



**BOLETÍN: DERIVADAS NO TRIGONOMÉTRICAS** *Solucionario*

Calcula la derivada de las siguientes funciones **simplificando** lo máximo posible:

$f(x) = 1 - x + x^3 - 2$ <span style="float: right;">1</span>	$y = (4x^3 - 2x^{-2} + 5x - 3)^5$ <span style="float: right;">2</span>	$f(x) = x^2 + \frac{2}{x} - 3\sqrt{x}$ <span style="float: right;">3</span>
$f(x) = \frac{2-x}{(1-x)^2}$ <span style="float: right;">4</span>	$f(x) = \frac{2-x}{x^2}$ <span style="float: right;">5</span>	$f(x) = \sqrt{x^2(1-x)}$ <span style="float: right;">6</span>
$f(x) = \frac{x+1}{x^2+3x+2}$ <span style="float: right;">7</span>	$f(x) = \frac{3x^3-2}{x^2}$ <span style="float: right;">8</span>	$f(x) = \frac{2x-x^2}{3x^3+1}$ <span style="float: right;">9</span>
$f(x) = \frac{e^x + e^{-x}}{2}$ <span style="float: right;">10</span>	$y = \frac{x^3}{\text{Ln}x}$ <span style="float: right;">11</span>	$f(x) = 10^{x^2}$ <span style="float: right;">12</span>
$f(x) = 2^{\frac{x-1}{x}}$ <span style="float: right;">13</span>	$f(x) = \frac{3e^x}{\sqrt{x}}$ <span style="float: right;">14</span>	$f(x) = \frac{1}{e^{2x}}$ <span style="float: right;">15</span>
$f(x) = \text{Log}(2x-1)$ <span style="float: right;">16</span>	$f(x) = \text{Log}(\sqrt{x})$ <span style="float: right;">17</span>	$f(x) = \text{Log}(x \cdot e^{2x})$ <span style="float: right;">18</span>
$f(x) = e^{\text{Log}(x^2)}$ <span style="float: right;">19</span>	$y = 5^x + x^5$ <span style="float: right;">20</span>	$f(x) = 2^x \cdot x^2$ <span style="float: right;">21</span>
$f(x) = e^{\text{Ln}\left(\frac{1}{x}\right)}$ <span style="float: right;">22</span>	$f(x) = \frac{e^{x^2} - e^{3x}}{3}$ <span style="float: right;">23</span>	$f(x) = \frac{e^x}{x^4}$ <span style="float: right;">24</span>
$f(x) = \sqrt[3]{x} \cdot (x^2+1)$ <span style="float: right;">25</span>	$f(x) = \frac{5^x \cdot x^5}{\sqrt{5x}}$ <span style="float: right;">26</span>	$f(x) = (2x+5)^x$ ● <span style="float: right;">27</span>
$f(x) = \text{Log}\left(\frac{2x-1}{3x}\right)$ <span style="float: right;">28</span>	$f(x) = \frac{(3-x)^2}{27-9x}$ <span style="float: right;">29</span>	$f(x) = 1 - \frac{e^x - 2}{x}$ <span style="float: right;">30</span>
$f(x) = e^x \cdot e^{1-x}$ <span style="float: right;">31</span>	$f(x) = \text{Log}(\sqrt{x^5})$ <span style="float: right;">32</span>	$f(x) = (x^2)^{\text{Ln}x}$ ● <span style="float: right;">33</span>
$y = \sqrt{e^x + x^3}$ <span style="float: right;">34</span>	$f(x) = \left(\frac{2}{x-1} - 1\right)^2$ <span style="float: right;">35</span>	$f(x) = \sqrt{\frac{1+x}{1-x}}$ <span style="float: right;">36</span>
$f(x) = \text{Log}(x^2 - 3)^3$ <span style="float: right;">37</span>	$f(x) = 4^{x^2-x+3}$ <span style="float: right;">38</span>	$f(x) = \left(\frac{1}{5}\right)^{x^2-2x}$ <span style="float: right;">39</span>
$f(x) = \sqrt[4]{x^{-1}} + \frac{2}{x} - 1$ <span style="float: right;">40</span>	$f(x) = -1 + \sqrt{2-x}$ <span style="float: right;">41</span>	$f(x) = \sqrt{\frac{1+e^x}{2}}$ <span style="float: right;">42</span>
$f(x) = 2^{x^2} \cdot 2^{\sqrt{x}}$ <span style="float: right;">43</span>	$f(x) = 7^{\text{Log}(x+2)}$ <span style="float: right;">44</span>	$f(x) = \sqrt[4]{4x^3 - 2x}$ <span style="float: right;">45</span>
$f(x) = x^2 \cdot e^x \cdot x^{1/2}$ <span style="float: right;">46</span>	$\frac{x^2}{5} + \frac{y^2}{7} = 1$ ● <span style="float: right;">47</span>	$y^7 - y^6 - 2x + 1 = 0$ ● <span style="float: right;">48</span>
$x^2 + y^2 - 5y - 1 = 0$ ● <span style="float: right;">49</span>	$y^2 - y \cdot x + 2x^2 - x + 2 = 0$ ● <span style="float: right;">50</span>	$x^3 - y \cdot x^2 - \sqrt[3]{x^2} = 1$ ● <span style="float: right;">51</span>

● Se resuelven con derivación logarítmica.

● Se resuelven con derivación implícita.

$$\boxed{1} \quad y = 1 - x + x^3 - 2, \quad y' = -1 + 3x^2$$

$$y' = n u^{n-1} \cdot u'$$

$$\boxed{2} \quad y = (4x^3 - 2x^{-2} + 5x - 3)^5$$

$$y' = 5 \cdot (4x^3 - 2x^{-2} + 5x - 3)^4 \cdot (12x^2 + 4x^{-3} + 5) = (4x^3 - \frac{2}{x^2} + 5x - 3)^4 (60x^2 + \frac{20}{x^3} + 25)$$

$$\boxed{3} \quad y = x^2 + \frac{2}{x} - 3\sqrt{x} = x^2 + 2 \cdot x^{-1} - 3 \cdot x^{1/2}$$

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$y' = 2x - 2 \cdot x^{-2} - 3 \cdot \frac{1}{2\sqrt{x}} = 2x - \frac{2}{x^2} - \frac{3}{2\sqrt{x}}$$

$$\frac{3}{2} x^{-1/2} = \frac{3}{2} \cdot \frac{1}{x^{1/2}} = \frac{3}{2} \cdot \frac{1}{\sqrt{x}} \uparrow$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - u \cdot v'}{v^2}$$

$$\boxed{4} \quad y = \frac{2-x}{(1-x)^2} \left(\frac{u}{v}\right) \quad y' = \frac{(-1) \cdot (1-x)^{-2} - (2-x) \cdot 2(1-x) \cdot (-1)}{(1-x)^4 \cdot 3} = \frac{-(1-x) + 2(2-x)}{(1-x)^3} = \frac{3-x}{(1-x)^3}$$

$$\boxed{5} \quad y = \frac{2-x}{x^2} \left(\frac{u}{v}\right), \quad y' = \frac{(-1) \cdot x^2 - (2-x) \cdot 2x}{(x^2)^2} = \frac{-x^2 - 4x + 2x^2}{x^4} = \frac{x^2 - 4x}{x^4} = \frac{x-4}{x^3}$$

$$\boxed{6} \quad y = \sqrt{x^2(1-x)}, \quad y' = \frac{2x(1-x) - x^2}{2\sqrt{x^2(1-x)}} = \frac{2x - 3x^2}{2\sqrt{x^2(1-x)}}$$

$$\boxed{7} \quad y = \frac{x+1}{x^2+3x+2}, \quad y' = \frac{1 \cdot (x^2+3x+2) - (x+1) \cdot (2x+3)}{(x^2+3x+2)^2} = \frac{x^2+3x+2-2x^2-3x-3}{(x^2+3x+2)^2} = \frac{-x^2-2x-1}{(x^2+3x+2)^2}$$

