1.-

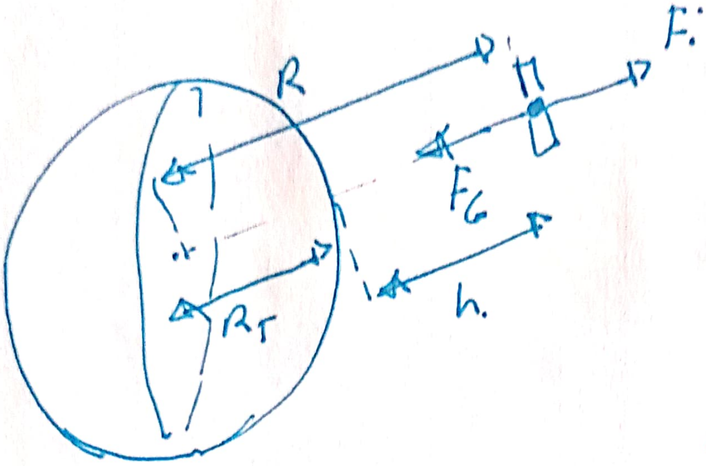
$m = 200\ Kg$

$V = 7'00\ Km \cdot s^{-1}$

DATOS

$g = 9'81\ m \cdot s^{-1}$ // $R_T = 6'37 \cdot 10^6\ m.$

a)



$$F_G = F_i$$

$$G \frac{M_T m}{R^2} = m \frac{v^2}{R}$$

$$G \frac{M_T}{R^2} = \frac{v^2}{\cancel{R}}$$

$$G \frac{M_T}{R} = v^2$$

$$v^2 = G \frac{M_T}{R} \quad ; \quad G??$$

$$g = G \frac{M_T}{R_T^2} \Rightarrow G = \frac{g \cdot R_T^2}{M_T}$$

$$v^2 = \frac{g\ R_T^2}{\cancel{M_T}} \cdot \frac{\cancel{M_T}}{R} \Rightarrow v^2 = \frac{g \cdot R_T^2}{R}$$

$$R = \frac{g \cdot R_T^2}{v^2} = \frac{(6'37 \cdot 10^6 m)^2 \cdot 9'81\ m/s^2}{(7 \cdot 10^3 m/s)^2} = 8'124 \cdot 10^6 m.$$

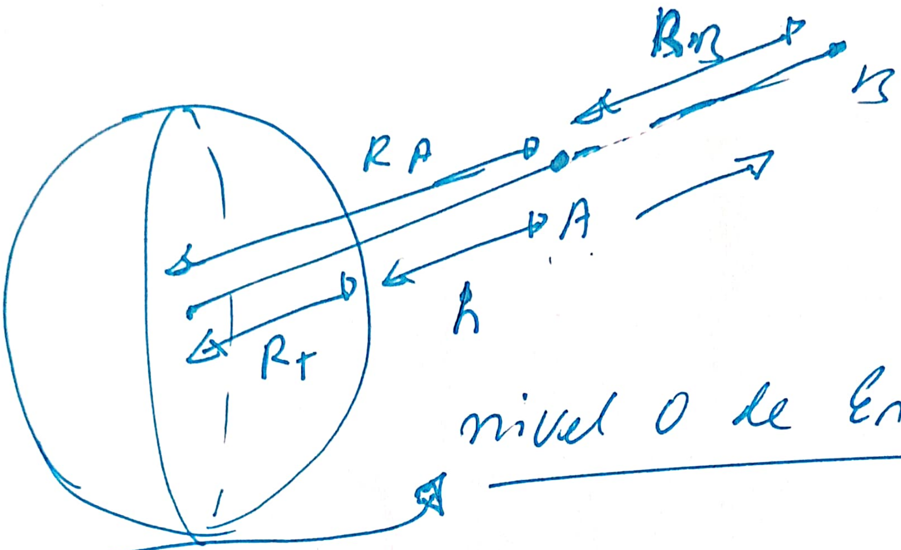
$$R = R_T + h \Rightarrow h = R - R_T = 8'124 \cdot 10^6 m - 6'37 \cdot 10^6 m.$$

$$h = 1'75 \cdot 10^6 m.$$

$$R_1 = \frac{1}{\frac{1}{8'124\,10^6 m} - \frac{(7\,10^3 m/s)^2}{9'81 \frac{m}{s^2} \cdot (6'37\,10^6 m)^2}} = - ??$$

Moito máis axeitado

b)



nivel 0 de Enerxía potencial A

Enerxía A = Enerxía B

$(E_p)_A + (E_c)_A = (E_p)_B + (E_c)_B$

$(E_c)_B = 0$ non continua o movemento

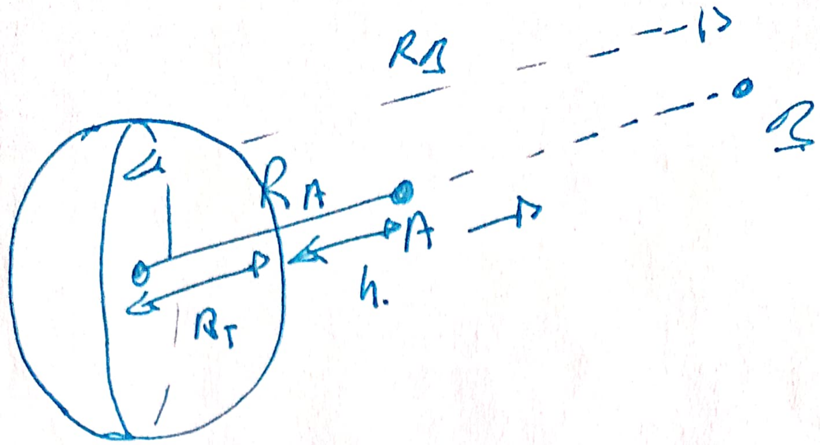
$(E_p)_A = 0$

$(E_c)_A = -(E_p)_B \;;\; \frac{1}{2} m v_A^2 = -G\frac{M.m.}{R_B}$

Valor de R_B negativo.

