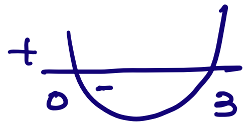
Examen Tema 9 (2/5/2024)

Ejercicio 1

$$f(x) = \frac{x^2 - x - 6}{x^2 - 3x} \qquad \mathrm{Dom} f = \mathbb{R} - \{0, 3\}$$

f es continua en $\mathbb{R} - \{0, 3\}$



Estudio en $x=0$

$0 \notin \mathrm{Dom} f$

$$\nexists \lim_{x \to 0} f(x) = \frac{-6}{0} = \pm\infty$$

$$\lim_{x \to 0^-} \frac{x^2 - x - 6}{x^2 - 3x} = \frac{-6}{0^+} = -\infty$$

$$\lim_{x \to 0^+} \frac{x^2 - x - 6}{x^2 - 3x} = \frac{-6}{0^-} = +\infty$$

f tiene en $x=0$ discontinuidad de salto ∞ ($x=0$ Asíntota vertical)

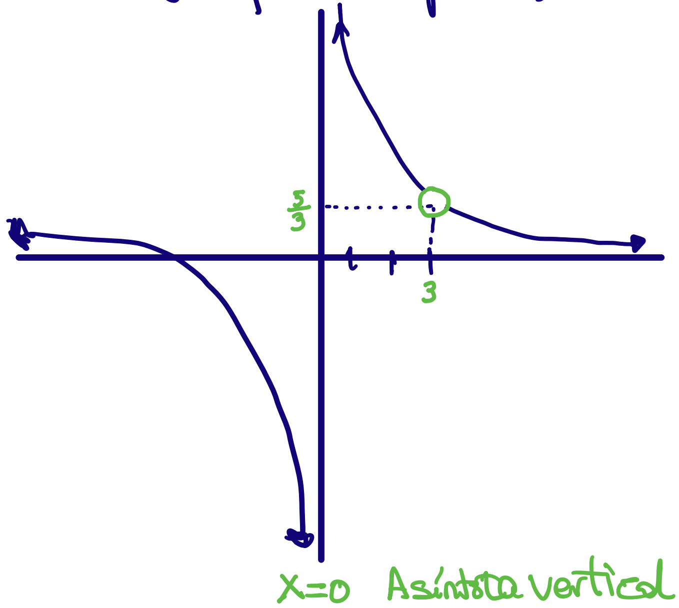
Estudio en $x=3$

$3 \notin \mathrm{Dom} f$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 3x} \underset{\frac{0}{0}}{=} \lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{x\cancel{(x-3)}} = \frac{3+2}{3} = \frac{5}{3}$$

f tiene en $x=0$ discontinuidad evitable (Le falta 1 punto)

Interpretación gráfica



Ejercicio 2

$$f(x)=\begin{cases}\dfrac{x^2-x}{x^2-3x+2} & x\neq 1\\ k & x=1\end{cases}$$

$\mathrm{Dom}\, f=\mathbb{R}-\{2\}$

f continua en $\mathbb{R}-\{1,2\}$

a) Si $x=1$

$$\lim_{x\to 1} f(x)=\lim_{x\to 1}\frac{x^2-x}{x^2-3x+2}\underset{\frac{0}{0}}{=}\lim_{x\to 1}\frac{x(x-1)}{(x-1)(x-2)}=\frac{1}{-1}=-1$$

$f(1)=k$

$\boxed{f \text{ continua en } x=1 \iff k=-1}$

Si $k=-1$ f es continua en $x=1$ $\Rightarrow$ f cont en $\mathbb{R}-\{2\}$

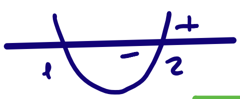
Si $k\neq -1$ f no es continua en $x=1$, tiene en $x=1$ una discontinuidad de salto finito ; f cont en $\mathbb{R}-\{1,2\}$

b) Asíntotas de f $(2\notin \mathrm{Dom}\, f)$

Si $x=2$ $\displaystyle\lim_{x\to 2} f(x)=\lim_{x\to 2}\frac{x^2-x}{x^2-3x+2}\underset{\frac{2}{0}}{=}$

$$\lim_{x\to 2^-}\frac{x^2-x}{x^2-3x+2}=\frac{2}{0^-}=-\infty$$

$$\lim_{x\to 2^+}\frac{x^2-x}{x^2-3x+2}=\frac{2}{0^+}=+\infty$$



No existe $\displaystyle\lim_{x\to 2} f(x)=\pm\infty$ $\Rightarrow$ $\boxed{x=2 \text{ Asíntota vertical}}$

