

SOLUCIONES:

HOJA 2 TEMA 3: POLINOMIOS.

10.- Simplifica:

$$a) \frac{x^5 + 6x^4 + 9x^3}{x^3 + 3x^2}$$

$$d) \frac{x^4 - 2x^3 - 3x^2}{x^4 - 9x^2}$$

$$g) \frac{1}{(x-1)^2} + \frac{2}{x-1} + \frac{1}{x^2-1}$$

$$j) \frac{(x-1)^2}{2} \cdot \frac{1}{x^2-1} - \frac{3x}{(x+1)^2}$$

$$m) \left(\frac{x}{x-1} - \frac{x+1}{x} \right) \cdot \frac{x^2+x}{2}$$

$$o) \frac{x+1}{x} - \frac{x}{x+1} - \frac{2x+2}{x^2+x}$$

$$r) \left(\frac{1}{2x-1} - \frac{1}{2x} \right) \cdot \frac{(2x-1)^2}{3x}$$

$$b) \frac{x^3 - x}{x^3 + 3x^2 + 2x}$$

$$e) \frac{x^3 - 3x^2 + 3x - 1}{x^3 - 2x^2 + x}$$

$$h) \left(\frac{2x-1}{x+1} - \frac{3x}{x-1} \right) \cdot \left(\frac{x^3 - x}{-x^2 - 6x + 1} \right)$$

$$k) \frac{2x-1}{x} + \frac{x+1}{x+2} - \frac{3x^2+2}{x^2+2x}$$

$$n) \frac{2x+3}{x+3} - \frac{3x^2+3}{x^2+3x} + \frac{x+1}{x}$$

$$p) \left(\frac{1}{x+1} : \frac{1}{x} \right) + \frac{2x+1}{(x+1)^2}$$

$$s) \frac{2x}{x+2} + \frac{3x+1}{x} - \frac{5x^2+7x}{x^2+2x}$$

$$c) \frac{x^3 - x^2 - 2x}{x^3 - 3x^2 + 2x}$$

$$f) \left(\frac{3}{x} - \frac{2x}{x+1} \right) \cdot \frac{x^2+x}{x-1}$$

$$i) \frac{2x}{x-2} + \frac{3x-1}{x+2} - \frac{1}{x^2-4}$$

$$l) \frac{x+4}{x-3} - \frac{2x^2+4x}{x^2-9} + \frac{x}{x+3}$$

$$\tilde{n}) \frac{1}{x+1} : \frac{1}{2x-1} - \frac{1}{x^2-1}$$

$$q) \frac{2x+1}{x-1} + \frac{3x}{x+1} - \frac{5x^2}{x^2-1}$$

Solución:

$$a) \frac{x^5 + 6x^4 + 9x^3}{x^3 + 3x^2} = \frac{x^3(x^2 + 6x + 9)}{x^2(x+3)} = \frac{x^3(x+3)^2}{x^2(x+3)} = x(x+3) = x^2 + 3x$$

$$b) \frac{x^3 - x}{x^3 + 3x^2 + 2x} = \frac{x(x^2 - 1)}{x(x^2 + 3x + 2)} = \frac{x(x-1)(x+1)}{x(x+1)(x+2)} = \frac{x-1}{x+2}$$

$$c) \frac{x^3 - x^2 - 2x}{x^3 - 3x^2 + 2x} = \frac{x(x^2 - x - 2)}{x(x^2 - 3x + 2)} = \frac{x(x-3)(x+1)}{x(x-2)(x-1)} = \frac{x+1}{x-1}$$

$$d) \frac{x^4 - 2x^3 - 3x^2}{x^4 - 9x^2} = \frac{x^2(x^2 - 2x - 3)}{x^2(x^2 - 9)} = \frac{x^2(x-3)(x+1)}{x^2(x-3)(x+3)} = \frac{x+1}{x+3}$$

$$e) \frac{x^3 - 3x^2 + 3x - 1}{x^3 - 2x^2 + x} = \frac{(x-1)^3}{x(x-1)} = \frac{x-1}{x}$$

