdash r \wedge w$ |
| 3 | $(s \vee t \rightarrow w) \rightarrow w$          |                      |                     |
| 4 | $r$                                               |                      |                     |
| 5 | $p \wedge q$                                      | $E \rightarrow 1, 4$ |                     |
| 6 | $s \vee t \rightarrow w$                          | $E \rightarrow 2, 5$ |                     |
| 7 | $w$                                               | $E \rightarrow 3, 6$ |                     |
| 8 | $r \wedge w$                                      | $I \wedge 4, 7$      |                     |

17)

|   |                            |  |                     |
|---|----------------------------|--|---------------------|
| 1 | $r \rightarrow s \wedge t$ |  |                     |
| 2 | $r$                        |  | $\vdash s \wedge q$ |
| 3 | $r \rightarrow p \wedge q$ |  |                     |

- $4 p \wedge q$        $E \rightarrow 3, 2$   
 $5 s \wedge t$        $E \rightarrow 1, 2$   
 $6 s$            $E \wedge 5$   
 $7 q$            $E \wedge 4$   
 $8 s \wedge q$        $I \wedge 6, 7$

18)

- $-1 r \rightarrow \neg \neg(s \rightarrow q)$   
 $-2 (s \wedge w) \wedge p$        $\vdash \neg \neg r \rightarrow q$   
 $3 \neg \neg r$   
 $4 r$                    $E \neg 3$   
 $5 \neg \neg(s \rightarrow q)$        $E \rightarrow 1, 4$   
 $6 s \rightarrow q$                $E \neg 5$   
 $7 s \wedge w$                $E \wedge 2$   
 $8 s$                    $E \wedge 7$   
 $9 q$                    $E \rightarrow 6, 8$   
 $10 \neg \neg r \rightarrow q$        $I \rightarrow 3-9$

19)

- $-1 r \wedge (s \vee t) \rightarrow w$   
 $-2 r$                    $\vdash p$   
 $-3 s \wedge t$   
 $-4 w \rightarrow p$   
 $5 s$                    $E \wedge 3$   
 $6 s \vee t$                $I \vee 5$   
 $7 r \wedge (s \vee t)$        $I \wedge 2, 6$   
 $8 w$                    $E \rightarrow 1, 7$   
 $9 p$                    $E \rightarrow 4, 8$

20)

 $\vdash (p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ 

- $1 (p \rightarrow q) \wedge (q \rightarrow r)$   
 $2 p$   
 $3 p \rightarrow q$                $E \wedge 1$   
 $4 q$                    $E \rightarrow 3, 2$   
 $5 q \rightarrow r$                $E \wedge 1$   
 $6 r$                    $E \rightarrow 5, 4$   
 $7 p \rightarrow r$                $I \rightarrow 2-6$   
 $8 (p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$        $I \rightarrow 1-7$

21)

- $-1 (p \wedge q \rightarrow r) \wedge (p \rightarrow s \wedge t)$   
 $-2 p \wedge t$                    $\vdash r \wedge (w \vee m)$   
 $-3 s \wedge t \rightarrow q \wedge (w \vee m)$   
 $4 p \wedge q \rightarrow r$                $E \wedge 1$   
 $5 p$                        $E \wedge 2$   
 $6 p \rightarrow s \wedge t$                $E \wedge 1$   
 $7 s \wedge t$                    $E \rightarrow 6, 5$   
 $8 q \wedge (w \vee m)$            $E \rightarrow 3, 7$   
 $9 w \vee m$                    $E \wedge 8$   
 $10 q$                        $E \wedge 8$   
 $11 p \wedge q$                    $I \wedge 5, 10$   
 $12 r$                        $E \rightarrow 4, 11$   
 $13 r \wedge (w \vee m)$            $I \wedge 12, 9$

22)