→ Comportamiento en $\pm\infty$

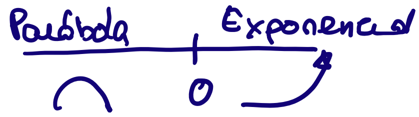
$$\lim_{x\to \infty} f(x)=\lim_{x\to\infty}\frac{x^2-x}{x^2-3x+2}=1$$

$$\lim_{x\to -\infty} f(x)=\lim_{x\to-\infty}\frac{x^2-x}{x^2-3x+2}=\lim_{x\to\infty}\frac{x^2+x}{x^2+3x+2}=\lim_{x\to\infty}\frac{x^2}{x^2}=1$$

$\boxed{y=1 \text{ Asíntota horizontal}}$

Ejercicio 3

$$f(x)=\begin{cases} 1-x^2 & x\leq 0 \\ 2^{x+1} & x>0 \end{cases}$$



a) $\mathrm{Dom} f = \mathbb{R}$

b) $y=1-x^2$ función polinómica $\Rightarrow$ f es continua en $(-\infty\ 0)$

$y=2^{x+1}$ función exponencial $\Rightarrow$ f continua en $(0,\infty)$

Estudio en $x=0$

- $f(0)=1-0^2=1$
- $\lim\limits_{x\to 0} f(x)$:
  - $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0} 1-x^2=1-0=1$
  - $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0} 2^{x+1}=2^{0+1}=2$