$$f) \left(\frac{3}{x} \cdot \frac{2x}{x+1} \right) \cdot \frac{x^2 + x}{x-1} = \frac{3(x+1) - 2x^2}{x(x+1)} \cdot \frac{x^2 + x}{x-1} = \frac{3x+3-2x^2}{x(x+1)} \cdot \frac{x(x+1)}{x-1} = \frac{-2x^2 + 3x + 3}{x-1}$$

$$g) \frac{1}{(x-1)^2} + \frac{2}{x-1} + \frac{1}{x^2-1} = \frac{1}{(x-1)^2} + \frac{2}{(x-1)} + \frac{1}{(x-1)(x+1)} = \frac{x+1+2(x^2-1)+(x-1)}{(x-1)^2(x+1)} = \frac{x+1+2x^2-2+x-1}{(x-1)^2(x+1)} = \frac{2x^2+2x-2}{(x-1)^2(x+1)}$$

$$h) \left(\frac{2x-1}{x+1} - \frac{3x}{x-1} \right) \cdot \left(\frac{x^3 - x}{-x^2 - 6x + 1} \right) = \frac{(2x-1)(x-1) - 3x(x+1)}{(x+1)(x-1)} \cdot \frac{x^3 - x}{-x^2 - 6x + 1} =$$

$$= \frac{2x^2 - 2x - x + 1 - 3x^2 - 3x}{(x+1)(x-1)} \cdot \frac{x(x-1)(x+1)}{-x^2 - 6x + 1} = \frac{-x^2 - 6x + 1}{(x+1)(x-1)} \cdot \frac{x(x-1)(x+1)}{-x^2 - 6x + 1} = x$$

$$i) \frac{2x}{x-2} + \frac{3x-1}{x+2} - \frac{1}{x^2-4} = \frac{2x(x+2)}{x^2-4} - \frac{(3x-1)(x-2)}{x^2-4} - \frac{1}{x^2-4} = \frac{2x^2+4x-3x^2+6x+x-2-1}{x^2-4} = \frac{-x^2+11x-3}{x^2-4}$$

$$j) \frac{(x-1)^2}{2} \cdot \frac{1}{x^2-1} - \frac{3x}{(x+1)^2} = \frac{(x-1)^2}{2(x-1)(x+1)} - \frac{3x}{(x+1)^2} = \frac{x-1}{2(x+1)} - \frac{3x}{(x+1)^2} = \frac{x^2-1-6x}{2(x+1)^2} = \frac{x^2-6x-1}{2(x+1)^2}$$

$$k) \frac{2x-1}{x} + \frac{x+1}{x+2} - \frac{3x^2+2}{x^2+2x} = \frac{(2x-1)(x+2)}{x^2+2x} + \frac{x(x+1)}{x^2+2x} - \frac{3x^2+2}{x^2+2x} = \frac{2x^2+4x-x-2+x^2+x-3x^2-2}{x^2+2x} = \frac{4x-4}{x^2+2x}$$

$$l) \frac{x+4}{x-3} - \frac{2x^2+4x}{x^2-9} + \frac{x}{x+3} = \frac{(x+4)(x+3)}{x^2-9} - \frac{2x^2+4x}{x^2-9} + \frac{x(x-3)}{x^2-9} = \frac{x^2+3x+4x+12-2x^2-4x+x^2-3x}{x^2-9} = \frac{12}{x^2-9}$$

$$m) \left(\frac{x}{x-1} - \frac{x+1}{x} \right) \cdot \frac{x^2+x}{2} = \frac{x^2-(x+1)(x-1)}{x(x-1)} \cdot \frac{x^2+x}{2} = \frac{x^2-(x^2-1)}{x(x-1)} \cdot \frac{x(x+1)}{2} = \frac{x^2-x^2+1}{x(x-1)} \cdot \frac{x(x+1)}{2} =$$

$$= \frac{1}{x(x-1)} \cdot \frac{x(x+1)}{2} = \frac{x(x+1)}{2x(x-1)} = \frac{x+1}{2(x-1)} = \frac{x+1}{2x-2}$$

$$n) \frac{2x+3}{x+3} - \frac{3x^2+3}{x^2+3x} + \frac{x+1}{x} = \frac{2x^2+3x}{x^2+3x} - \frac{3x^2+3}{x^2+3x} + \frac{(x+1)(x+3)}{x^2+3x} = \frac{2x^2+3x-3x^2-3+x^2+3x+x+3}{x^2+3x} = \frac{7x}{x^2+3x} = \frac{7x}{x(x+3)} = \frac{7}{x+3}$$