$r = \infty$

b)



Enerxía A = Enerxía B

$$(E_p)_A + (E_c)_A = (E_p)_B + (E_c)_A \quad \Big| \quad \frac{1}{2} m v_A^2 - G\frac{Mm}{R_A} = \frac{1}{2} m v_B^2 - G\frac{M.m.}{R_B}$$

$$\frac{1}{2} \cancel{m} v_A^2 - G\frac{M\cancel{m}}{R_A} = \frac{1}{2}\cancel{m} \cancel{v_B^2} - G\frac{M\cancel{m}.}{R_B}$$

$$v_B \to 0$$

porque non sigue. Calcular $\Rightarrow R_B$?

$$\frac{1}{2} v_A^2 - \frac{GM.}{R_A} = -G\frac{M.}{R_B} \;;\; -\frac{1}{2} v_A^2 + G\frac{M.}{R_A} = G\frac{M.}{R_B}$$

$$\left(-\frac{1}{2} v_A^2 + \frac{GM.}{R_A}\right) R_B = GM. \;;$$

$$\frac{1}{\cancel{GM.}}\left(-\frac{1}{2} v_A^2 + \frac{\cancel{GM.}}{R_A}\right) R_B = \cancel{GM.}\,\frac{1}{\cancel{GM}}$$

$$\left(-\frac{v_A^2}{GM} + \frac{1}{R_A}\right) R_B = 1 \Rightarrow R_B = \frac{1}{\frac{1}{R_A} - \frac{v_A^2}{GM}}$$

$$R_B = \frac{1}{\frac{1}{R_A} - \frac{v_A^2}{\frac{g R_T^2}{\cancel{M_T}} \cancel{M_T}}} \;;\; R_B = \frac{1}{\frac{1}{R_A} - \frac{v_A^2}{g.R_T^2}}$$

2.–

DATOS

$T_X = 12 T_T$

a)? $R_X = n\, T_T$

relación

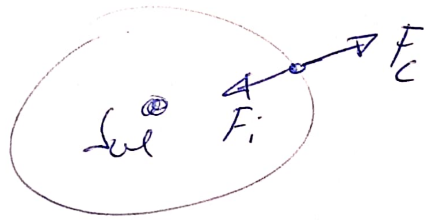
lei de Kepler $\frac{T^2}{R^3} = cte$: $\frac{T_X^2}{R_X^3} = \frac{T_T^2}{R_T^3}$

$R_X^3 \frac{T_T^2}{R_T^3} = T_X^2 \Rightarrow R_X^3 = \frac{T_X^2}{T_T^2} \cdot R_T^3$

$R_X^3 = \frac{(12)^2 T_T^2}{T_T^2} \cdot R_T^3 \Rightarrow R_X = 5'24 R_T$

b) aceleracións nas órbitas?

a aceleración normal de xiro en torno a Xupiter.



$F_c = m \cdot \frac{V^2}{r} \Rightarrow a_T = \frac{V_T^2}{R_T}$; $a_X = \frac{V_X^2}{R_X}$

$\frac{V^2}{R} = \frac{w^2 \cdot R^2}{R} = \frac{4\pi^2 R}{T^2}$; $a_X = \frac{4\pi^2 R_X}{T_X^2}$

$a_X = \frac{4\pi^2 (5'24 R_T)^2}{(12 T_T)^2} = 4\pi^2 0'036 \frac{R_T}{T_T^2}$ | $a_X = 0'036\, a_T$

$a_T = 4\pi^2 \frac{R_T}{T_T^2}$

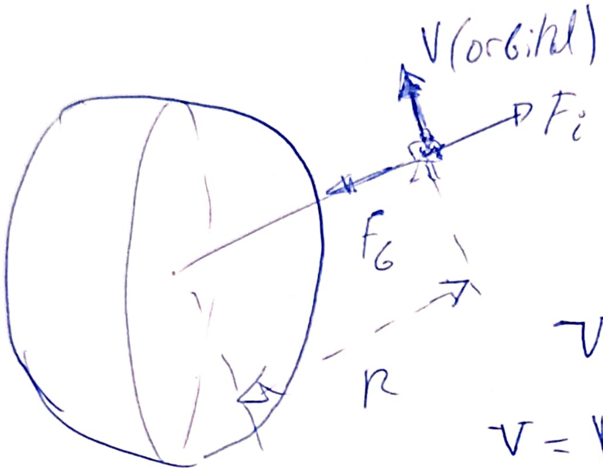
3.-

$R = 1850\,km = 1'850 \cdot 10^6\,m$ a) v?

$M_L = 7'36 \cdot 10^{22}\,kg$ b) T?

Dato $G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$

a)



$|F_G| = |F_i|$

$G \frac{Mm}{R^2} = m \frac{v^2}{R}$

$v^2 = \frac{GM}{R} = \frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 7'36 \cdot 10^{22} kg}{1'850 \cdot 10^6 m}$

$v = \sqrt{\frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 7'36 \cdot 10^{22} kg}{1'850 \cdot 10^6 m}}$

$v = 1629 \frac{m}{s}$

b)

$v = \omega \cdot R \Rightarrow v = \frac{2\pi}{T} \cdot R \Rightarrow T = \frac{2\pi R}{v}$

$T = \frac{2\pi \cdot 1'850 \cdot 10^6 m}{1629 \frac{m}{s}} = 7136\,s$

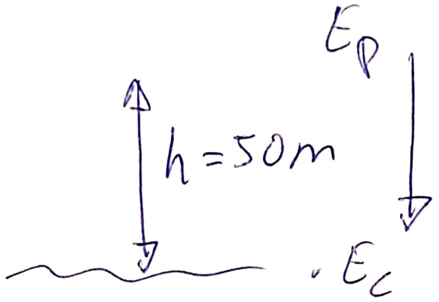
4.-

$M_M = 0'107\, M_T$

$R_M = 0'533\, R_T$

DATOS: $g_T = 9'81\, m \cdot s^{-1}$; $R_T = 6'37 \cdot 10^6\, m$

a) $h = 50\, m$. tiempo que tarda t



$$E_c = E_p \Rightarrow \frac{1}{2}\cancel{m}v^2 = \cancel{m}gh \Rightarrow v = \sqrt{2gh}$$