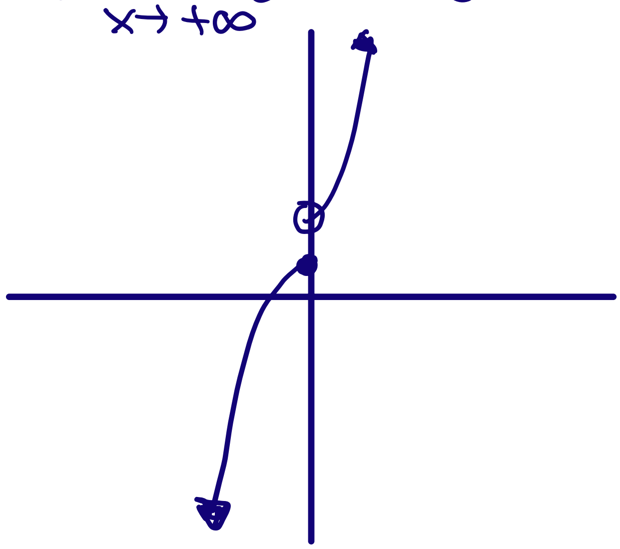
$\nexists \lim\limits_{x\to 0} f(x)$

f no es continua en $x=0$

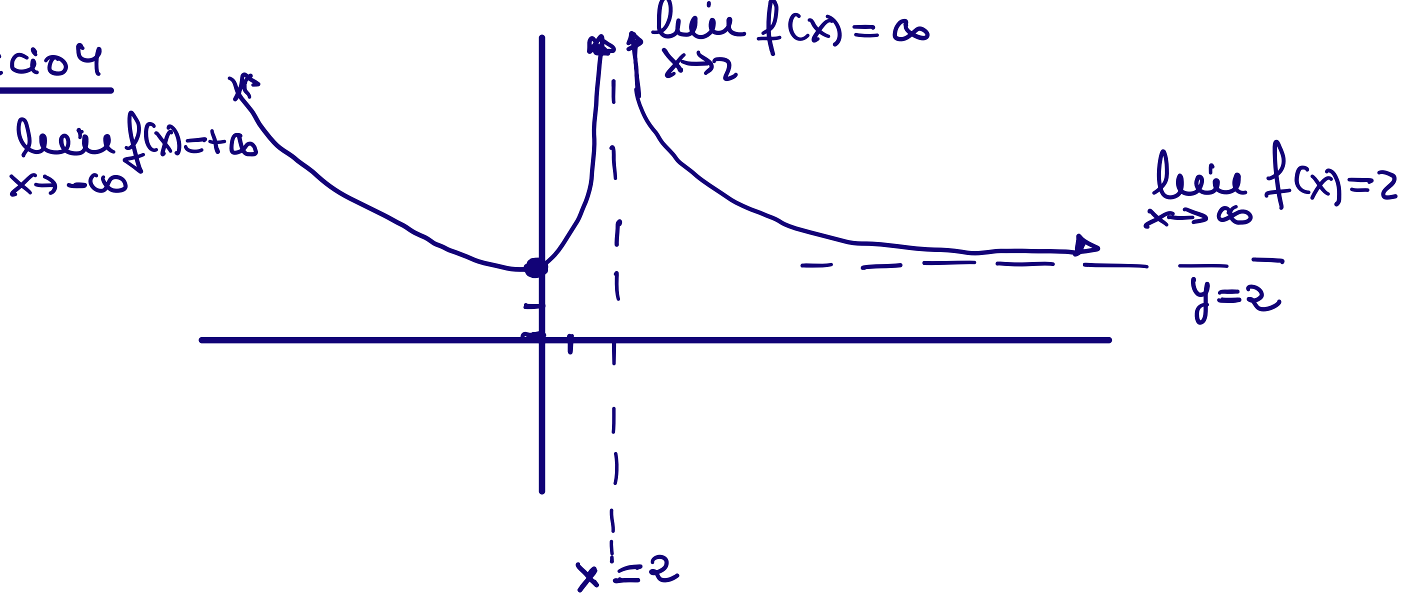
$\boxed{f \text{ es continua en } \mathbb{R}-\{0\}}$

c) $\lim\limits_{x\to -\infty} f(x)=\lim\limits_{x\to -\infty} 1-x^2=\lim\limits_{x\to \infty} 1-x^2=-\infty$

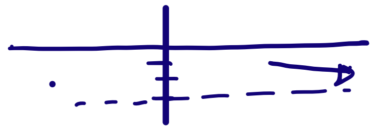
$\lim\limits_{x\to +\infty} f(x)=\lim\limits_{x\to +\infty} 2^{x+1}=2^{\infty}=\infty$



Ejercicio 4



Ejercicio 5

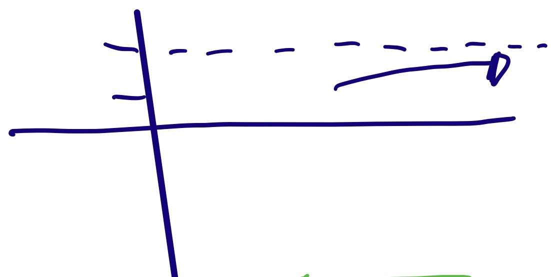


a) $\lim_{x\to\infty}\left(\frac{3x^2}{x+1}-\frac{3x^3}{x^2-1}\right) \underset{\infty-\infty}{=} \lim_{x\to\infty}\left(\frac{3x^2(x-1)-3x^3}{x^2-1}\right)=\lim_{x\to\infty}\frac{\cancel{3x^3}-3x^2-\cancel{3x^3}}{x^2-1}=-3$

b) $\lim_{x\to\infty}\left(\sqrt{x^2-3}-\sqrt{x^2-4x}\right) \underset{\infty-\infty}{=} \lim_{x\to\infty}\left(\sqrt{x^2-3}-\sqrt{x^2-4x}\right)\cdot\frac{\sqrt{x^2-3}+\sqrt{x^2-4x}}{\sqrt{x^2-3}+\sqrt{x^2-4x}}=$

