ACTIVIDAD DE MATEMÁTICAS I - 1º BACHILLERATO - Deriva y simplifica Solucionario

1. $y = \left(4x^3 - 2x^{-2} + 5x - 3\right)^5$

   $y' = 5\left(4x^3 - 2x^{-2} + 5x - 3\right)^4\left(12x^2 + 4x^{-3} + 5\right)$

2. $y = \sin^5 x - \cos x^5$ $\qquad y' = 5\sin^4 x\cos x + 5x^4\sin\left(x^5\right)$

3. $y = \dfrac{1 + \sin x}{1 - \sin x}$ $\qquad y' = \dfrac{2\cos x}{\left(1 - \sin x\right)^2}$

4. $y = \dfrac{\cos x}{\ln x}$ $\qquad y' = -\dfrac{\cos x + x\ln x\sin x}{x\ln^2 x}$

5. $y = 5^x + x^5$ $\qquad y' = 5^x\ln\left(5\right) + 5x^4$

6. $y = 3^x x^3$ $\qquad y' = 3^x x^2\left(x\ln(3) + 3\right)$

7. $y = arctan\sqrt{1 + x^2}$ $\qquad y' = \dfrac{x}{\sqrt{x^2 + 1}\left(x^2 + 2\right)}$

8. $y = e^x\cos x$ $\qquad y' = e^x\left(\cos x - \sin x\right)$

9. $y = 2^{\sin\sqrt{x}}$ $\qquad y' = \dfrac{2^{\sin\sqrt{x}}\ln(2)\cos\sqrt{x}}{2\sqrt{x}}$

10. $y = \ln\left(\sqrt{\sin x}\right)$ $\qquad y' = \dfrac{\cot x}{2}$

11. $y = \sin\left(\ln\sqrt{x}\right)$ $\qquad y' = \dfrac{\cos\left(\dfrac{\ln x}{2}\right)}{2x}$

12. $y = \dfrac{1}{2\cos^2 x} + \ln\left(\cos x\right)$ $\qquad y' = \dfrac{\sin x}{\cos^3 x} - \tan x = \tan^3 x$

13. $y = (\cos x)^x$ $\qquad y' = (\cos x)^x\left[\ln\left(\cos x\right) - x\tan x\right]$

14. $y = \left(\log x\right)^{\tan x}$ $\qquad y' = \left(\log x\right)^{\tan x}\left[\left(1 + \tan^2 x\right)\ln\left(\log x\right) + \dfrac{\tan x}{x\log x}\log e\right]$

15. $x^2 + y^2 - 5y - 1 = 0$ $\qquad y' = \dfrac{-2x}{2y - 5}$

16. $e^{x\cdot y} - \ln x + \tan y = -5\cos x$ $\qquad y' = \dfrac{5\sin x + \dfrac{1}{x} - ye^{xy}}{xe^{xy} + 1 + \tan^2 y}$

1. $y=\left(4x^3-2x^{-2}+5x-3\right)^5$; $\boxed{y=u^n;\ n\in\mathbb{R}}$; $\boxed{y'=n\cdot u^{n-1}\cdot u'}$

$$y'=5\cdot(4x^3-2x^{-2}+5x-3)^4\cdot(12x^2+4x^{-3}+5)=(4x^3-2x^{-2}+5x-3)^4\cdot\left(60x^2+\frac{20}{x^3}+25\right)$$

2. $y=\sin^5 x-\cos x^5$

Nótese que $\text{sen}^5 x=(\text{sen}\ x)^5$ y que $\cos(x^5)$ tiene x^5 dentro del coseno.

$\boxed{y=\text{sen}\ u}$; $\boxed{y'=u'\cdot\cos u}$; $\boxed{y=\cos u}$; $\boxed{y'=-u'\cdot\text{sen}\ u}$

$$y'=5\cdot\text{sen}^4 x\cdot\cos x+\underbrace{\text{sen}\ x^5}_{\text{sen}\ u}\cdot\underbrace{(5x^4)}_{u'}$$

3. $y=\dfrac{1+\sin x}{1-\sin x}$; $\boxed{y=\dfrac{u}{v}}$; $\boxed{y'=\dfrac{u'\cdot v-u\cdot v'}{v^2}}$

$$y'=\frac{\cos x\cdot(1-\text{sen}\ x)-(1+\text{sen}\ x)\cdot(-\cos x)}{(1-\text{sen}\ x)^2}=\frac{\cos x-\text{sen}\ x\cos x+\cos x+\text{sen}\ x\cos x}{(1-\text{sen}\ x)^2}=\frac{2\cdot\cos x}{(1-\text{sen}\ x)^2}$$

4. $y=\dfrac{\cos x}{\ln x}$; $\boxed{y=\ln u}$; $\boxed{y'=\dfrac{u'}{u}}$

$$y'=\frac{-\text{sen}\ x\cdot\ln x-\cos x\cdot\frac{1}{x}}{(\ln x)^2}=\frac{-x\cdot\text{sen}\ x\cdot\ln x-\cos x}{x\cdot(\ln x)^2}$$

5. $y=5^x+x^5$; $y'=5^x\cdot\ln 5+5x^4$ $\boxed{y=a^u;\ a\in\mathbb{R}^+}$; $\boxed{y'=a^u\cdot\ln a\cdot u'}$

6. $y=3^x x^3$; $\boxed{y=u\cdot v}$; $\boxed{y'=u'\cdot v+u\cdot v'}$

$$y'=3^x\cdot\ln 3\cdot x^3+3^x\cdot 3x^2=3^x\cdot(\ln 3\cdot x^3+3x^2)=3^x\cdot x^2\cdot(x\cdot\ln 3+3)$$