(simplificar)

Método II: Simplificamos primero. Factorizamos el denominador.

$$x^2+3x+2=0 \Rightarrow x = \frac{-3 \pm \sqrt{9-4 \cdot 1 \cdot 2}}{2} < -1 \Rightarrow y = \frac{(x+1)}{(x+1)(x+2)} = \frac{1}{x+2} = (x+2)^{-1}, \quad y' = -\frac{1}{(x+2)^2}$$

$$\boxed{8} \quad y = \frac{3x^3-2}{x^2}, \quad y' = \frac{9x^2 \cdot x^2 - (3x^3-2) \cdot 2x}{x^4} = \frac{9x^4 - 6x^4 + 4x}{x^4} = 3 + \frac{4}{x^3}$$

$$\boxed{9} \quad y = \frac{2x-x^2}{3x^3+1}, \quad y' = \frac{(2-2x) \cdot (3x^3+1) - (2x-x^2) \cdot 9x^2}{(3x^3+1)^2} = \frac{3x^4 - 12x^3 - 2x + 2}{(3x^3+1)^2}$$

(simplificar)

$$\boxed{10} \quad y = \frac{e^x + e^{-x}}{2} \Rightarrow y' = \frac{1}{2} \cdot (e^x - e^{-x})$$

$$y' = e^u \cdot u'$$

$$(e^x)' = e^x \cdot 1 \quad (e^{-x})' = e^{-x} \cdot (-1)$$

$$\boxed{11} \quad y = \frac{x^3}{\ln x}, \quad y' = \frac{3x^2 \cdot \ln x - x^3 \cdot \frac{1}{x}}{(\ln x)^2} = \frac{3x^2 \cdot \ln x - x^2}{(\ln x)^2}$$

$$(\ln u)' = \frac{u'}{u}$$

$$\boxed{12} \quad y = 10^{x^2} \Rightarrow y' = 10^{x^2} \cdot \ln 10 \cdot 2x \quad y' = a^u \cdot \ln a \cdot u'$$

$$\boxed{13} \quad y = 2^{\frac{x-1}{x}} \Rightarrow y' = 2^{\frac{x-1}{x}} \cdot \ln 2 \cdot \frac{1 \cdot x - (x-1) \cdot 1}{x^2} = 2^{\frac{x-1}{x}} \cdot \frac{\ln 2}{x^2}$$

$$\boxed{14} \quad y = \frac{3e^x}{\sqrt{x}}, \quad y' = \frac{3e^x \sqrt{x} - 3e^x \frac{1}{2\sqrt{x}}}{x} = \frac{3e^x \cdot (2x-1)}{2x\sqrt{x}}$$

$$\boxed{15} \quad y = \frac{1}{e^{2x}} = e^{-2x} \Rightarrow y' = -2 \cdot e^{-2x} = -\frac{2}{e^{2x}}$$

$$\boxed{16} \quad y = \log(2x-1) \quad y' = \frac{2}{2x-1} \cdot \log e = \frac{2}{(2x-1)\ln 10}$$

$$\boxed{17} \quad y = \log \sqrt{x} \Rightarrow y' = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}} \cdot \log_{10} e = \frac{1}{2x} \cdot \log_{10} e = \frac{1}{2x \ln 10}$$

$$(\log_a u)' = \frac{u'}{u} \cdot \log_a e$$

Método II:  $y = \log x^{1/2} = \frac{1}{2} \cdot \log x \Rightarrow y' = \frac{1}{2} \cdot \frac{1}{x} \cdot \log_{10} e = \frac{1}{2x \ln 10}$

$$(\log_a u)' = \frac{u'}{u \cdot \ln a}$$

$$\boxed{18} \quad y = \log(x \cdot e^{2x}) \Rightarrow y' = \frac{1 \cdot e^{2x} + x \cdot 2 \cdot e^{2x}}{x \cdot e^{2x}} \cdot \log_{10} e = \frac{1+2x}{x} \cdot \log_{10} e = \frac{1+2x}{x \cdot \ln 10}$$

$$\boxed{19} \quad y = e^{\log(x^2)} \Rightarrow y' = e^{\log(x^2)} \cdot \frac{2x}{x^2} \cdot \log e = e^{\log(x^2)} \cdot \frac{2}{x} \cdot \log e = e^{\log(x^2)} \cdot \frac{2}{x \ln 10}$$

$$\boxed{20} \quad y = 5^x + x^5 \Rightarrow y' = 5^x \cdot \ln 5 \cdot 1 + 5x^4$$

$$\boxed{21} \quad y = 2^x \cdot x^2 \Rightarrow y' = 2^x \cdot \ln 2 \cdot 1 \cdot x^2 + 2^x \cdot 2x = 2^x (x^2 \ln 2 + 2x)$$

$$y' = x \cdot 2^x (x \ln 2 + 2)$$

$$\boxed{22} \quad y = e^{\ln(\frac{1}{x})}, \quad y' = e^{\ln(\frac{1}{x})} \cdot \left(\ln \frac{1}{x}\right)' = e^{\ln(\frac{1}{x})} \cdot \frac{-\frac{1}{x^2}}{\frac{1}{x}} = e^{\ln(\frac{1}{x})} \cdot \left(-\frac{1}{x}\right) = -\frac{1}{x^2}$$

Método rápido:  $y = e^{\ln(\frac{1}{x})} = \frac{1}{x} \Rightarrow y' = -\frac{1}{x^2}$

$$\boxed{23} \quad y = \frac{e^{x^2} - e^{3x}}{3} \Rightarrow y' = \frac{2x e^{x^2} - 3e^{3x}}{3}$$

$$\boxed{24} \quad f(x) = \frac{e^x}{x^4}, \quad f'(x) = \frac{e^x \cdot x^4 - e^x \cdot 4x^3}{(x^4)^2} = \frac{e^x (x^4 - 4x^3)}{x^8} = \frac{e^x (x-4)}{x^5}$$

$$\boxed{25} \quad f(x) = \sqrt[3]{x} \cdot (x^2+1) = x^{1/3} \cdot (x^2+1)$$

$$f'(x) = \frac{1}{3} \cdot x^{\frac{1}{3}-1} \cdot (x^2+1) + x^{1/3} \cdot 2x = \frac{1}{3} x^{-2/3} (x^2+1) + 2x \sqrt[3]{x} = \frac{x^2+1}{2\sqrt[3]{x^2}} + 2x \sqrt[3]{x}$$

$$\boxed{26} \quad f(x) = \frac{5^x \cdot x^5}{\sqrt{5x}},$$

$$f'(x) = \frac{(5^x \cdot \ln 5 \cdot x^5 + 5^x \cdot 5x^4) \cdot \sqrt{5x} - 5^x \cdot x^5 \cdot \frac{5}{2\sqrt{5x}}}{\sqrt{5x}^2} = \frac{5^x \cdot (\ln 5 \cdot x^5 \sqrt{5x} + 5x^4 \sqrt{5x} - x^5 \cdot \frac{5}{2\sqrt{5x}})}{5x}$$

 Derivación logarítmica y derivación implícita

27  $y = (2x+5)^x$  Tomamos logaritmos neperianos en ambos miembros

Llamo  $y = f(x)$  por comodidad.