- $-1 (r \rightarrow \neg w) \rightarrow (\neg q \rightarrow t)$   
 $-2 \neg w \wedge p$                $\vdash \neg q \rightarrow s \vee r$   
 $-3 t \rightarrow m \wedge s$   
 $4 \neg q$   
 $5 r$   
 $6 \neg w$                    $E \wedge 2$   
 $7 r \rightarrow \neg w$                $I \rightarrow 5-6$   
 $8 \neg q \rightarrow t$                $E \rightarrow 1, 7$   
 $9 t$                        $E \rightarrow 8, 4$   
 $10 m \wedge s$                $E \rightarrow 3, 9$   
 $11 s$                        $E \wedge 10$   
 $12 s \vee r$                $I \vee 11$   
 $13 \neg q \rightarrow s \vee r$        $I \rightarrow 4-12$

23)

- $-1 r \wedge s \rightarrow t \wedge m$        $\vdash (t \wedge m) \vee (w \rightarrow m \wedge s)$   
 $-2 r \wedge s$   
 $3 t \wedge m$                    $E \rightarrow 1, 2$   
 $4 (t \wedge m) \vee (w \rightarrow m \wedge s)$        $I \vee 3$

24)

- $-1 r \wedge p \wedge q$                $\vdash w \vee s$   
 $-2 r \wedge q \rightarrow w$   
 $3 r$                        $E \wedge 1$

- 4  $p \wedge q$  E $\wedge$  1  
 5  $q$  E $\wedge$  4  
 6  $r \wedge q$  I $\wedge$  3, 5  
 7  $w$  E $\rightarrow$  2, 6  
 8  $w \vee s$  I $\vee$  7

25)

- 1  $r \rightarrow (s \rightarrow t \wedge m)$   $\vdash s \rightarrow (r \rightarrow t \vee q \vee p)$   
 2  $s$   
 3  $r$   
 4  $s \rightarrow t \wedge m$  E $\rightarrow$  1, 3  
 5  $t \wedge m$  E $\rightarrow$  4, 2  
 6  $t$  E $\wedge$  5  
 7  $t \vee q \vee p$  I $\vee$  6  
 8  $r \rightarrow t \vee q \vee p$  I $\rightarrow$  3-7  
 9  $s \rightarrow (r \rightarrow t \vee q \vee p)$  I $\rightarrow$  2-8

26)

- 1  $q \wedge p$   $\vdash s \vee (t \rightarrow r)$   
 —2  $p \vee r \rightarrow s$   
 3  $p$  E $\wedge$  1  
 4  $p \vee r$  I $\vee$  3  
 5  $s$  E $\rightarrow$  2, 4  
 6  $s \vee (t \rightarrow r)$  I $\vee$  5

27)

- 1  $p \rightarrow r \wedge q$   
 —2  $r \wedge m \rightarrow t$   $\vdash t \vee m$   
 —3  $q \rightarrow m$   
 —4  $p \wedge s$   
 5  $p$  E $\wedge$  4  
 6  $r \wedge q$  E $\rightarrow$  1, 5  
 7  $q$  E $\wedge$  6  
 8  $m$  E $\rightarrow$  3, 7  
 9  $t \vee m$  I $\vee$  8

28)

- 1  $t \vee \neg q \rightarrow (q \vee s) \wedge (\neg r \vee t)$   
 —2  $q \rightarrow r$   $\vdash t \rightarrow r$   
 —3  $s \rightarrow r$   
 4  $t$   
 5  $t \vee \neg q$  I $\vee$  4  
 6  $(q \vee s) \wedge (\neg r \vee t)$  E $\rightarrow$  1, 5  
 7  $q \vee s$  E $\wedge$  6  
 8  $q$   
 9  $r$  E $\rightarrow$  2, 8  
 10  $s$   
 11  $r$  E $\rightarrow$  3, 10  
 12  $r$  E $\vee$  7, 8-9, 10-11  
 13  $t \rightarrow r$  I $\rightarrow$  4-12

29)