$$g_M = \frac{G M_M}{R_M^2} \; ; \; g_T = \frac{G M_T}{R_T^2} \; ; \; \frac{g_M}{g_T} = \frac{\frac{\cancel{G} M_M}{R_M^2}}{\frac{\cancel{G} M_T}{R_T^2}} \Rightarrow \frac{g_M}{g_T} = \frac{R_T^2 M_M}{R_M^2 M_T}$$

$$g_M = g_T \frac{M_M}{M_T}\frac{R_T^2}{R_M^2} \; ; \; g_M = g_T \frac{0'107\, M_T}{M_T}\frac{\cancel{R_T^2}}{(0'533)^2 \cancel{R_T^2}}$$

$$g_M = 0'377\, g_T \Rightarrow g_M = 3'69\, \frac{m}{s^2}$$

$$v = \sqrt{2gh} = \sqrt{2 \cdot 3'69\frac{m}{s^2} \cdot 50m} = 19'59\, m/s$$

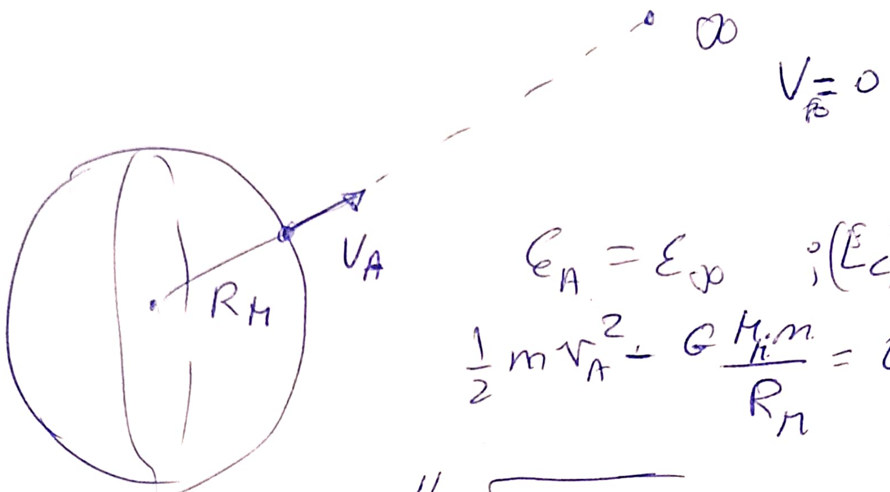
t?

$$v = v_0 + gt \; ; \; \cancel{e = v_0 t - \frac{1}{2}gt^2} \;\; e = e_0 + v_0 t - \frac{1}{2}gt^2$$

$$-e_0 = -\frac{1}{2}gt^2 \Rightarrow t = \left(\frac{2e_0}{g}\right)^{1/2} \; ; \; t = \left(\frac{2 \cdot 50m}{3'69\frac{m}{s^2}}\right)^{1/2}$$

$$t = 5'21\, s$$

b) V de escape



$E_A = E_\infty \; ; (E_c)_A + (E_p)_A = (E_c)_\infty + (E_p)_\infty$

$$\frac{1}{2} m v_A^2 - G \frac{M_M m}{R_M} = 0$$

$$v = \left(\frac{2}{\cancel{m}} \frac{G M_M \cdot \cancel{m}}{R_M} \right)^{1/2} = \sqrt{\frac{2 \cdot G M_M}{R_M}}$$

$$G?? \Rightarrow g_M = G \frac{M_M}{R_M^2} \Rightarrow G = \frac{g_M \cdot R_M^2}{M_M}$$

$M_M = 0'107 \, M_T \; ; \; R_M = 0'533 \cdot R_T$

$$v = \left(\frac{2 \cdot \frac{g_M \cdot R_M^{\cancel{2}}}{\cancel{M_M}} \cdot \cancel{M_M}}{\frac{\cancel{R_M}}{1}} \right)^{1/2} = (2 \cdot g_M \cdot R_M)^{1/2}$$

$$v = (2 \cdot 3'69 \, m/s^2 \cdot 0'533 \cdot 6'37 \cdot 10^6 \, m)^{1/2}$$

$$v = 2'242 \cdot 10^3 \, m/s$$

5.-

$g_{planeta} = 7'2 . m. s^{-2}$

$R_{planeta} = 4100 Km = 4'1\ 10^6 m.$

Datos

$G = 6'67 . 10^{-11} \frac{N . m^2}{Kg^2}$

a) M_p ?

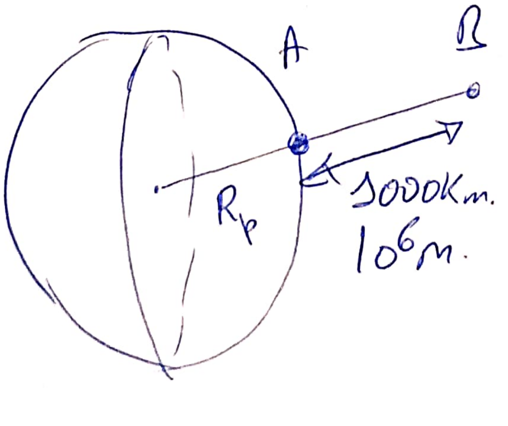
$$g_p = G \frac{M_p}{R_p^2} \Rightarrow M_p = \frac{g_p R_p^2}{G}$$

$$M_p = \frac{7'2\ m . s^{-2} (4'1 . 10^6 m)^2}{6'67 . 10^{-11} \frac{N . m^2}{Kg^2}} = 1'81\ 10^{24}\ Kg.$$

Unidades

$$\frac{m . s^{-2}\ m^2}{N . \frac{m^2}{Kg^2}} = \frac{\frac{m . s^{-2} . \cancel{m^2}}{1}}{\frac{Kg . m . s^{-2} . m^2}{Kg^2}} = \frac{\cancel{m} . \cancel{s^{-2}} . m^2\ Kg^2}{Kg\ \cancel{m} . \cancel{s^{-2}} . \cancel{m^2}} = Kg$$

b)



Enería para ir de A. B.