$\ln y = \ln (2x+5)^x$  bajamos el exponente (propiedad de los logaritmos)

$\ln y = x \cdot \ln (2x+5)$  derivamos implícitamente; recordamos que  $(\ln u)' = \frac{u'}{u}$

$\frac{y'}{y} = 1 \cdot \ln(2x+5) + x \cdot \frac{2}{2x+5}$  despejamos  $y'$

$y' = y \cdot \left[ \ln(2x+5) + \frac{2x}{2x+5} \right]$  sustituimos  $y$  por el enunciado:  $y = (2x+5)^x$

$y' = (2x+5)^x \cdot \left[ \ln(2x+5) + \frac{2x}{2x+5} \right]$

28  $y = \log \left( \frac{2x-1}{3x} \right)$   $y' = \frac{\frac{1}{3x^2}}{\frac{2x-1}{3x}} \cdot \log e = \frac{\log e}{x(2x-1)} = \frac{1}{x(2x-1) \ln 10}$

$\left( \frac{2x-1}{3x} \right)' = \frac{2 \cdot 3x - (2x-1) \cdot 3}{(3x)^2} = \frac{6x - 6x + 3}{9x^2} = \frac{1}{3x^2}$

29  $g(x) = \frac{(3-x)^2}{27-9x} = \frac{(3-x)^2}{9(3-x)} = \frac{1}{9} \cdot (3-x)$ , Método II:  $y' = \frac{1}{9} \cdot (-1) = -\frac{1}{9}$

Método directo:  $g'(x) = \frac{-2 \cdot (3-x) \cdot (27-9x) - (3-x)^2 \cdot (-9)}{(27-9x)^2} = \frac{-2 \cdot (3-x) \cdot (27-9x) + 9(3-x)^2}{(27-9x)^2} =$

$9(3-x)^2 = (9-3x)^2$ ;  $(27-9x)^2 = 3^2(9-3x)^2 = 3^2 \cdot 3^2 \cdot (3-x)^2$

$= \frac{-2 \cdot (3-x) \cdot 3^2(3-x) + 3^2(3-x)^2}{3^2 \cdot 3^2 \cdot (3-x)^2} = \frac{-2+1}{3^2} = -\frac{1}{9}$

30  $g(x) = 1 - \frac{e^x - 2}{x}$ ,  $g'(x) = 0 - \frac{e^x \cdot x - (e^x - 2) \cdot 1}{x^2} = \frac{e^x(x-1) + 2}{x^2}$

31  $f(x) = e^x \cdot e^{1-x}$ ,  $f'(x) = e^x \cdot e^{1-x} + e^x \cdot e^{1-x} (-1) = 0$

32  $f(x) = \log \sqrt{x^5} = \log x^{5/2} = \frac{5}{2} \cdot \log x$ ,  $f'(x) = \frac{5}{2} \cdot \frac{1}{x \ln 10}$

33  $y = (x^2)^{\ln x}$  Tomamos logaritmos neperianos en ambos miembros  $y = f(x)$

$\ln y = \ln (x^2)^{\ln x}$  bajamos el exponente (propiedad de los logaritmos)

$\ln y = \ln x \cdot \ln (x^2)$  derivamos implícitamente; recordamos que  $(\ln u)' = \frac{u'}{u}$

$\frac{y'}{y} = \frac{1}{x} \cdot \ln(x^2) + \ln x \cdot \frac{2x}{x^2} = \frac{2}{x} \ln x + \frac{2}{x} \ln x = \frac{4}{x} \ln x$  despejamos  $y'$

$y' = y \cdot \frac{4}{x} \ln x$  sustituimos  $y$  por el enunciado:  $y = (x^2)^{\ln x}$

$y' = (x^2)^{\ln x} \cdot \frac{4}{x} \ln x$

$$34 \quad y = \sqrt{e^x + x^3}, \quad y' = \frac{e^x + 3x^2}{2 \cdot \sqrt{e^x + x^3}}$$

$$35 \quad f(x) = \left(\frac{2}{x-1} - 1\right)^2 \Rightarrow f'(x) = 2 \cdot \left(\frac{2}{x-1} - 1\right) \cdot \frac{-2 \cdot 1}{(x-1)^2} = \frac{2 - (x-1)}{x-1} \cdot \frac{-4}{(x-1)^2} = \frac{4x-12}{(x-1)^3}$$

$$36 \quad y = \sqrt{\frac{1+x}{1-x}}, \quad y' = \frac{1}{2 \cdot \sqrt{\frac{1+x}{1-x}}} \cdot \frac{1 \cdot (1-x) - (1+x)(-1)}{(1-x)^2} = \frac{1}{2 \cdot \sqrt{\frac{1+x}{1-x}}} \cdot \frac{2}{(1-x)^2} = \sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{(1-x)^2}$$

$$37 \quad y = \log_8 (x^2 - 3)^3, \quad y' = \frac{3 \cdot (x^2 - 3)^2}{(x^2 - 3)^3} \cdot 2x \cdot \log_8 e = \frac{6x \log_8 e}{x^2 - 3} = \frac{6x}{(x^2 - 3) \cdot \ln 10}$$

$$38 \quad y = 4^{x^2 - x + 3}, \quad y' = 4^{x^2 - x + 3} \cdot \ln 4 \cdot (2x - 1) = (2x - 1) \cdot \ln 4 \cdot 4^{x^2 - x + 3}$$

$$39 \quad y = \left(\frac{1}{5}\right)^{x^2 - 2x}, \quad y' = \left(\frac{1}{5}\right)^{x^2 - 2x} \cdot \ln\left(\frac{1}{5}\right) \cdot (2x - 2)$$

$$40 \quad y = \sqrt[4]{x^{-1}} + \frac{2}{x} - 1 = x^{-1/4} + 2x^{-1} - 1$$

$$y' = -\frac{1}{4} x^{-5/4} + 2 \cdot (-1) \cdot x^{-2} + 0 = -\frac{1}{4} x^{-5/4} - 2x^{-2} = -\frac{1}{4 \sqrt[4]{x^5}} - \frac{2}{x^2} = -\frac{1}{4x \sqrt[4]{x}} - \frac{2}{x^2}$$

$$41 \quad y = -1 + \sqrt{2-x}, \quad y' = 0 + \frac{-1}{2 \cdot \sqrt{2-x}}$$

$$42 \quad y = \sqrt{\frac{1+e^x}{2}}, \quad y' = \frac{\frac{e^x}{2}}{2 \cdot \sqrt{\frac{1+e^x}{2}}} = \frac{e^x}{4 \cdot \sqrt{\frac{1+e^x}{2}}} = \frac{e^x}{\sqrt{1+e^x}}$$

$$43 \quad y = 2^{x^2} \cdot 2^{\sqrt{x}} = 2^{x^2 + \sqrt{x}}, \quad y' = 2^{x^2 + \sqrt{x}} \cdot \ln 2 \cdot \left[2x + \frac{1}{2\sqrt{x}}\right]$$

$$44 \quad f(x) = 7^{\log(x+2)}, \quad f'(x) = 7^{\log(x+2)} \cdot \ln 7 \cdot \frac{1}{(x+2) \cdot \ln 10}$$

$$45 \quad f(x) = \sqrt[4]{4x^3 - 2x} = (4x^3 - 2x)^{1/4}$$

$$f'(x) = \frac{1}{4} \cdot (4x^3 - 2x)^{1/4 - 1} \cdot (12x^2 - 2) = \frac{1}{4} \cdot (4x^3 - 2x)^{-3/4} \cdot (12x^2 - 2) = \frac{6x^2 - 1}{2 \cdot \sqrt[4]{(4x^3 - 2x)^3}}$$

$$46 \quad f(x) = x^2 \cdot e^x \cdot x^{1/2} \Rightarrow f(x) = e^x \cdot x^{2+1/2} = e^x \cdot x^{5/2}$$

$$f'(x) = e^x \cdot x^{5/2} + e^x \cdot \frac{5}{2} x^{5/2-1} = e^x \cdot x^{5/2} + e^x \cdot \frac{5}{2} x^{3/2} = e^x \cdot \left(\sqrt{x^5} + \frac{5}{2} \cdot \sqrt{x^3}\right)$$