- 1  $(p \vee t) \wedge (m \rightarrow w) \rightarrow s \vee q$   
 —2  $(p \vee t) \wedge (s \rightarrow r)$   $\vdash r$   
 —3  $(m \rightarrow w) \wedge (q \rightarrow r)$   
 4  $p \vee t$  E $\wedge$  2  
 5  $m \rightarrow w$  E $\wedge$  3  
 6  $(p \vee t) \wedge (m \rightarrow w)$  I $\wedge$  4, 5  
 7  $s \vee q$  E $\rightarrow$  1, 6  
 8  $s \rightarrow r$  E $\wedge$  2  
 9  $q \rightarrow r$  E $\wedge$  3  
 10  $s$   
 11  $r$  E $\rightarrow$  8, 10  
 12  $q$   
 13  $r$  E $\rightarrow$  9, 12  
 14  $r$  E $\vee$  7, 10-11, 12-13



30)

|    |                            |                        |
|----|----------------------------|------------------------|
| —1 | $t \vee m$                 |                        |
| —2 | $t \rightarrow p$          |                        |
| —3 | $p \rightarrow s$          |                        |
| —4 | $m \rightarrow q$          |                        |
| —5 | $q \rightarrow w \wedge t$ |                        |
|    |                            |                        |
| 6  | $t$                        |                        |
| 7  | $p$                        | $E \rightarrow 2, 6$   |
| 8  | $s$                        | $E \rightarrow 3, 7$   |
| 9  | $s \vee (w \wedge t)$      | $I \vee 8$             |
|    |                            |                        |
| 10 | $m$                        |                        |
| 11 | $q$                        | $E \rightarrow 4, 10$  |
| 12 | $w \wedge t$               | $E \rightarrow 5, 11$  |
| 13 | $s \vee (w \wedge t)$      | $I \vee 12$            |
| 14 | $s \vee (w \wedge t)$      | $E \vee 1, 6-9, 10-13$ |

31)

|    |                                   |                      |
|----|-----------------------------------|----------------------|
| —1 | $s \vee t \rightarrow r$          |                      |
| —2 | $r \vee q \rightarrow m \wedge w$ |                      |
|    |                                   |                      |
| 3  | $s$                               |                      |
| 4  | $s \vee t$                        | $I \vee 3$           |
| 5  | $r$                               | $E \rightarrow 1, 4$ |
| 6  | $r \vee q$                        | $I \vee 5$           |
| 7  | $m \wedge w$                      | $E \rightarrow 2, 6$ |
| 8  | $m$                               | $E \wedge 7$         |
| 9  | $m \vee p$                        | $I \vee 8$           |
| 10 | $s \rightarrow m \vee p$          | $I \rightarrow 3-9$  |

32)

|    |                                |                      |
|----|--------------------------------|----------------------|
| —1 | $(w \vee r) \wedge (q \vee r)$ |                      |
| —2 | $q \wedge p \rightarrow s$     |                      |
| —3 | $w \vee r \rightarrow t$       |                      |
|    |                                |                      |
| 4  | $w \vee r$                     | $E \wedge 1$         |
| 5  | $t$                            | $E \rightarrow 3, 4$ |
| 6  | $s \vee t$                     | $I \vee 5$           |

33)

|    |                                       |                         |
|----|---------------------------------------|-------------------------|
| —1 | $r \wedge t \rightarrow p \vee s$     |                         |
| —2 | $r \rightarrow t$                     |                         |
| —3 | $r \wedge (p \rightarrow q \wedge m)$ |                         |
| —4 | $s \rightarrow w \wedge n$            |                         |
|    |                                       |                         |
| 5  | $r$                                   | $E \wedge 3$            |
| 6  | $t$                                   | $E \rightarrow 2, 5$    |
| 7  | $r \wedge t$                          | $I \wedge 5, 6$         |
| 8  | $p \vee s$                            | $E \rightarrow 1, 7$    |
|    |                                       |                         |
| 9  | $p$                                   |                         |
| 10 | $p \rightarrow q \wedge m$            | $E \wedge 3$            |
| 11 | $q \wedge m$                          | $E \rightarrow 10, 9$   |
| 12 | $q$                                   | $E \wedge 11$           |
| 13 | $q \vee w$                            | $I \vee 12$             |
|    |                                       |                         |
| 14 | $s$                                   |                         |
| 15 | $w \wedge n$                          | $E \rightarrow 4, 14$   |
| 16 | $w$                                   | $E \wedge 15$           |
| 17 | $q \vee w$                            | $I \vee 16$             |
| 18 | $q \vee w$                            | $E \vee 8, 9-13, 14-17$ |