é a enerxía cinética

$$(E_c)_A + (E_p)_A = \cancel{(E_c)_B} + (E_p)_B$$

$\cancel{\Delta}$ 0

cando chegue a B non ten velocidade porque non segue movendose

$$(E_c)_A = (E_p)_B - (E_p)_A$$

$$(E_c)_A = -\frac{G M.m}{r_B} - \left(-\frac{G M.m}{r_A}\right)$$

$$(E_c)_A = G M.m.\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$(E_c)_A = 6'67.10^{-11} \frac{N.m^2}{\cancel{Kg^2}} 3 Kg . 5'81\,10^{24} Kg \left(\frac{1}{4'1\,10^6 m} - \frac{1}{5'1\,10^6 m}\right)$$

$$(E_c)_A = 5'73\,10^7 J$$

Unidades

$$N.\frac{m^{\cancel{2}}}{\cancel{Kg^2}}.\cancel{Kg}.\cancel{Kg}\,\cancel{m^{-1}} = N.m = J$$

6.-

$m = 150\,Kg$

$V = 30\,Km.s^{-1} = 3\,10^{4}\,m\,s^{-1}$

DATOS

$G = 6'67 \cdot 10^{-11}\,\frac{N.m^2}{Kg^2}$

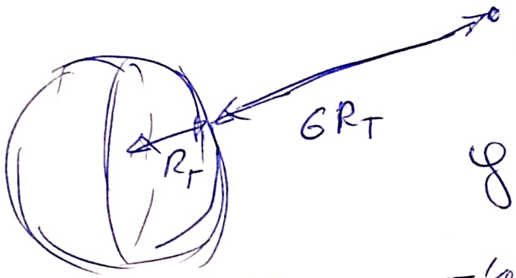
$M_T = 5'98\;10^{24}\,Kg$

$R_T = 6'37 \cdot 10^{6}\,m.$

a) Peso ?

b) Em ?

a) Peso ; $Peso = m \cdot g$



g ?

$$g = G\,\frac{M}{R^2}$$

$$g = 6'67 \cdot 10^{-11}\,\frac{N.m^2}{Kg^2} \cdot \frac{5'98\,10^{24}\,Kg}{(7 \cdot 6'37\,10^{6}\,m)^2} = 0'20\,m/s^2$$

$$Peso = m.g = 150\,Kg \cdot 0'20\,\frac{m}{s^2} = 30'1\,N$$

b) $$E_m = (E_c + E_p) = \frac{1}{2}\,m\,v^2 + \left(-G\,\frac{M.m}{r}\right)$$

$$E_m = m \cdot \left(\frac{1}{2}v^2 - G\,\frac{M}{r}\right) = 150\,Kg\left[\frac{1}{2}\left(3 \cdot 10^{4}\,\frac{m}{s}\right)^2 - \frac{6'67 \cdot 10^{-11}\,N.m^2/Kg^2 \cdot 5'98\,10^{24}\,Kg}{7 \cdot 6'37\,10^{6}\,m}\right]$$

$$E_m = 6'6\;10^{10}\,J$$

7.-

$m_1 = 2\ Kg \qquad x = 0\ m$

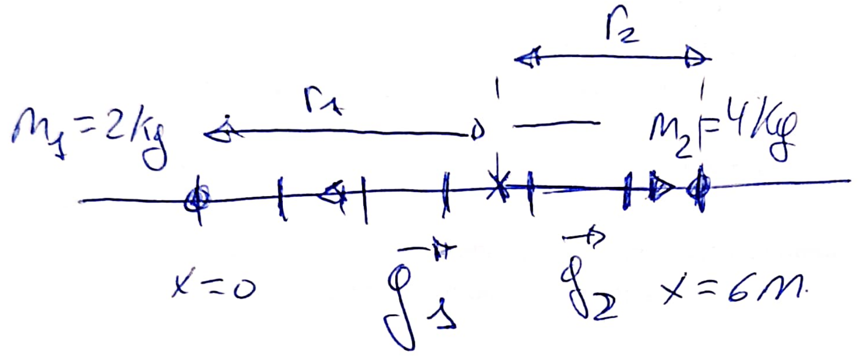
$m_2 = 4\ Kg \qquad x = 6\ m.$

a) $g = 0$? onde

b) V? en $x=2$

c) $W_{x=2}^{\infty}$ desde $x=2m$ a infinito

a)



de onde estará máis cerca o punto da masa de 4 Kg ou da de 2 Kg

$$\vec{g}_1 = -G\frac{m_1}{r_1^2}\vec{i} \qquad \vec{g}_2 = G\frac{m_2}{r_2^2}\vec{i} \qquad \left|\vec{g}_1\right| = \left|\vec{g}_2\right|$$

$$\cancel{G}\frac{m_1}{r_1^2} = \cancel{G}\frac{m_2}{r_2^2} \Rightarrow \frac{m_1}{r_1^2} = \frac{m_2}{r_2^2}\ ;\ r_1 + r_2 = 6m.$$

$\frac{m_1}{r_1^2} = \frac{m_2}{r_2^2} \Rightarrow$ extraigo a raiz para non ter ecuacións de 2º grado

$$\frac{m_1^{1/2}}{r_1} = \frac{m_2^{1/2}}{r_2}\ ;\ m_1^{1/2} r_2 = m_2^{1/2}\cdot r_1\ ;\quad r_1 = 6 - r_2$$

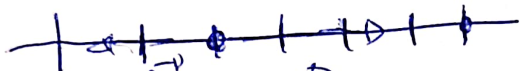
$$\frac{m_1^{1/2}}{m_2^{1/2}} r_2 = 6 - r_2 \Rightarrow \left(\frac{m_1^{1/2}}{m_2^{1/2}} + 1\right) r_2 = 6m.$$

$$r_2 = \frac{Gm}{\left[\left(\frac{m_1}{m_2}\right)^{1/2}+1\right]} \Rightarrow r_2 = \frac{Gm}{\left[\left(\frac{2}{4}\right)^{1/2}+1\right]} =$$

$r_2 = 3'51\,m.$; $r_1 = 2'49\,m.$

b) g en $x = 2\,m.$

g?