$$\tilde{n}) \frac{1}{x+1} : \frac{1}{2x-1} - \frac{1}{x^2-1} = \frac{2x-1}{x+1} \cdot \frac{1}{x^2-1} = \frac{(2x-1)(x-1)}{x^2-1} \cdot \frac{1}{x^2-1} = \frac{2x^2-2x-x+1}{x^2-1} \cdot \frac{1}{x^2-1} = \frac{2x^2-3x+1-1}{x^2-1} = \frac{2x^2-3x}{x^2-1}$$

$$o) \frac{x+1}{x} - \frac{x}{x+1} - \frac{2x+2}{x^2+x} = \frac{(x+1)^2}{x^2+x} - \frac{x^2}{x^2+x} - \frac{2x+2}{x^2+x} = \frac{x^2+2x+1-x^2-2x-2}{x^2+x} = \frac{-1}{x^2+x}$$

$$p) \frac{1}{x+1} : \frac{1}{x} + \frac{2x+1}{(x+1)^2} = \frac{x}{x+1} + \frac{2x+1}{(x+1)^2} = \frac{x(x+1)}{(x+1)^2} + \frac{2x+1}{(x+1)^2} = \frac{x^2+x+2x+1}{(x+1)^2} = \frac{x^2+3x+1}{(x+1)^2}$$

$$q) \frac{2x+1}{x-1} + \frac{3x}{x+1} - \frac{5x^2}{x^2-1} = \frac{(2x+1)(x+1)}{x^2-1} + \frac{3x(x-1)}{x^2-1} - \frac{5x^2}{x^2-1} = \frac{2x^2+2x+x+1+3x^2-3x-5x^2}{x^2-1} = \frac{1}{x^2-1}$$

$$r) \left(\frac{1}{2x-1} - \frac{1}{2x} \right) \cdot \frac{(2x-1)^2}{3x} = \frac{2x-(2x-1)}{(2x-1) \cdot 2x} \cdot \frac{(2x-1)^2}{3x} = \frac{2x-2x+1}{2x(2x-1)} \cdot \frac{(2x-1)^2}{3x} = \frac{1}{2x(2x-1)} \cdot \frac{(2x-1)^2}{3x} = \frac{2x-1}{6x^2}$$

$$s) \frac{2x}{x+2} + \frac{3x+1}{x} - \frac{5x^2+7x}{x^2+2x} = \frac{2x^2}{x^2+2x} + \frac{(3x+1)(x+2)}{x^2+2x} - \frac{5x^2+7x}{x^2+2x} = \frac{2x^2+3x^2+6x+x+2-5x^2-7x}{x^2+2x} = \frac{2}{x^2+2x}$$

12.- Dados los polinomios:

$$P(x) = 3x^4 + 8x^3 - 15x^2 - 32x + 12$$

$$Q(x) = 2x^4 + x^3 - 16x^2 + 3x + 18$$

determina los polinomios $A(x)$ y $B(x)$ de menor grado que cumplan que:

$$P(x) \cdot A(x) + Q(x) \cdot B(x) = 0$$

$$P(x) \cdot A(x) + Q(x) \cdot B(x) = 0 \rightarrow P(x) \cdot A(x) = -Q(x) \cdot B(x) \rightarrow \frac{A(x)}{B(x)} = -\frac{Q(x)}{P(x)}$$

$$P(x) = 3x^4 + 8x^3 - 15x^2 - 32x + 12 = (x - 2)(x + 2)(x + 3)(3x - 1)$$

$$Q(x) = 2x^4 + x^3 - 16x^2 + 3x + 18 = (x - 2)(x + 1)(x + 3)(2x - 3)$$

$$\frac{A(x)}{B(x)} = -\frac{Q(x)}{P(x)} = -\frac{(x - 2)(x + 1)(x + 3)(2x - 3)}{(x - 2)(x + 2)(x + 3)(3x - 1)} = -\frac{(x + 1)(2x - 3)}{(x + 2)(3x - 1)}$$

$$\text{Así, } A(x) = -2x^2 + x + 3 \text{ y } B(x) = 3x^2 + 5x - 2.$$