34)

|    |                                       |                      |
|----|---------------------------------------|----------------------|
| —1 | $(q \wedge w) \wedge (q \vee r)$      |                      |
| —2 | $r \wedge (q \wedge s) \rightarrow s$ |                      |
| —3 | $(q \wedge w) \vee r \rightarrow t$   |                      |
|    |                                       |                      |
| 4  | $q \wedge w$                          | $E \wedge 1$         |
| 5  | $(q \wedge w) \vee r$                 | $I \vee 4$           |
| 6  | $t$                                   | $E \rightarrow 3, 5$ |
| 7  | $s \vee t$                            | $I \vee 6$           |

35)

|    |                                            |                      |
|----|--------------------------------------------|----------------------|
| —1 | $u \vee r \rightarrow p \vee q$            |                      |
| —2 | $q \rightarrow t$                          |                      |
| —3 | $p \rightarrow m \wedge (s \rightarrow t)$ |                      |
| —4 | $s \rightarrow w \wedge o$                 |                      |
|    |                                            |                      |
| 5  | $u$                                        |                      |
| 6  | $u \vee r$                                 | $I \vee 5$           |
| 7  | $p \vee q$                                 | $E \rightarrow 1, 6$ |

|    |                              |                         |
|----|------------------------------|-------------------------|
| 8  | $p$                          |                         |
| 9  | $m \wedge (s \rightarrow t)$ | $E \rightarrow 3, 8$    |
| 10 | $m$                          | $E \wedge 9$            |
| 11 | $t \vee m$                   | $I \vee 10$             |
| 12 | $q$                          |                         |
| 13 | $t$                          | $E \rightarrow 2, 12$   |
| 14 | $t \vee m$                   | $I \vee 13$             |
| 15 | $t \vee m$                   | $E \vee 7, 8-11, 12-14$ |
| 16 | $u \rightarrow t \vee m$     | $I \rightarrow 5-15$    |

36)

|    |                                     |                                          |
|----|-------------------------------------|------------------------------------------|
| 1  | $p \vee (r \wedge m) \rightarrow s$ |                                          |
| 2  | $q \wedge s \rightarrow t$          | $\vdash p \rightarrow (q \rightarrow r)$ |
| 3  | $s \wedge t \rightarrow r$          |                                          |
| 4  | $p$                                 |                                          |
| 5  | $q$                                 |                                          |
| 6  | $p \vee (r \wedge m)$               | $I \vee 4$                               |
| 7  | $s$                                 | $E \rightarrow 1, 6$                     |
| 8  | $q \wedge s$                        | $I \wedge 5, 7$                          |
| 9  | $t$                                 | $E \rightarrow 2, 8$                     |
| 10 | $s \wedge t$                        | $I \wedge 7, 9$                          |
| 11 | $r$                                 | $E \rightarrow 3, 10$                    |
| 12 | $q \rightarrow r$                   | $I \rightarrow 5-11$                     |
| 13 | $p \rightarrow (q \rightarrow r)$   | $I \rightarrow 4-12$                     |