 Derivación logarítmica y derivación implícita

47 ●  $\frac{x^2}{5} + \frac{y^2}{7} = 1$  derivamos implícitamente

$\frac{2x}{5} + \frac{2yy'}{7} = 0$  ; despejamos  $y'$  :

$\frac{2yy'}{7} = -\frac{2x}{5} \Rightarrow y' = -\frac{7x}{5y}$

Explícitamente es más complejo y habría que tener en cuenta el signo de las raíces:

$y = \left[ \left(1 - \frac{x^2}{5}\right) \cdot 7 \right]^{1/2} \Rightarrow y' = \frac{1}{2} \left(7 - \frac{7}{5}x^2\right)^{\frac{1}{2}-1} \cdot \left(-\frac{7}{5}\right) \cdot 2x = \frac{-\frac{7}{5}x}{\sqrt{7 - \frac{7}{5}x^2}}$

48 ●  $y^7 - y^6 - 2x + 1 = 0$  derivamos implícitamente

$7y^6y' - 6y^5y' - 2 + 0 = 0$  ; despejamos  $y'$  :

$y' \cdot (7y^6 - 6y^5) = 2$

$y' = \frac{2}{7y^6 - 6y^5} = \frac{2}{y^5(7y - 6)}$

Si consideramos las coordenadas de un punto  $(x, y)$ , podemos calcular su derivada  $y'$

Ej:  $P(0, 1) \rightarrow y' = \frac{2}{1(7-6)} = 2$

49 ●  $x^2 + y^2 - 5y - 1 = 0$  derivamos implícitamente

$2x + 2yy' - 5y' - 0 = 0$

$y'(2y - 5) = -2x$

$y' = \frac{-2x}{(2y - 5)}$

Ej:  $P(2, 1) \rightarrow y' = \frac{-2 \cdot 2}{2 \cdot 1 - 5} = \frac{-4}{-3} = \frac{4}{3}$

50 ●  $y^2 - yx + 2x^2 - x + 2 = 0$  derivamos implícitamente

$2yy' - y'x - y \cdot 1 + 4x - 1 + 0 = 0$

$(2y - x)y' = 1 - 4x + y$

$y' = \frac{1 - 4x + y}{2y - x}$

Ej:  $P(2, 1) \rightarrow y' = \frac{1 - 4 \cdot 2 + 1}{2 \cdot 1 - 2} = \frac{-6}{0} = -\infty \neq$

$P(0, 1) \rightarrow y' = \frac{1 - 4 \cdot 0 + 1}{2 \cdot 1 - 0} = \frac{2}{2} = 1$

51 ●  $x^3 - yx^2 - \sqrt[3]{x^2} = 1$

$3x^2 - y'x^2 - y \cdot 2x - \frac{2}{3\sqrt[3]{x}} = 0$

$\sqrt[3]{x^2} = x^{2/3}$

$(x^{2/3})' = \frac{2}{3} \cdot x^{2/3-1} \cdot 1 = \frac{2}{3} \cdot x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

$y'x^2 = 3x^2 - y \cdot 2x - \frac{2}{3\sqrt[3]{x}}$

$y' = \frac{3x^2}{x^2} - \frac{y \cdot 2x}{x^2} - \frac{2}{3\sqrt[3]{x} \cdot x^2}$

$y' = 3 - \frac{y \cdot 2}{x} - \frac{2}{x^2 \cdot 3\sqrt[3]{x}}$



**BOLETÍN: DERIVADAS TRIGONOMÉTRICAS** *Solucionario*

Calcula la derivada de las siguientes funciones **simplificando** lo máximo posible:

$f(x) = e^{\text{Log}(\text{sen } x)}$ <span style="float: right;">1</span>	$f(x) = \sqrt{\frac{1 + \text{sen } x}{2}}$ <span style="float: right;">2</span>	$f(x) = (1 + \text{tg}^4 x)^6$ <span style="float: right;">3</span>
$f(x) = (\text{sen } x)^{\text{tg } x}$ ● <span style="float: right;">4</span>	$f(x) = 2^{\text{sen } x \cdot \text{tg } x}$ <span style="float: right;">5</span>	$f(x) = \text{arctg}\left(\frac{e^{2x}}{7}\right)$ <span style="float: right;">6</span>
$f(x) = \text{arc cos } x^3$ <span style="float: right;">7</span>	$f(x) = \text{Log}(x \cdot \text{sen } x)$ <span style="float: right;">8</span>	$2x^4 - \cos(xy) - 2 = 0$ ● <span style="float: right;">9</span>
$f(x) = \text{arc sen}\left(\frac{x}{4}\right)$ <span style="float: right;">10</span>	$f(x) = \text{arc sen}\sqrt{1 - 4x^2}$ <span style="float: right;">11</span>	$f(x) = \text{tg } 3x$ <span style="float: right;">12</span>
$f(x) = \text{tg}(x^3)$ <span style="float: right;">13</span>	$f(x) = \frac{4^x \cdot \text{sen } x}{2^x}$ <span style="float: right;">14</span>	$f(x) = \text{sen}(\text{sen}(\text{sen } x^2))$ <span style="float: right;">15</span>
$f(x) = \text{Log}(\sqrt{\cos x})$ <span style="float: right;">16</span>	$f(x) = (\cos x)^{\text{sen } x}$ ● <span style="float: right;">17</span>	$y = \sqrt{\text{sen } x + x^3}$ <span style="float: right;">18</span>
$f(x) = \cos^3(4x^2 - 3)$ <span style="float: right;">19</span>	$f(x) = \frac{\text{sen } x}{x^2}$ <span style="float: right;">20</span>	$f(x) = (\text{sen}(2x - 1))^{1/2}$ <span style="float: right;">21</span>
$f(x) = \text{cosec } x \cdot \text{tg } x$ <span style="float: right;">22</span>	$f(x) = \frac{1}{\sqrt[3]{x^2}} + \cos(2x)$ <span style="float: right;">23</span>	$f(x) = e^x \cdot \text{sen}(x + 1)$ <span style="float: right;">24</span>
$f(x) = 2 \cdot \text{tg}(2x + 2)$ <span style="float: right;">25</span>	$f(x) = \text{Log}(\text{sen } x^2)$ <span style="float: right;">26</span>	$f(x) = \text{arc sen}\left(\frac{x}{4}\right)$ <span style="float: right;">27</span>
$f(x) = (\text{arc tg } x)^{\text{Log } x}$ ● <span style="float: right;">28</span>	$f(x) = \text{sen}(x - \text{Log}(\cot(x - 1)))$ <span style="float: right;">29</span>	$y^3 - 2 \cdot \text{sen } y + \cos x = 4$ ● <span style="float: right;">30</span>
$f(x) = x \cdot \text{tg}(x + 2)$ <span style="float: right;">31</span>	$f(x) = \sqrt{\frac{1 - \text{sen } x}{1 + \text{sen } x}}$ <span style="float: right;">32</span>	$f(x) = e^{\text{sen } x^2 - 1}$ <span style="float: right;">33</span>
$\text{Log } y - \text{sen } x^2 - \frac{1}{y} = 7x$ ● <span style="float: right;">34</span>	$f(x) = \cos x \cdot \text{sen}(2x + 2)$ <span style="float: right;">35</span>	$f(x) = \frac{1 + \text{sen } x - \cos x}{2}$ <span style="float: right;">36</span>
$f(x) = \text{tg}\left(x + \frac{1}{x}\right)$ <span style="float: right;">37</span>	$f(x) = (x - 1)^{\text{tg } x - \text{Log } x}$ ● <span style="float: right;">38</span>	$f(x) = x^2 - \text{sen}\left(\frac{\pi}{2} - 1\right)$ <span style="float: right;">39</span>
$y^2 - y \cdot x + 2x^2 - \cos x + 2 = 0$ ● <span style="float: right;">40</span>	$f(x) = \frac{1 - \text{sen } x + \cos x}{2}$ <span style="float: right;">41</span>	$f(x) = \text{Log}_a(\text{sen } x)$ <span style="float: right;">42</span>
$f(x) = \text{sen}(\text{arc sen}(x - 2))$ <span style="float: right;">43</span>	$f(x) = 2^{\text{sen } x}$ <span style="float: right;">44</span>	$f(x) = \frac{1}{\text{sen}(x - 2)}$ <span style="float: right;">45</span>
$f(x) = \text{tg } x \cdot \text{sec } x$ <span style="float: right;">46</span>	$f(x) = -1 - \cot^2 x$ <span style="float: right;">47</span>	