7. $y=\arctan\sqrt{1+x^2}$; $\boxed{y=\text{arc tg}\ u}$; $\boxed{y'=\dfrac{u'}{1+u^2}}$; $\boxed{y=\sqrt{u}}$; $\boxed{y'=\dfrac{u'}{2\cdot\sqrt{u}}}$

$$y'=\frac{\frac{2x}{2\sqrt{1+x^2}}}{1+\sqrt{1+x^2}^{\,2}}=\frac{\frac{x}{\sqrt{1+x^2}}}{1+1+x^2}=\frac{x}{(2+x^2)\cdot\sqrt{1+x^2}}$$

8. $y=e^x\cos x$; $\boxed{y=e^u}$; $\boxed{y'=e^u\cdot u'}$

$$y'=e^x\cdot 1\cdot\cos x+e^x\cdot(-\text{sen}\ x)=e^x\cdot(\cos x-\text{sen}\ x)$$

9. $y = 2^{\sin\sqrt{x}}$; $\boxed{y = a^u \ ; \ a \in \mathbb{R}^+}$; $\boxed{y' = a^u \cdot \ln a \cdot u'}$

$$y' = 2^{\operatorname{sen}\sqrt{x}} \cdot \ln 2 \cdot \cos\sqrt{x} \cdot \frac{1}{2\cdot\sqrt{x}} = \frac{2^{\operatorname{sen}\sqrt{x}} \cdot \ln 2 \cdot \cos\sqrt{x}}{2\cdot\sqrt{x}}$$

10. $y = \ln\left(\sqrt{\sin x}\right)$; $\boxed{y = \ln u}$; $\boxed{y' = \frac{u'}{u}}$

$$y' = \frac{\frac{\cos x}{2\cdot\sqrt{\operatorname{sen} x}}}{\sqrt{\operatorname{sen} x}} = \frac{\cos x}{2\cdot \operatorname{sen} x} = \frac{\operatorname{cotg} x}{2}$$

11. $y = \sin\left(\ln\sqrt{x}\right)$; $y' = \cos\left(\ln\sqrt{x}\right)\cdot\frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}} = \cos\left(\ln\sqrt{x}\right)\cdot\frac{1}{2\cdot x}$

Escribo $\ln\sqrt{x} = \ln x^{1/2} = \frac{1}{2}\cdot\ln x = \frac{\ln x}{2}$ y por eso resulta $y' = \dfrac{\cos\left(\frac{\ln x}{2}\right)}{2x}$

12. $y = \dfrac{1}{2\cos^2 x} + \ln(\cos x)$ lo convierto en potencia $y = \frac{1}{2}\cdot(\cos x)^{-2} + \ln(\cos x)$

$$y' = \frac{1}{2}\cdot(-2)\cdot(\cos x)^{-3}(-\operatorname{sen} x) + \frac{-\operatorname{sen} x}{\cos x} = \frac{\operatorname{sen} x}{\cos^3 x} - \operatorname{tg} x = \frac{\operatorname{tg} x}{\cos^2 x} - \operatorname{tg} x = \operatorname{tg} x\left(\frac{1}{\cos^2 x} - 1\right) =$$

$$= \operatorname{tg} x\cdot\frac{1-\cos^2 x}{\cos^2 x} = \operatorname{tg} x\,\frac{\operatorname{sen}^2 x}{\cos^2 x} = \operatorname{tg}^3 x$$

13. $y = (\cos x)^x$ Derivación logarítmica

$\ln y = \ln(\cos x)^x = x\cdot\ln(\cos x)$ Derivación implícita.

$$\frac{y'}{y} = 1\cdot\ln(\cos x) + x\cdot\frac{-\operatorname{sen} x}{\cos x}$$

$$y' = y\cdot\left[\ln(\cos x) - x\cdot\operatorname{tg} x\right] = (\cos x)^x\cdot\left[\ln(\cos x) - x\cdot\operatorname{tg} x\right]$$

14. $y = (\log x)^{\tan x}$ Derivación logarítmica

$\ln y = \operatorname{tg} x\cdot\ln(\log x)$ Derivación implícita.

$$\frac{y'}{y} = (1+\operatorname{tg}^2 x)\cdot\ln(\log x) + \operatorname{tg} x\cdot\frac{\frac{1}{x\cdot\ln 10}}{\log x} = (1+\operatorname{tg}^2 x)\cdot\ln(\log x) + \frac{\operatorname{tg} x}{x\cdot\ln 10\cdot\log x}$$

$$y' = \underbrace{(\log x)^{\operatorname{tg} x}}_{y}\cdot\left[(1+\operatorname{tg}^2 x)\cdot\ln(\log x) + \frac{\operatorname{tg} x}{x\cdot\ln 10\cdot\log x}\right]$$

$$\log e = \frac{\ln e}{\ln 10} = \frac{1}{\ln 10}$$

15. $x^2 + y^2 - 5y - 1 = 0$ Derivación implícita.

$2x \cdot 1 + 2y \cdot y' - 5 \cdot y' - 0 = 0$ Despejamos y'

$y'(2y-5) = -2x$

$y' = \dfrac{-2x}{2y-5} = \dfrac{2x}{5-2y}$

16. $e^{x \cdot y} - \ln x + \tan y = -5 \cos x$ Derivación implícita.

$e^{x \cdot y} \cdot (1 \cdot y + x \cdot y') - \dfrac{1}{x} + (1 + tg^2 y) \cdot y' = +5 \text{ sen } x$

$(e^{x \cdot y} + 1 + tg^2 y) \cdot y' = 5 \text{ sen } x + \dfrac{1}{x} - y \cdot e^{xy}$ Despejamos y'

$$y' = \dfrac{5 \text{ sen } x + \frac{1}{x} - y \cdot e^{xy}}{e^{x \cdot y} + 1 + tg^2 y}$$