37)

|    |                                     |                                            |
|----|-------------------------------------|--------------------------------------------|
| 1  | $w \vee r$                          |                                            |
| 2  | $w \rightarrow q \wedge s$          | $\vdash m \vee (s \vee (r \rightarrow t))$ |
| 3  | $r \rightarrow p \wedge t$          |                                            |
| 4  | $w$                                 |                                            |
| 5  | $q \wedge s$                        | $E \rightarrow 2, 4$                       |
| 6  | $s$                                 | $E \wedge 5$                               |
| 7  | $s \vee (r \rightarrow t)$          | $I \vee 6$                                 |
| 8  | $m \vee (s \vee (r \rightarrow t))$ | $I \vee 7$                                 |
| 9  | $r$                                 |                                            |
| 10 | $r$                                 |                                            |
| 11 | $p \wedge t$                        | $E \rightarrow 3, 10$                      |
| 12 | $t$                                 | $E \wedge 11$                              |
| 13 | $r \rightarrow t$                   | $I \rightarrow 10-12$                      |
| 14 | $s \vee (r \rightarrow t)$          | $I \vee 13$                                |

|    |                                     |                       |
|----|-------------------------------------|-----------------------|
| 15 | $m \vee (s \vee (r \rightarrow t))$ | $I \vee 14$           |
| 16 | $m \vee (s \vee (r \rightarrow t))$ | $E \vee 1, 4-8, 9-15$ |

38)

|   |                    |                   |
|---|--------------------|-------------------|
| 1 | $\neg(r \wedge t)$ | $\vdash t \vee p$ |
| 2 | $r \wedge t$       | $E \neg 1$        |
| 3 | $t$                | $E \wedge 2$      |
| 4 | $t \vee p$         | $I \vee 3$        |

39)

|    |                                                       |                                            |
|----|-------------------------------------------------------|--------------------------------------------|
| 1  | $\neg(r \rightarrow (r \rightarrow p) \wedge \neg q)$ | $\vdash r \rightarrow (p \wedge q) \vee s$ |
| 2  | $r$                                                   |                                            |
| 3  | $\neg(r \rightarrow (r \rightarrow p) \wedge \neg q)$ | $E \neg 1$                                 |
| 4  | $\neg$                                                |                                            |
| 5  | $r \wedge \neg$                                       | $I \wedge 2, 4$                            |
| 6  | $\neg$                                                | $I \neg 4-5$                               |
| 7  | $\neg(r \rightarrow (r \rightarrow p) \wedge \neg q)$ | $E \rightarrow 3, 6$                       |
| 8  | $\neg$                                                | $E \wedge 7$                               |
| 9  | $\neg q$                                              | $E \wedge 7$                               |
| 10 | $p$                                                   | $E \neg 8$                                 |
| 11 | $q$                                                   | $E \neg 9$                                 |
| 12 | $p \wedge q$                                          | $I \wedge 10, 11$                          |
| 13 | $(p \wedge q) \vee s$                                 | $I \vee 12$                                |
| 14 | $r \rightarrow (p \wedge q) \vee s$                   | $I \rightarrow 2-13$                       |

40)

|   |                                |                      |
|---|--------------------------------|----------------------|
| 1 | $q \rightarrow \neg(p \vee s)$ |                      |
| 2 | $\neg(p \vee s) \rightarrow r$ | $\vdash \neg q$      |
| 3 | $\neg$                         |                      |
| 4 | $q$                            |                      |
| 5 | $\neg(p \vee s)$               | $E \rightarrow 1, 4$ |
| 6 | $r$                            | $E \rightarrow 2, 5$ |
| 7 | $r \wedge \neg$                | $I \wedge 6, 3$      |
| 8 | $\neg q$                       | $I \neg 4-7$         |

41)

|   |                                 |                                                                |
|---|---------------------------------|----------------------------------------------------------------|
| 1 | $s \wedge t \rightarrow \neg q$ | $\vdash q \rightarrow \neg(s \wedge t) \vee (r \rightarrow t)$ |
| 2 | $q$                             |                                                                |
| 3 | $s \wedge t$                    |                                                                |
| 4 | $\neg q$                        | $E \rightarrow 1, 3$                                           |
| 5 | $q \wedge \neg q$               | $I \wedge 2, 4$                                                |