● Se resuelven con derivación logarítmica.

● Se resuelven con derivación implícita.

$$\boxed{1} \quad f(x) = e^{\log(\operatorname{sen} x)}$$

$$f'(x) = e^{\log(\operatorname{sen} x)} \cdot \frac{\cos x}{\operatorname{sen} x \cdot \ln 10}$$

$$\boxed{y = e^u} ; \boxed{y' = e^u \cdot u'}$$

$$\boxed{2} \quad f(x) = \sqrt{\frac{1 + \operatorname{sen} x}{2}} \quad \frac{d}{dx} \left[ \frac{1}{2} (1 + \operatorname{sen} x) \right] = \frac{1}{2} \cdot \cos x$$

$$f'(x) = \frac{1}{2 \cdot \sqrt{\frac{1 + \operatorname{sen} x}{2}}} \cdot \frac{1}{2} \cdot \cos x = \frac{\cos x}{4 \cdot \sqrt{\frac{1 + \operatorname{sen} x}{2}}}$$

$$\boxed{3} \quad f(x) = (1 + \operatorname{tg}^4 x)^6$$

$$f'(x) = 6 \cdot (1 + \operatorname{tg}^4 x)^5 \cdot 4 \cdot \operatorname{tg}^3 x \cdot (1 + \operatorname{tg}^2 x) \cdot 1$$

$$\boxed{y = \operatorname{tg} u} ;$$

$$\boxed{y' = u' \cdot \sec^2 u} \quad \boxed{y' = u' \cdot (1 + \operatorname{tg}^2 u)}$$

$$\boxed{4} \quad f(x) = (\operatorname{sen} x)^{\operatorname{tg} x} ; \text{ Tomamos logaritmos neperianos en ambos miembros}$$

Llamo  $y = f(x)$  por comodidad.

$$\ln y = \ln (\operatorname{sen} x)^{\operatorname{tg} x} \Rightarrow \ln y = \operatorname{tg} x \cdot \ln (\operatorname{sen} x) ; \text{ derivamos implícitamente}$$

$$\frac{y'}{y} = \frac{1}{\cos^2 x} \cdot \ln (\operatorname{sen} x) + \operatorname{tg} x \cdot \underbrace{\frac{\cos x}{\operatorname{sen} x}}_1 \quad (\operatorname{tg} x)' = \sec^2 x = \frac{1}{\cos^2 x} = 1 + \operatorname{tg}^2 x$$

$$y' = y \cdot \left( \frac{\ln (\operatorname{sen} x)}{\cos^2 x} + 1 \right) \Rightarrow y' = \operatorname{sen} x^{\operatorname{tg} x} \cdot \left( \frac{\ln (\operatorname{sen} x)}{\cos^2 x} + 1 \right)$$

$$\boxed{5} \quad f(x) = 2^{\operatorname{sen} x \cdot \operatorname{tg} x} \quad \boxed{y = \operatorname{tg} u} ; \boxed{y' = u' \cdot \sec^2 u} \quad \boxed{y' = u' \cdot (1 + \operatorname{tg}^2 u)}$$

$$f'(x) = 2^{\operatorname{sen} x \cdot \operatorname{tg} x} \cdot \ln 2 \cdot \left( \cos x \cdot \operatorname{tg} x + \operatorname{sen} x \cdot \frac{1}{\cos^2 x} \right) =$$

$$= 2^{\operatorname{sen} x \cdot \operatorname{tg} x} \cdot \ln 2 \cdot \left( \operatorname{sen} x + \frac{\operatorname{tg} x}{\cos x} \right)$$

También podemos escribir:  $f'(x) = 2^{\operatorname{sen} x \cdot \operatorname{tg} x} \cdot \ln 2 \cdot \frac{d}{dx} (\operatorname{sen} x \cdot \operatorname{tg} x)$

donde  $\frac{d}{dx} (\operatorname{sen} x \cdot \operatorname{tg} x) = (\operatorname{sen} x \cdot \operatorname{tg} x)' = \cos x \cdot \operatorname{tg} x + \operatorname{sen} x \cdot \frac{1}{\cos^2 x}$

$$\boxed{6} \quad f(x) = \operatorname{arctg} \left( \frac{e^{2x}}{7} \right)$$

$$\boxed{y = \operatorname{arctg} u} \quad ; \quad \boxed{y' = \frac{u'}{1+u^2}}$$

$$\boxed{y = e^u} \quad ; \quad \boxed{y' = e^u \cdot u'}$$

$$f'(x) = \frac{\frac{e^{2x}}{7} \cdot 2}{1 + \left(\frac{e^{2x}}{7}\right)^2} = \frac{\frac{e^{2x}}{7} \cdot 2}{1 + \frac{e^{4x}}{49}} = \frac{\frac{e^{2x}}{7} \cdot 2 \cdot 49}{49 + e^{4x}} = \frac{14 e^{2x}}{49 + e^{4x}}$$

$$\boxed{7} \quad f(x) = \operatorname{arc} \cos x^3$$

$$\boxed{y = \operatorname{arc} \cos u} \quad ; \quad \boxed{y' = -\frac{u'}{\sqrt{1-u^2}}}$$

$$f'(x) = -\frac{3x^2}{\sqrt{1-x^6}}$$

$$\boxed{8} \quad f(x) = \log(x \cdot \operatorname{sen} x)$$

$$f'(x) = \frac{\operatorname{sen} x + x \cdot \cos x}{x \cdot \operatorname{sen} x} \cdot \log_{10} e = \left( \frac{1}{x} + \cotg x \right) \cdot \log_{10} e = \frac{1 + x \cdot \cotg x}{x \cdot \ln 10}$$

$$\boxed{9} \quad 2x^4 - \cos(x \cdot y) - 2 = 0 \quad ; \quad \text{derivamos implícitamente}$$

$$8x^3 + \operatorname{sen}(x \cdot y) \cdot \frac{d}{dx}(x \cdot y) + 0 = 0$$

$$8x^3 + \operatorname{sen}(x \cdot y) \cdot (1 \cdot y + x \cdot y') = 0 \quad ; \quad \text{reordenamos:}$$