$m_1 = 2\,Kg$ $\vec{g}_1$ $\vec{g}_2$ $m_2 = 4\,Kg$

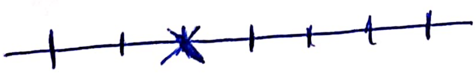
$$\vec{g}_1 = -G\frac{m_1}{r_1^2}\,\vec{i} \quad ; \quad \vec{g}_2 = +G\frac{m_2}{r_2^2}\,\vec{i}$$

$$\vec{g}_T = G\left(\frac{m_2}{r_2^2} - \frac{m_1}{r_1^2}\right)\vec{i}$$

$$\vec{g}_T = 6'67\cdot 10^{-11}\,\frac{N\cdot m^2}{Kg^2}\left(\frac{4\,Kg}{(4\,m)^2} - \frac{2\,Kg}{(2\,m)^2}\right)\vec{i}$$

$$\vec{g}_T = -1'67\cdot 10^{-11}\,\frac{N}{Kg}\,\vec{i}$$

V en $x = 2\,m.$



$m_1 = 2\,Kg$ $m_2 = 4\,Kg$

$$V_1 = -G\frac{m_1}{r_1} \quad ; \quad V_2 = -G\frac{m_2}{r_2}$$

$$V_T = V_1 + V_2 = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$V_T = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}\left(\frac{2\,Kg}{2\,m} + \frac{4\,Kg}{4\,m}\right)$$

$$V_T = -1'33\ 10^{-10}\ \frac{J}{Kg}$$

unidades

$$\frac{N \cdot m^{\cancel{2}}}{Kg^{\cancel{2}}} \cdot \frac{\cancel{Kg}}{\cancel{m}} = \frac{N \cdot m}{Kg} \quad [N \cdot m = J\ (\text{julio})]$$

o traballos $W = F \cdot e\ (N \cdot m)$

c) $W = -\Delta E_p$; $m' = 6\,Kg$

m_1 $x=2$ m_2

$U = E_p = V \cdot m'$

o potencial gravitatorio pola masa é a enerxía potencial

$$W_{x=2}^{\infty} = -\Delta E_p = -\left[\cancel{(E_p)_{\infty}} - (E_p)_{x=2}\right]$$

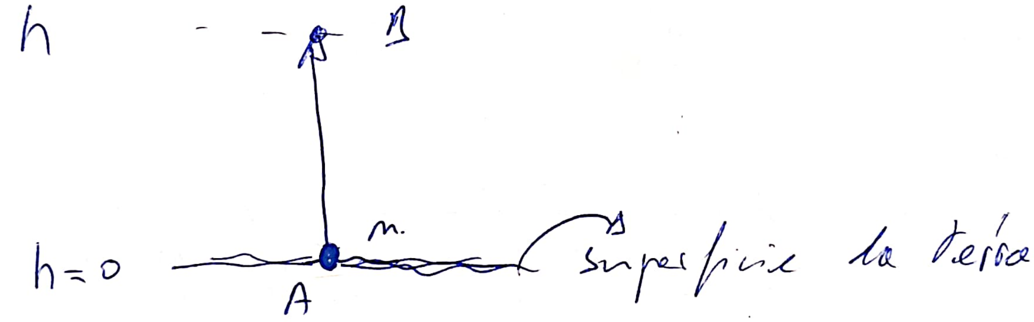
0

$$W_{x=2}^{\infty} = (E_p)_{x=2} = -V_{x=2} \cdot m'$$

$$m' = 6\,Kg$$

$$W_2^{\infty} = -1'33 \cdot 10^{-10} \frac{J}{Kg} \cdot 6\,Kg = -7'98 \cdot 10^{-10}\,J$$

interpretación do signo, Dependendo de si o signo do traballo é positivo o negativo o traballo é realizado polo campo ou contra o campo.



Si subimos unha masa do punto A o punto B, temos que facelo co noso traballo, imos ver o signo

$$W_A^B = -\Delta E_p = -(U_B - U_A) = -\left[-\frac{M.m.}{R_T+h} - \left(-\frac{G\,Mm}{R_T} \right) \right]$$

$$\left| \frac{Mm.}{R_T+h} \right| < \left| \frac{GMm.}{R_T} \right|$$

$W_A^B = -$ negativo, feito contra o campo

8.- $h = 350\,Km = 3'5\,10^5 m.$

a) v?

b) T?

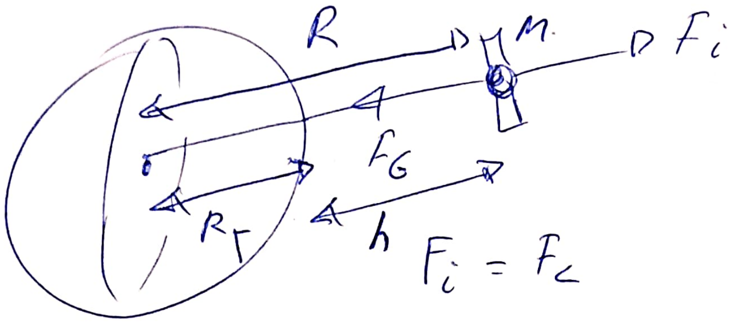
c) comparar a_c con g_T

DATOS

$r_T = 6'37\,10^6 m.$

$g_0 = g_T = 9'81 \frac{m}{s^2}$

a)



$R = 3'5\,10^5 m + 6'37\,10^6 m$

$R = 6'72 . 10^6 m.$

$F_i = F_c$

$$G \frac{M_T \cancel{m}}{R^2} = \cancel{m} \frac{v^2}{\cancel{R}} \Rightarrow v^2 = \frac{G M_T}{R}$$

G en función de g_0; $g_0 = g_T$

$$g_0 = G \frac{M_T}{R_T^2} \Rightarrow G = \frac{g_0 R_T^2}{M_T}$$

$$v^2 = \frac{g_0 R_T^2 \cancel{M_T}}{R \cancel{M_T}} ; \quad v = \left(\frac{9'81 m/s^2 . (6'37\,10^6 m)^2}{6'72\,10^6 m} \right)^{1/2}$$

$$v = 7'7 . 10^3 m/s$$

b)

$$v = w . R \Rightarrow v = \frac{2\pi}{T} . R \Rightarrow T = \frac{2\pi R}{v}$$

$$T = \frac{2 . \pi . 6'72 . 10^6 m}{7'7 . 10^3 \frac{m}{s}} = 5'45\,10^3 s$$

c) $$a_c = \frac{v^2}{R} = \frac{(7'7\,10^3 m/s)^2}{6'72\,10^6 m} = 8'82\, m/s^2$$