- |                                                                                                                                                           |                                                                            |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------|
| $\begin{array}{l} 6 \neg(s \wedge t) \\ 7 \neg(s \wedge t) \vee (r \rightarrow t) \\ 8 q \rightarrow \neg(s \wedge t) \vee (r \rightarrow t) \end{array}$ | $\begin{array}{l} I \neg 3-5 \\ I \vee 6 \\ I \rightarrow 2-7 \end{array}$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------|

42)

- |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                          |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{l} -1 q \vee r \rightarrow \neg(p \wedge s) \\ -2 t \vee m \rightarrow k \vee m \\ -3 k \rightarrow q \\ -4 m \rightarrow r \\ -5 \neg(p \wedge s) \rightarrow \neg(t \vee m) \\ 6 t \vee m \\ 7 k \vee m \\ 8 k \\ 9 q \\ 10 q \vee r \\ 11 \neg(p \wedge s) \\ 12 \neg(t \vee m) \\ 13 (t \vee m) \wedge \neg(t \vee m) \\ 14 m \\ 15 r \\ 16 q \vee r \\ 17 \neg(p \wedge s) \\ 18 \neg(t \vee m) \\ 19 (t \vee m) \wedge \neg(t \vee m) \\ 20 (t \vee m) \wedge \neg(t \vee m) \\ 21 \neg(t \vee m) \end{array}$ | $\begin{array}{l} \vdash \neg(t \vee m) \\ E \rightarrow 2, 6 \\ E \rightarrow 3, 8 \\ I \vee 9 \\ E \rightarrow 1, 10 \\ E \rightarrow 5, 11 \\ I \wedge 6, 12 \\ E \rightarrow 4, 14 \\ I \vee 15 \\ E \rightarrow 1, 16 \\ E \rightarrow 5, 17 \\ I \wedge 6, 18 \\ E \vee 7, 8-13, 14-19 \\ I \neg 6-20 \end{array}$ |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

43)

- |                                                                                                                                                                                               |                                                                                                                                                                     |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{l} -1 s \wedge r \\ -2 r \wedge s \rightarrow \neg t \\ 3 t \\ 4 r \\ 5 s \\ 6 r \wedge s \\ 7 \neg t \\ 8 t \wedge \neg t \\ 9 \neg r \\ 10 t \rightarrow \neg r \end{array}$ | $\begin{array}{l} \vdash t \rightarrow \neg r \\ E \wedge 1 \\ I \wedge 4, 5 \\ E \rightarrow 2, 6 \\ I \wedge 3, 7 \\ I \neg 4-8 \\ I \rightarrow 3-9 \end{array}$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|

44)

- |                                                                                                                                                                                                                                         |                                                                                                                                                                               |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{l} -1 t \vee p \rightarrow q \wedge r \\ -2 q \vee w \rightarrow (t \vee p \rightarrow s) \\ 3 t \vee p \\ 4 q \wedge r \\ 5 q \\ 6 q \vee w \\ 7 t \vee p \rightarrow s \\ 8 s \\ 9 t \vee p \rightarrow s \end{array}$ | $\begin{array}{l} \vdash t \vee p \rightarrow s \\ E \rightarrow 1, 3 \\ E \wedge 4 \\ I \vee 5 \\ E \rightarrow 2, 6 \\ E \rightarrow 7, 3 \\ I \rightarrow 3-8 \end{array}$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

45)

- |                                                                                                                                                                                                                                               |                                                                                                                                                                                       |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{l} -1 s \vee p \rightarrow q \\ -2 q \rightarrow (w \rightarrow m) \\ -3 r \rightarrow s \\ -4 r \wedge \neg r \vee w \\ 5 r \\ 6 s \\ 7 s \vee p \\ 8 q \\ 9 w \rightarrow m \\ 10 \neg r \vee w \\ 11 w \\ 12 m \end{array}$ | $\begin{array}{l} \vdash m \\ E \wedge 4 \\ E \rightarrow 3, 5 \\ I \vee 6 \\ E \rightarrow 1, 7 \\ E \rightarrow 2, 8 \\ E \wedge 4 \\ E \neg 10 \\ E \rightarrow 9, 11 \end{array}$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