$$8x^3 + (y + x \cdot y') \cdot \operatorname{sen}(x \cdot y) = 0 \quad ; \quad \text{despejamos } y':$$

$$x \cdot y' \operatorname{sen}(x \cdot y) = -8x^3 - y \operatorname{sen}(x \cdot y)$$

$$y' = -\frac{8x^3 + y \operatorname{sen}(x \cdot y)}{x \cdot \operatorname{sen}(x \cdot y)} = -\frac{8x^2}{\operatorname{sen}(x \cdot y)} - \frac{y}{x}$$

$$\boxed{10} \quad f(x) = \operatorname{arc} \operatorname{sen} \left( \frac{x}{4} \right)$$

$$\boxed{y = \operatorname{arc} \operatorname{sen} u} \quad ; \quad \boxed{y' = \frac{u'}{\sqrt{1-u^2}}}$$

$$f'(x) = \frac{\frac{1}{4}}{\sqrt{1 - \left(\frac{x}{4}\right)^2}} = \frac{1}{\sqrt{16 - x^2}}$$

$$\boxed{11} \quad f(x) = \operatorname{arc} \operatorname{sen} \sqrt{1 - 4x^2}$$

$$f'(x) = \frac{\frac{-8x}{2\sqrt{1-4x^2}}}{\sqrt{1 - (1-4x^2)}} = \frac{-4x}{\sqrt{1-4x^2}} = \frac{-2}{\sqrt{1-4x^2}}$$

$$12) f(x) = \operatorname{tg} 3x \Rightarrow f'(x) = 3 \cdot (1 + \operatorname{tg}^2 3x)$$

$$13) f(x) = \operatorname{tg}(x^3) \Rightarrow f'(x) = 3x^2 \cdot [1 + \operatorname{tg}^2(x^3)]$$

$$14) f(x) = \frac{4^x \cdot \operatorname{sen} x}{2^x} = \frac{(2^x)^2 \cdot \operatorname{sen} x}{2^x} = 2^x \cdot \operatorname{sen} x$$

$4^x = (2^2)^x = (2^x)^2$

$$f'(x) = 2^x \cdot \ln 2 \cdot \operatorname{sen} x + 2^x \cdot \cos x = 2^x \cdot (\ln 2 \cdot \operatorname{sen} x + \cos x)$$

$$15) f(x) = \operatorname{sen}(\operatorname{sen}(\operatorname{sen} x^2))$$

$$f'(x) = \underbrace{\cos(\operatorname{sen}(\operatorname{sen} x^2))}_{\frac{d}{dx} \operatorname{sen}(\operatorname{sen}(\operatorname{sen} x^2))} \cdot \underbrace{\cos(\operatorname{sen} x^2)}_{\frac{d}{dx} \operatorname{sen}(\operatorname{sen} x^2)} \cdot \underbrace{\cos x^2}_{\frac{d}{dx} \operatorname{sen} x^2} \cdot \underbrace{2x}_{\frac{d}{dx} x^2}$$

Reordenando:  $f'(x) = 2x \cdot \cos x^2 \cdot \cos(\operatorname{sen} x^2) \cdot \cos(\operatorname{sen}(\operatorname{sen} x^2))$

$$16) f(x) = \log(\sqrt{\cos x})$$

$$f'(x) = \frac{-\operatorname{sen} x}{2\sqrt{\cos x}} \cdot \log e = -\frac{\operatorname{sen} x}{2\cos x} \cdot \log e = -\frac{\operatorname{tg} x}{2} \cdot \log e = \frac{-\operatorname{tg} x}{2 \cdot \ln 10}$$

$$17) f(x) = (\cos x)^{\operatorname{sen} x} \quad ; \quad \text{Tomamos logaritmos neperianos en ambos miembros}$$

$$\ln f(x) = \ln (\cos x)^{\operatorname{sen} x} = \operatorname{sen} x \cdot \ln \cos x$$

Llamo  $y = f(x)$  por comodidad. ; derivamos implícitamente

$$\frac{y'}{y} = \cos x \cdot \ln \cos x + \operatorname{sen} x \cdot \frac{-\operatorname{sen} x}{\cos x} = \cos x \cdot \ln \cos x - \operatorname{sen} x \cdot \operatorname{tg} x$$

$$y' = (\cos x)^{\operatorname{sen} x} \cdot (\cos x \cdot \ln \cos x - \operatorname{sen} x \cdot \operatorname{tg} x)$$

$$18) f(x) = \sqrt{\operatorname{sen} x + x^3} \Rightarrow f'(x) = \frac{\cos x + 3x^2}{2 \cdot \sqrt{\operatorname{sen} x + x^3}}$$

$$19) f(x) = \cos^3(4x^2 - 3) \Rightarrow f'(x) = 3 \cos^2(4x^2 - 3) \cdot (-\operatorname{sen}(4x^2 - 3)) \cdot 8x$$

$$= -24x \cdot \operatorname{sen}(4x^2 - 3) \cdot \cos^2(4x^2 - 3)$$

$$\boxed{20} \quad f(x) = \frac{\operatorname{sen} x}{x^2} \Rightarrow f'(x) = \frac{\cos x \cdot x^2 - \operatorname{sen} x \cdot 2x}{x^4} = \frac{x \cos x - 2 \operatorname{sen} x}{x^3}$$

$$\boxed{21} \quad f(x) = [\operatorname{sen}(2x-1)]^{1/2}$$

$$f'(x) = \frac{1}{2} [\operatorname{sen}(2x-1)]^{1/2-1} \cdot 2 \cdot \cos(2x-1) = \frac{2 \cdot \cos(2x-1)}{2 \sqrt{\operatorname{sen}(2x-1)}} = \frac{\cos(2x-1)}{\sqrt{\operatorname{sen}(2x-1)}}$$

$\uparrow$   
 $-1/2$

$$\boxed{22} \quad f(x) = \operatorname{cosec} x \cdot \operatorname{tg} x = \frac{1}{\operatorname{sen} x} \cdot \frac{\operatorname{sen} x}{\cos x} = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$f'(x) = -\cos x^{-1-1} \cdot (-\operatorname{sen} x) = \frac{\operatorname{sen} x}{\cos^2 x} = \frac{\operatorname{tg} x}{\cos x}$$

$$\boxed{23} \quad f(x) = \frac{1}{\sqrt[3]{x^2}} + \cos(2x) = x^{-2/3} + \cos(2x)$$

$$f'(x) = -\frac{2}{3} \cdot x^{-2/3-1} - \operatorname{sen}(2x) \cdot 2 = -\frac{2}{3} \cdot x^{-5/3} - 2 \cdot \operatorname{sen}(2x) = -\frac{2}{3 \sqrt[3]{x^5}} - 2 \cdot \operatorname{sen}(2x)$$

$$\boxed{24} \quad f(x) = e^x \cdot \operatorname{sen}(x+1), \text{ derivada de un producto.}$$

$$f'(x) = e^x \cdot \operatorname{sen}(x+1) \cdot 1 + e^x \cdot \cos(x+1) \cdot 1 = e^x [\operatorname{sen}(x+1) + \cos(x+1)]$$

$$\boxed{25} \quad f(x) = 2 \cdot \operatorname{tg}(2x+2) \quad \boxed{y = \operatorname{tg} u} ; \quad \boxed{y' = u' \cdot \sec^2 u} \quad \boxed{y' = u' \cdot (1 + \operatorname{tg}^2 u)}$$

$$f'(x) = 2 \cdot [1 + \operatorname{tg}^2(2x+2)] \cdot 2 = 4 \cdot [1 + \operatorname{tg}^2(2x+2)]$$

$$\text{Alternativamente } f'(x) = 2 \cdot 2 \cdot \sec^2(2x+2) = 4 \cdot \sec^2(2x+2)$$

$$\boxed{26} \quad f(x) = \log(\operatorname{sen} x^2), \quad f'(x) = \frac{\cos x^2 \cdot 2x}{\operatorname{sen} x^2 \cdot \ln 10} = \frac{2x \cdot \cos x^2}{\ln 10 \cdot \operatorname{sen} x^2}$$

Reordeno para que se vea bien qué números están fuera de las funciones trigonométricas.

$$\boxed{27} \quad f(x) = \operatorname{arc} \operatorname{sen}\left(\frac{x}{4}\right) \quad \text{Es el ejercicio } \boxed{10} \text{ repetido por error.}$$

28  $f(x) = (\operatorname{arctg} x)^{\log x}$  Función elevada a función  $\Rightarrow$  derivación logarítmica.