9.- $\frac{2 \text{ voltas}}{24 h}$

DATOS

$G = 6'67 \, 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$M_T = 5'98 \, 10^{24} \, Kg$

$R_T = 6'37 \cdot 10^6 \, m$

$m_{sate} = 150 \, Kg$

a) h?

b) Em?

c) T? para 2h.

a)

$$w = \frac{2 \text{ voltas}}{24 h} \cdot \frac{2\pi \text{ rad}}{1 \text{ volta}} \cdot \frac{1 h}{3600 s} = 1'45 \cdot 10^{-4} \frac{rad}{s}$$

$$G \frac{M_T \cancel{m}}{R^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{R}} \; ; \; G \frac{M_T}{R} = w^2 R^2$$

$$R^3 = \frac{G M_T}{w^2}$$

Comprobamos se esta ben despexado, UNIDADES

$$\frac{G M_T}{w^2} = \frac{\frac{N \cdot m^2}{Kg^{\cancel{2}}} \cdot \cancel{Kg}}{s^{-2}} = \frac{N \cdot m^2}{Kg \, s^{-2}} = \frac{\cancel{Kg} \, m \cdot \cancel{s^{-2}} \cdot m^2}{\cancel{Kg} \, \cancel{s^{-2}}} = m^3$$

R^3

$$R = \left(\frac{6'67 \cdot 10^{-11} N \cdot m^2 / Kg^2 \cdot 5'98 \, 10^{24} Kg}{\left(1'45 \cdot 10^{-4} \frac{rad}{s}\right)^2} \right)^{1/3}$$

$R = 2'67 \cdot 10^7 \, m.$

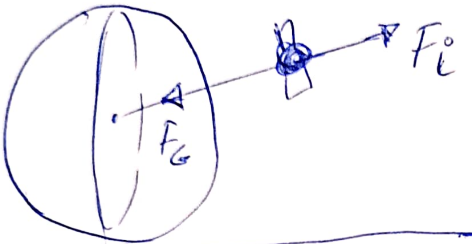
$h = R - R_T = 2'03 \cdot 10^7 \, m.$

b)

$$E_m = E_c + E_p$$

$$E_c = \frac{1}{2} m v^2 \ ; \ E_p = U = -G\frac{M\cdot m}{r}$$

para un obxecto que xira en torno a outro



$$G\frac{Mm}{R^{\cancel{2}}} = m\frac{v^2}{\cancel{R}}$$

$$mv^2 = G\frac{Mm}{R}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}G\frac{M\cdot m}{R}$$

$$E_m = E_c + E_p = \frac{1}{2}mv^2 - G\frac{Mm}{R}$$

$$E_m = \frac{1}{2}G\frac{Mm}{R} - G\frac{M\cdot m}{R} = -\frac{1}{2}G\frac{Mm}{R}$$

$$E_m = -\frac{1}{2}\,6'67\cdot10^{-11}\frac{N\cdot m^2}{Kg^2}\,\frac{5'98\cdot10^{24}Kg\cdot150Kg}{(2'67\cdot10^7 m}$$

$$E_m = -1'12\,10^9\,J$$

c) $$h' = 2\times2'67\cdot10^7 m = 5'34\cdot10^7 m$$

$$R = R_T + h' = 5'98\cdot10^7 m.$$

$$G\frac{M_T m}{R^2} = m\frac{v^2}{R} \ ; \ G\frac{M_T}{R} = \frac{4\pi^2}{T^2}\cdot R^2 \ ; \ T^2 = \frac{4\pi^2R^3}{GM_T}$$

$$T = \left(\frac{4\pi^2(5'98\cdot10^7 m)^3}{6'67\cdot10^{-11}\frac{N\cdot m^2}{Kg^2}\cdot5'98\,10^{24}Kg}\right)^{1/2} = 1'45\,10^5 s$$

$T = 40'41\,h.$

10.-

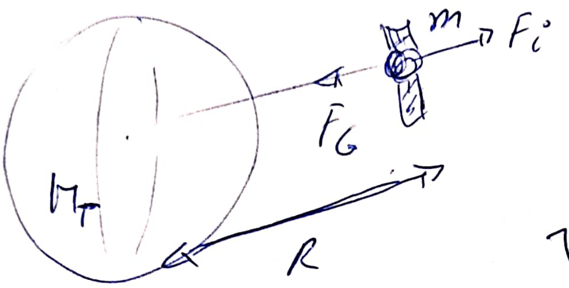
$R = 2R_T$

DATOS

$R_T = 6370\ km = 6'37 \cdot 10^6 m$

$g = 9'81\ m/s$; $g_T = g_0 = 9'81 \frac{m}{s^2}$

a) v?



$F_G = F_i$

$\cancel{m} \frac{v^2}{R} = G \frac{M_T \cancel{m}}{R^2}$

$v^2 = \frac{G M_T}{R}$

$g_T = G \frac{M_T}{R_T^2} \Rightarrow v^2 = \frac{G M_T}{2R_T}$

$G = \frac{g_T R_T^2}{M_T}$

$v^2 = \frac{g_T R_T^{\cancel{2}}}{\cancel{M_T}} \frac{\cancel{M_T}}{2\cancel{R_T}} = \frac{g_T R_T}{2}$

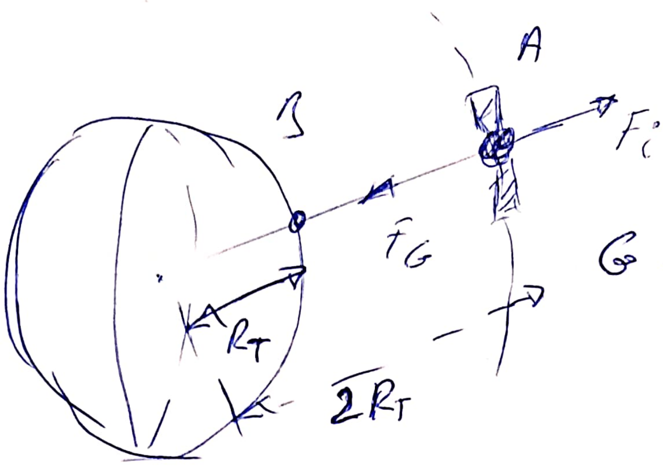
$v^2 = \frac{9'81\ m/s^2 \cdot 6'37 \cdot 10^6 m}{2}$; $v^2 = 3'124 \cdot 10^6 \frac{m}{s^2}$