46)

- |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                               |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{l} -1 \neg(\neg k \vee \neg o) \rightarrow \neg s \\ -2 \neg(h \vee m) \wedge (r \rightarrow t) \\ -3 \neg r \vee \neg q \leftrightarrow o \wedge k \\ -4 (\neg p \rightarrow p) \rightarrow (k \rightarrow w) \\ -5 (\neg p \rightarrow m) \wedge (m \rightarrow n) \\ 6 \neg r \vee \neg q \\ 7 \neg p \\ 8 \neg q \\ 9 \neg p \rightarrow m \\ 10 m \\ 11 h \vee m \\ 12 \neg(h \vee m) \\ 13 (h \vee m) \wedge \neg(h \vee m) \\ 14 \neg p \\ 15 p \end{array}$ | $\begin{array}{l} \vdash \neg r \vee \neg q \rightarrow w \wedge (r \rightarrow t) \\ E \wedge 5 \\ E \rightarrow 9, 8 \\ I \vee 10 \\ E \wedge 2 \\ I \wedge 11, 12 \\ I \neg 8-13 \\ E \neg 14 \end{array}$ |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|



|    |                                                                                                                      |                        |
|----|----------------------------------------------------------------------------------------------------------------------|------------------------|
| 16 | $\neg p \rightarrow p$                                                                                               | $I \rightarrow 7-15$   |
| 17 | $k \rightarrow w$                                                                                                    | $E \rightarrow 4, 16$  |
| 18 | $[(\neg r \vee \neg q) \rightarrow (o \wedge k)] \wedge$<br>$\wedge [(o \wedge k) \rightarrow (\neg r \vee \neg q)]$ | $Df \leftrightarrow 3$ |
| 19 | $(\neg r \vee \neg q) \rightarrow (o \wedge k)$                                                                      | $E \wedge 18$          |
| 20 | $o \wedge k$                                                                                                         | $E \rightarrow 19, 6$  |
| 21 | $k$                                                                                                                  | $E \wedge 20$          |
| 22 | $w$                                                                                                                  | $E \rightarrow 17, 21$ |
| 23 | $r \rightarrow t$                                                                                                    | $E \wedge 2$           |
| 24 | $w \wedge (r \rightarrow t)$                                                                                         | $I \wedge 22, 23$      |
| 25 | $\neg r \vee \neg q \rightarrow w \wedge (r \rightarrow t)$                                                          | $I \rightarrow 6-24$   |

47)

|    |                                                             |                        |
|----|-------------------------------------------------------------|------------------------|
| —1 | $p \wedge q \rightarrow (\neg s \vee \neg r \rightarrow s)$ |                        |
| —2 | $\neg(\neg p \vee \neg q)$                                  | $\vdash \neg t \vee s$ |
| —3 | $\neg p$                                                    |                        |
| 4  | $\neg(\neg t \vee s)$                                       |                        |
| 5  | $\neg p \vee \neg q$                                        | $I \vee 3$             |
| 6  | $(\neg p \vee \neg q) \wedge \neg(\neg p \vee \neg q)$      | $I \wedge 5, 2$        |
| 7  | $\neg(\neg t \vee s)$                                       | $I \rightarrow 4-6$    |
| 8  | $\neg t \vee s$                                             | $E \neg 7$             |

## BLOQUE III

(NOTA: A continuación se presenta una posible solución a cada uno de los ejercicios propuestos, pero evidentemente caben otras múltiples soluciones alternativas.)