$y = (\operatorname{arctg} x)^{\log x}$ ; Tomamos logaritmo neperiano en ambos términos. Llamamos  $y = f(x)$ .

$\ln y = \ln (\operatorname{arctg} x)^{\log x}$ ; Bajo el exponente por las propiedades de los logaritmos.

$\ln y = \log x \cdot \ln (\operatorname{arctg} x)$ ; Derivamos implícitamente este producto; recordamos que  $(\ln u)' = \frac{u'}{u}$

$$\frac{y'}{y} = \frac{1}{x \cdot \ln 10} \cdot \ln (\operatorname{arctg} x) + \log x \cdot \frac{1}{1+x^2} \cdot \frac{1}{\operatorname{arctg} x}$$

$$y = \operatorname{arctg} u; \quad y' = \frac{u'}{1+u^2}$$

Despejamos  $y'$ ; sustituimos  $y$  por el enunciado:  $y = (\operatorname{arctg} x)^{\log x}$

$$y' = (\operatorname{arctg} x)^{\log x} \cdot \left[ \frac{1}{x \cdot \ln 10} \cdot \ln (\operatorname{arctg} x) + \log x \cdot \frac{1}{(1+x^2) \cdot \operatorname{arctg} x} \right]$$

29  $f(x) = \operatorname{sen} (x - \log (\operatorname{cotg} (x-1)))$

$$\operatorname{cotg} (x-1) = \frac{\cos (x-1)}{\operatorname{sen} (x-1)} = \operatorname{tg}^{-1} (x-1)$$

$$f(x) = \operatorname{sen} \left( x - \log \left[ \operatorname{tg}^{-1} (x-1) \right] \right)$$

$$y = \operatorname{tg} u; \quad y' = u' \cdot (1 + \operatorname{tg}^2 u)$$

$$f'(x) = \cos \left( x - \log \left[ \operatorname{tg}^{-1} (x-1) \right] \right) \cdot \left[ 1 - \frac{-\operatorname{tg}^{-2} (x-1) \cdot [1 + \operatorname{tg}^2 (x-1)] \cdot 1}{\operatorname{tg}^{-1} (x-1) \cdot \ln 10} \right] =$$

$$= \cos \left( x - \log \left[ \operatorname{tg}^{-1} (x-1) \right] \right) \cdot \left[ 1 - \frac{-\operatorname{tg}^{-2} (x-1) - 1}{\operatorname{tg}^{-1} (x-1) \cdot \ln 10} \right] \quad \text{Hay otros métodos}$$

30  $y^3 - 2 \cdot \operatorname{sen} y + \cos x = 4$ ; Derivamos implícitamente

$$3y^2 \cdot y' - 2 \cdot y' \cdot \cos y - \operatorname{sen} x = 0; \quad \text{Despejamos } y'$$

$$y' (3y^2 - 2 \cos y) = \operatorname{sen} x$$

$$y' = \frac{\operatorname{sen} x}{3y^2 - 2 \cos y}$$

$$\boxed{31} \quad f(x) = x \cdot \operatorname{tg}(x+2)$$

$$f'(x) = 1 \cdot \operatorname{tg}(x+2) + x \cdot [1 + \operatorname{tg}^2(x+2)] \cdot 1 = \operatorname{tg}(x+2) + x + x \cdot \operatorname{tg}^2(x+2)$$

$$\boxed{32} \quad f(x) = \sqrt{\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x}}$$

$$\boxed{(\sqrt{u})' = \frac{u'}{2\sqrt{u}}} \quad \boxed{\left(\frac{u}{v}\right)' = \frac{u'v - u \cdot v'}{v^2}}$$

$$f'(x) = \frac{\frac{-\cos x(1 + \operatorname{sen} x) - (1 - \operatorname{sen} x)\cos x}{(1 + \operatorname{sen} x)^2}}{2 \cdot \sqrt{\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x}}} = \frac{-\cos x - \cancel{\operatorname{sen} x \cos x} - \cos x + \cancel{\operatorname{sen} x \cos x}}{(1 + \operatorname{sen} x)^2 \cdot 2 \cdot \sqrt{\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x}}}$$

$$= \frac{-2 \cos x}{2 \cdot \sqrt{\frac{(1 - \operatorname{sen} x)}{1 + \operatorname{sen} x}} (1 + \operatorname{sen} x)^{4/3}} = \frac{\cos x}{(1 + \operatorname{sen} x) \cdot \sqrt{(1 - \operatorname{sen} x)(1 + \operatorname{sen} x)}} = \frac{\cos x}{(1 + \operatorname{sen} x) \cdot \cancel{\cos x}} =$$

$$1 - \operatorname{sen}^2 x = \cos^2 x$$

$$f'(x) = \frac{1}{1 + \operatorname{sen} x}$$

$$\boxed{33} \quad f(x) = e^{\operatorname{sen} x \cdot 2x-1} = e^{(2x-1) \cdot \operatorname{sen} x} \quad (\text{multiplicamos los exponentes})$$

$$f'(x) = e^{(2x-1) \cdot \operatorname{sen} x} \cdot [2 \cdot \operatorname{sen} x + (2x-1) \cos x]$$

Si el enunciado fuera  $e^{\operatorname{sen}(x^{2x-1})}$ , habría que usar la técnica de derivación logarítmica.

$$\boxed{34} \quad \log y - \operatorname{sen} x^2 - \frac{1}{y} = 7x \quad ; \text{ Expreso } \frac{1}{y} = y^{-1} \quad \text{lo derivo como potencia.}$$

$$\log y - \operatorname{sen} x^2 - y^{-1} = 7x \quad ; \text{ Derivamos implícitamente}$$

$$\frac{y'}{y \cdot \ln 10} - 2x \cdot \cos x^2 + y^{-2} \cdot y' = 7 \quad ; \text{ Despejamos } y'$$

$$y' \left( \frac{1}{y \cdot \ln 10} + \frac{1}{y^2} \right) = 7 + 2x \cdot \cos x^2$$

$$y' = \frac{7 + 2x \cdot \cos x^2}{\left( \frac{1}{y \cdot \ln 10} + \frac{1}{y^2} \right)}$$

$$\boxed{35} \quad f(x) = \cos x \cdot \operatorname{sen}(2x+2)$$

$$f'(x) = -\operatorname{sen} x \cdot \operatorname{sen}(2x+2) + \cos x \cdot \cos(2x+2) \cdot 2$$

$$f'(x) = -\operatorname{sen} x \cdot \operatorname{sen}(2x+2) + 2 \cdot \cos x \cdot \cos(2x+2)$$

$$\boxed{36} \quad f(x) = \frac{1 + \operatorname{sen} x - \cos x}{2} = \frac{1}{2} \cdot (1 + \operatorname{sen} x - \cos x)$$

$$f'(x) = \frac{1}{2} \cdot (0 + \cos x - (-\operatorname{sen} x)) = \frac{\cos x + \operatorname{sen} x}{2}$$

$$\boxed{37} \quad f(x) = \operatorname{tg}\left(x + \frac{1}{x}\right) = \operatorname{tg}\left(x + x^{-1}\right)$$

$$f'(x) = \left[1 + \operatorname{tg}^2\left(x + x^{-1}\right)\right] \cdot (1 - x^{-2}) = \left[1 + \operatorname{tg}^2\left(x + \frac{1}{x}\right)\right] \cdot \left(1 - \frac{1}{x^2}\right)$$

$$\boxed{38} \quad f(x) = (x-1)^{\operatorname{tg} x - \log x}$$

Función elevada a función  $\Rightarrow$  derivación logarítmica.