$v = 5'589 \cdot 10^3 m/s$

b) $g_{2R} = G \frac{M_T}{(2R_T)^2}$; $g_{2R} = \frac{g_T}{4} = \frac{9'81\ m/s^2}{4}$

$g_{2R} = 2'45 \frac{m}{s^2}$

c)



$$G\frac{M}{R^2} = m\frac{v^2}{R}$$

Enerxía A = Enerxía B

$$(E_c)_A + (E_p)_A = (E_c)_B + (E_p)_B$$

$$\frac{1}{2}\cancel{m}v_A^2 - G\frac{M\cancel{m}}{2R_T} = \frac{1}{2}\cancel{m}v_B^2 - G\frac{M\cancel{m}}{R_T}$$

$$\frac{1}{2}v_A^2 - G\frac{M}{2R_T} = \frac{1}{2}v_B^2 - G\frac{M}{R_T}$$

$$v_B^2 = 2\left[\frac{1}{2}v_A^2 + \frac{GM}{R_T}\left(-\frac{1}{2}+1\right)\right] \Rightarrow -\frac{1}{2}+1 = +\frac{1}{2}$$

$$v_A^2 = v_B^2 + \frac{GM}{R_T} \Rightarrow G = \frac{g_T R_T^2}{M_T}$$

$$v_A^2 = (40\,m/s)^2 + \frac{g_T R_T^{\cancel{2}}}{\cancel{M_T}}\frac{\cancel{M_T}}{\cancel{R_T}}$$

$$v_A^2 = (40\,m/s)^2 + 9'81\,m/s^2 \cdot 6'37 \cdot 10^6\,m$$

$$v_A^2 = 6'25\,10^6\,m^2/s^2 \; ; \; v_A = 7'905\,10^3\,m/s$$

11.- $m = 10^2\ kg$

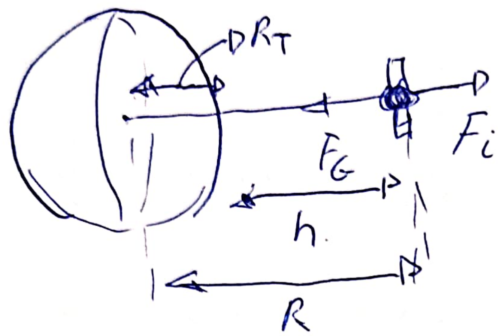
$h = 4 \cdot 10^3\ Km = 4 \cdot 10^6\ m$

DATOS

$R_T = 6'37 \cdot 10^6\ m$

$g_0 = 9'81\ \frac{m}{s^2}$

a) v? ; T? ; g



$|F_G| = |F_i|$

$R = 6'37 \cdot 10^6\ m + 4 \cdot 10^6\ m = 1'037 \cdot 10^7\ m$

$G \frac{M_T \cancel{m}}{R^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{R}}$; $v^2 = \frac{G\ M_T}{R}$

$g_T = G \frac{M_T}{R_T^2} \Rightarrow G = \frac{g_T R_T^2}{M_T} \longrightarrow v^2 = \frac{g_T R_T^2}{\cancel{M_T}} \cdot \frac{\cancel{M_T}}{R}$

$v = R_T \left(\frac{g_T}{R}\right)^{1/2} \Rightarrow v = 6'37 \cdot 10^6\ m \left(\frac{9'81\ m/s^2}{1'037 \cdot 10^7\ m}\right)^{1/2} = 6196\ \frac{m}{s}$

$v = \omega \cdot R \Rightarrow v = \frac{2\pi}{T} \cdot R \Rightarrow T = \frac{2 \cdot \pi R}{v} = \frac{2 \cdot \pi \cdot 1'037 \cdot 10^7 m}{6196\ m/s}$

$T = 10.525\ s$ (2 h, 55 min)

$a_c = g = G \frac{M_T}{R^2} = \frac{g_T R_T^2}{\cancel{M_T}} \cdot \frac{\cancel{M_T}}{R^2} = g_T \left(\frac{R_T}{R}\right)^2 = 9'81\ \frac{m}{s^2} \left(\frac{6'37 \cdot 10^6}{1'037 \cdot 10^7}\right)^2$

$a_c = g = 3'70\ m/s^2$

b) $\vec{L} = \vec{r} \wedge \vec{p}$; $|\vec{L}| = |\vec{r}| \cdot |\vec{p}| \cdot \text{sen}\ 90°$

$|\vec{L}| = |\vec{r}|\ |\vec{p}| = r \cdot mv = R \cdot m \cdot v = 1'037 \cdot 10^7 m \cdot 10^2 kg \cdot 6196\ \frac{m}{s}$

$|\vec{L}| = 6'425 \cdot 10^{12}\ kg\ \frac{m^2}{s}$

12.- $v = 7'62 \frac{Km}{s}$

a) h?

b) T?

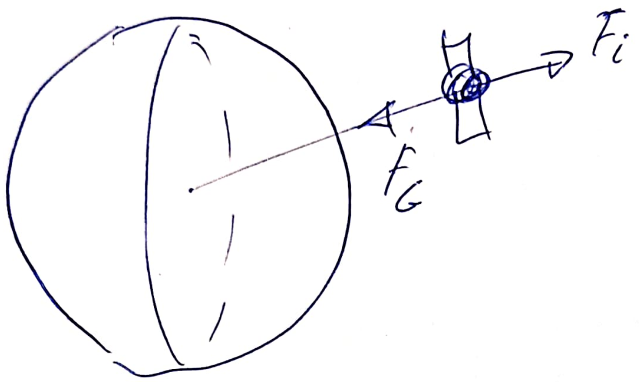
c) voltas en 24 horas

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$R_T = 6370\ Km$

$M_T = 5'93 \cdot 10^{24}\ Kg$

a)