1)

|    |                                   |                                          |
|----|-----------------------------------|------------------------------------------|
| —1 | $p \rightarrow q$                 | $\vdash s \rightarrow (p \rightarrow r)$ |
| —2 | $q \rightarrow r$                 |                                          |
| 3  | $s$                               |                                          |
| 4  | $p \rightarrow r$                 | $Tr \rightarrow 1, 2$                    |
| 5  | $s \rightarrow (p \rightarrow r)$ | $I \rightarrow 3-4$                      |

2)

|    |                                                           |                                 |
|----|-----------------------------------------------------------|---------------------------------|
| —1 | $p \rightarrow (q \rightarrow r)$                         | $\vdash t$                      |
| —2 | $\neg(p \rightarrow r) \wedge \neg(s \rightarrow \neg q)$ |                                 |
| 3  | $\neg(p \rightarrow r)$                                   | $E \wedge 2$                    |
| 4  | $\neg(s \rightarrow \neg q)$                              | $E \wedge 2$                    |
| 5  | $p \wedge \neg r$                                         | $Def \rightarrow \rightarrow 3$ |
| 6  | $s \wedge q$                                              | $Def \rightarrow \rightarrow 4$ |

|    |                   |                      |
|----|-------------------|----------------------|
| 7  | $q$               | $E \wedge 6$         |
| 8  | $p$               | $E \wedge 5$         |
| 9  | $q \rightarrow r$ | $E \rightarrow 1, 8$ |
| 10 | $r$               | $E \rightarrow 9, 7$ |
| 11 | $\neg r$          | $E \wedge 5$         |
| 12 | $r \wedge \neg r$ | $I \wedge 10, 11$    |
| 13 | $t$               | $ECQ 12$             |

3)

|    |                                                   |                                          |
|----|---------------------------------------------------|------------------------------------------|
| —1 | $p \rightarrow [(q \rightarrow r) \rightarrow s]$ |                                          |
| —2 | $(r \wedge \neg s) \vee (s \wedge t)$             | $\vdash w \rightarrow (p \rightarrow t)$ |
| 3  | $p$                                               |                                          |
| 4  | $(q \rightarrow r) \rightarrow s$                 | $E \rightarrow 1, 3$                     |
| 5  | $r \wedge \neg s$                                 |                                          |
| 6  | $\neg s$                                          | $E \wedge 5$                             |
| 7  | $\neg(q \rightarrow r)$                           | $MT 4, 6$                                |
| 8  | $q \wedge \neg r$                                 | $Def \rightarrow \rightarrow 7$          |
| 9  | $r$                                               | $E \wedge 5$                             |
| 10 | $\neg r$                                          | $E \wedge 8$                             |
| 11 | $r \wedge \neg r$                                 | $I \wedge 9, 10$                         |
| 12 | $t$                                               | $ECQ 11$                                 |
| 13 | $s \wedge t$                                      |                                          |
| 14 | $t$                                               | $E \wedge 13$                            |
| 15 | $t$                                               | $E \vee 2, 5-12, 13-14$                  |
| 16 | $p \rightarrow t$                                 | $I \rightarrow 3-15$                     |
| 17 | $w \rightarrow (p \rightarrow t)$                 | $Cr Pr 16$                               |

4)

|    |                                          |                          |
|----|------------------------------------------|--------------------------|
| —1 | $p \vee \neg q \vee r \rightarrow s$     |                          |
| —2 | $p \vee r \rightarrow (t \rightarrow s)$ | $\vdash t \rightarrow q$ |
| —3 | $\neg s \vee q$                          |                          |
| 4  | $\neg s$                                 |                          |
| 5  | $\neg(p \vee \neg q \vee r)$             | $MT 1, 4$                |
| 6  | $\neg p \wedge \neg(\neg q \vee r)$      | $DM \vee \wedge 5$       |
| 7  | $\neg(\neg q \vee r)$                    | $E \wedge 6$             |
| 8  | $q \wedge \neg r$                        | $DM \vee \wedge 7$       |
| 9  | $q$                                      | $E \wedge 8$             |
| 10 | $q$                                      |                          |
| 11 | $q$                                      | $Id 10$                  |
| 12 | $q$                                      | $E \vee 3, 4-9, 10-11$   |
| 13 | $t \rightarrow q$                        | $Cr Pr 12$               |