$$y = (x-1)^{\operatorname{tg} x - \log x} \quad \text{Tomamos logaritmo neperiano en ambos términos. Llamo } y=f(x).$$

$$\ln y = \ln (x-1)^{\operatorname{tg} x - \log x} \quad ; \quad \text{Bajo el exponente por las propiedades de los logaritmos.}$$

$$\ln y = (\operatorname{tg} x - \log x) \cdot \ln (x-1) \quad ; \quad \text{Derivamos implícitamente este producto ;}$$

$$\text{recordamos que } (\ln u)' = \frac{u'}{u}$$

$$\frac{y'}{y} = \left(1 + \operatorname{tg}^2 x - \frac{1}{x \ln 10}\right) \cdot \ln (x-1) + (\operatorname{tg} x - \log x) \cdot \frac{1}{x-1}$$

$$\text{Despejamos } y' ; \text{ sustituimos } y \text{ por el enunciado: } y = (x-1)^{\operatorname{tg} x - \log x}$$

$$y' = (x-1)^{\operatorname{tg} x - \log x} \cdot \left[ \left(1 + \operatorname{tg}^2 x - \frac{1}{x \ln 10}\right) \cdot \ln (x-1) + \frac{\operatorname{tg} x - \log x}{x-1} \right]$$

$$\boxed{39} \quad f(x) = x^2 - \operatorname{sen}\left(\frac{\pi}{2} - 1\right) \quad ; \quad \text{¡Ojo! } \operatorname{sen}\left(\frac{\pi}{2} - 1\right) \text{ es una constante, luego su derivada es } 0.$$

$$f'(x) = 2x$$

$$\boxed{40} \quad y^2 - y \cdot x + 2x^2 - \cos x + 2 = 0 \quad ; \quad \text{Derivamos implícitamente}$$

$$2y \cdot y' - (y' \cdot x + y \cdot 1) - (-\operatorname{sen} x) + 0 = 0 \quad ; \quad \text{Despejamos } y'$$

$$2y \cdot y' - y' \cdot x - y + \operatorname{sen} x = 0$$

$$y' (2y - x) = y - \operatorname{sen} x$$

$$y' = \frac{y - \operatorname{sen} x}{2y - x}$$

$$\boxed{41} \quad f(x) = \frac{1 - \operatorname{sen} x + \cos x}{2} = \frac{1}{2} \cdot (1 - \operatorname{sen} x + \cos x)$$

$$f'(x) = \frac{1}{2} \cdot (0 - \cos x - \operatorname{sen} x) = -\frac{\cos x + \operatorname{sen} x}{2}$$

$$\boxed{42} \quad f(x) = \log_a(\operatorname{sen} x)$$

$$f'(x) = \frac{\cos x}{\operatorname{sen} x \cdot \ln a} = \frac{1}{\operatorname{tg} x \cdot \ln a}$$

$$\boxed{43} \quad f(x) = \operatorname{sen}(\operatorname{arc} \operatorname{sen}(x-2))$$

$\operatorname{sen}$  y  $\operatorname{arc} \operatorname{sen}$ , son funciones recíprocas, luego:

$$f(x) = \operatorname{sen}(\operatorname{arc} \operatorname{sen}(x-2)) = (x-2) \Rightarrow f'(x) = 1$$

Método II: Si no te das cuenta de la relación anterior, se puede derivar directamente.

$$\boxed{y = \operatorname{arc} \operatorname{sen} u} \quad ; \quad \boxed{y' = \frac{u'}{\sqrt{1-u^2}}}$$

$$f'(x) = \cos(\operatorname{arc} \operatorname{sen}(x-2)) \cdot \frac{1}{\sqrt{1-(x-2)^2}} = \frac{\cos(\operatorname{arc} \operatorname{sen}(x-2))}{\sqrt{1-(x-2)^2}}$$

pero como  $\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \cos \alpha = \sqrt{1 - \operatorname{sen}^2 \alpha}$ , luego:

$$f'(x) = \frac{\sqrt{1 - \operatorname{sen}^2(\operatorname{arc} \operatorname{sen}(x-2))}}{\sqrt{1-(x-2)^2}} = \frac{\sqrt{1-(x-2)^2}}{\sqrt{1-(x-2)^2}} = 1 \quad (\text{es muchísimo más largo})$$

$$\boxed{44} \quad f(x) = 2^{\operatorname{sen} x}, \quad f'(x) = 2^{\operatorname{sen} x} \cdot \ln 2 \cdot \cos x$$

$$\boxed{45} \quad f(x) = \frac{1}{\operatorname{sen}(x-2)} = \operatorname{sen}^{-1}(x-2), \quad \text{lo derivo como una potencia.}$$

$$f'(x) = -\operatorname{sen}^{-2}(x-2) \cdot \cos(x-2) \cdot 1 = -\frac{\cos(x-2)}{\operatorname{sen}^2(x-2)} = -\frac{\operatorname{cotg}(x-2)}{\operatorname{sen}(x-2)}$$

$$\boxed{46} \quad f(x) = \operatorname{tg} x \cdot \sec x = \frac{\operatorname{tg} x}{\cos x} = \frac{\operatorname{sen} x}{\cos^2 x}$$

$$f'(x) = \frac{\cancel{\cos} x \cdot \cos^2 x - \operatorname{sen} x \cdot 2 \cdot \cancel{\cos} x \cdot (-\operatorname{sen} x)}{\cos^4 x^3} = \frac{\cos^2 x + 2 \operatorname{sen}^2 x}{\cos^3 x}$$

$$f'(x) = \frac{\cos^2 x + \operatorname{sen}^2 x + \operatorname{sen}^2 x}{\cos^3 x} = \frac{1 + \operatorname{sen}^2 x}{\cos^3 x} = \frac{1}{\cos^3 x} + \frac{\operatorname{sen}^2 x}{\cos^3 x} = \sec^3 x + \frac{\operatorname{tg}^2 x}{\cos x}$$

$$f'(x) = \sec^3 x + \operatorname{tg}^2 x \cdot \sec x = \sec x \cdot (\sec^2 x + \operatorname{tg}^2 x)$$

$$\boxed{47} \quad f(x) = -1 - \operatorname{cotg}^2 x = -1 - \frac{1}{\operatorname{tg}^2 x} = -1 - \operatorname{tg}^{-2} x$$

$$f'(x) = 0 + 2 \cdot \operatorname{tg}^{-3} x \cdot (1 + \operatorname{tg}^2 x) = 2 \cdot \operatorname{tg}^{-3} x + 2 \cdot \operatorname{tg}^{-1} x$$

$$f'(x) = 2 \cdot \operatorname{cotg}^3 x + 2 \cdot \operatorname{cotg} x = 2 \cdot \operatorname{cotg} x (\operatorname{cotg}^2 x + 1)$$