$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2-3}\right)^2-\left(\sqrt{x^2-4x}\right)^2}{\sqrt{x^2-3}+\sqrt{x^2-4x}}=\lim_{x\to\infty}\frac{x^2-3-x^2+4x}{\sqrt{x^2-3}+\sqrt{x^2-4x}}=\lim_{x\to\infty}\frac{-3+4x}{\sqrt{x^2-3}+\sqrt{x^2-4x}}=$

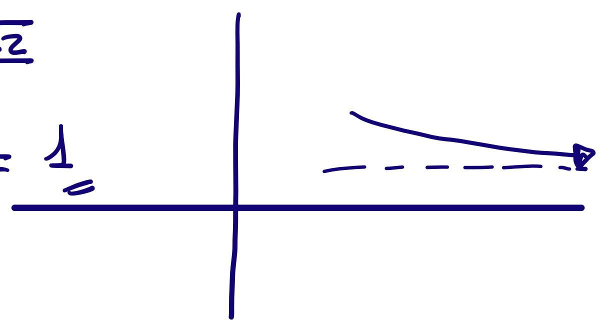
$=\lim_{x\to\infty}\frac{4x}{\sqrt{x^2}+\sqrt{x^2}}=\lim_{x\to\infty}\frac{4x}{2x}=\frac{4}{2}=2$



c) $\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-2}\right)^{2x}=\lim_{x\to\infty}\left(1+\frac{x^2+1}{x^2-2}-1\right)^{2x}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x^2-2}{3}}\right)^{\frac{x^2-2}{3}\cdot\frac{3}{x^2-2}\cdot 2x}=$

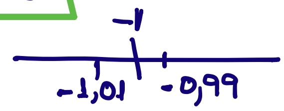
$\frac{x^2+1}{x^2-2}-1=\frac{x^2+1-x^2+2}{x^2-2}=\frac{3}{x^2-2}=\frac{1}{\frac{x^2-2}{3}}$

$=e^{\lim_{x\to\infty}\frac{6x}{x^2-2}}=e^{\lim_{x\to\infty}\frac{6}{x}}=e^0=1$



Ejercicio 6

$f(x)=\dfrac{2x^2}{x+1}$ $\boxed{\text{Dom} f = \mathbb{R}-\{-1\}}$



- f continua en $\mathbb{R}-\{-1\}$

$$\lim_{x\to -1} f(x) = \frac{2}{0} \quad \begin{cases} \displaystyle\lim_{x\to -1^-} \frac{2x^2}{x+1} = \frac{2}{0^-} = -\infty \\ \displaystyle\lim_{x\to -1^+} \frac{2x^2}{x+1} = \frac{2}{0^+} = +\infty \end{cases}$$

$\boxed{x=-1 \text{ A. vertical}}$

- Comportamiento en $\pm\infty$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{2x^2}{x+1} = \lim_{x\to\infty}\frac{2x^{\cancel{2}}}{\cancel{x}} = +\infty$$ (No hay AH)

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\frac{2x^2}{x+1} = \lim_{x\to+\infty}\frac{2x^2}{-x+1} = \lim_{x\to\infty}\frac{2x^{\cancel{2}}}{-\cancel{x}} = -\infty$$

Asíntota oblicua $y=mx+n$

$$m=\lim_{x\to\infty}\frac{f(x)}{x}$$

$$m=\lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{\frac{2x^2}{x+1}}{x} = \lim_{x\to\infty}\frac{2x^2}{x^2+x} = 2 \qquad \boxed{m=2}$$

$$n=\lim_{x\to\infty}\left(f(x)-mx\right)$$

$$n=\lim_{x\to\infty}\left(\frac{2x^2}{x+1}-2x\right) = \lim_{x\to\infty}\frac{\cancel{2x^2}-\cancel{2x^2}-2x}{x+1} = -2 \qquad \boxed{n=-2}$$

$\boxed{\text{Asíntota oblicua } y=2x-2}$