$F_G = F_i$; $G \frac{M_T \cancel{m}}{R^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{R}}$

$$R = \frac{G M_T}{v^2} = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{5'93 \cdot 10^{24}\ Kg}{\left(7620 \frac{m}{s}\right)^2}$$

$R = 6'812 \cdot 10^6\ m$

$h = R - R_T = 6'812 \cdot 10^6 m - 6'370 \cdot 10^6 m = 4'42 \cdot 10^5 m$

b) $V = \omega \cdot R \Rightarrow V = \frac{2\pi}{T} \cdot R$

$$T = \frac{2\pi R}{V} = \frac{2 \cdot \pi \cdot 6'812 \cdot 10^6 m}{7620 \frac{m}{s}} = 5617\ s\ (1h, 33 minutos)$$

c) $f = \frac{1}{T} = 1'78 \cdot 10^{-4} \frac{1}{\cancel{s}} \cdot 24 \times 3600 \cancel{s} = 15'4$ voltas

13.- $m = 500\ kg$

$h = 5000\ Km.$ ⟶ $5 \cdot 10^6\ m$

DATOS

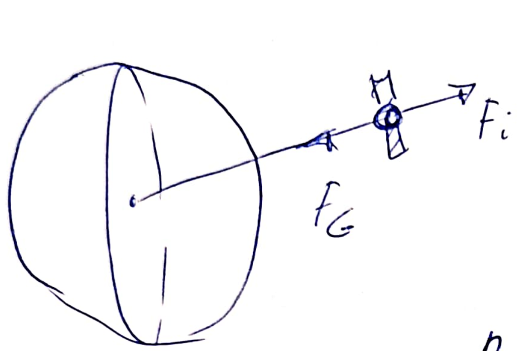
$R_T = 6370\ Km. = 6'37 \cdot 10^6\ m.$

$g_0 = 9'8\ ms^{-2}$

a) v?

b) E_m?

c) Enerxía para acadar o infinito



$$|\vec{F}_G| = |\vec{F}_i|$$

$$G\frac{M_T \cancel{m}}{R^2} = \frac{\cancel{m} v^2}{\cancel{R}}$$

$$R = 6'37 \cdot 10^6 m + 5 \cdot 10^6 m = 1'14 \cdot 10^7 m.$$

$$v^2 = \frac{G M_T}{R} \quad ; \quad g_0 = \frac{G M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$$

$$v^2 = \frac{g_0 R_T^2}{R} \; ; \; v = \sqrt{\frac{g_0 R_T^2}{R}} = \left(\frac{9'8 m/s^2 . (6'37 \cdot 10^6 m)^2}{1'14 \cdot 10^7 m}\right)^{1/2}$$

$$v = 5906\ m/s$$

b)

$$E_m = E_c + U = \frac{1}{2} m v^2 - G\frac{Mm}{R}$$

tendo en conta $G\frac{M_T m}{R^2} = \frac{m v^2}{\cancel{R}}$

$$m \cdot v^2 = G\frac{M_T m}{R}$$

$$\Rightarrow E_m = \frac{1}{2}\frac{G M_T m}{R} - G\frac{Mm}{R} = -\frac{1}{2} G\frac{Mm}{R}$$

$$E_m = -\frac{1}{2}\frac{G M_T m}{R} \; ; \; g_0 = \frac{G M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$$

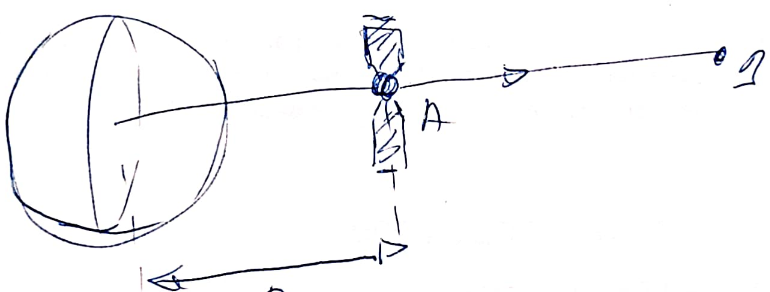
$$E_m = -\frac{1}{2}\frac{g_0 R_T^2 \cdot m}{R}$$

$$E_m = -\frac{1}{2}\frac{9'81\, m/s^2\, (6'37 \cdot 10^6 m)^2\, 500\, Kg}{1'14\, 10^7 m} = -8'73\, 10^4\, J$$

unidades $= \dfrac{\frac{m}{s^2} \cdot m^2 \cdot Kg}{\frac{m}{1}}$ $\Big\}$ $Kg \dfrac{m}{s^2} = N$

$$\frac{N \cdot m^2}{\cancel{m}} = N \cdot m = J$$

c)



Enerxia en A = Enerxia ∞

$$\frac{1}{2} m v_A^2 - \frac{G M_T m}{R} = \frac{1}{2} m \underbrace{v_\infty^2}_{0} - \underbrace{\frac{G M_T m}{\infty}}_{0}$$

igual o do apartado B

a enerxia que hai que facer $8'73\, 10^4\, J$

14.-

$113 \text{ Km} = 1'13\ 10^5 m$

a) periodo da orbita

b) v e w

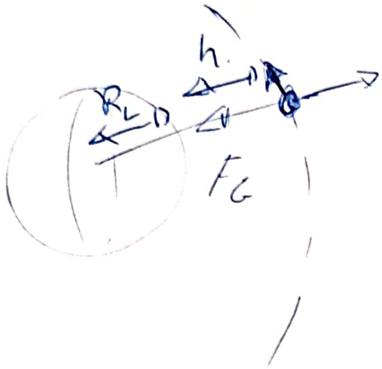
c) v_e

DATOS:

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$R_L = 1740 \text{ Km} = 1'740\ 10^6 m$

$M_L = 7'36 \cdot 10^{22} Kg$



$$G \frac{M_L m}{R^2} = m \frac{v^2}{R}$$

$R = 1'740\ 10^6 m + 1'13\ 10^5 m =$

$$v^2 = \frac{G M_L}{R} = \frac{6'67 \cdot 10^{-11} N \cdot m^2/Kg^2 \cdot 7'36 \cdot 10^{22} Kg}{1'853 \cdot 10^6 m}$$

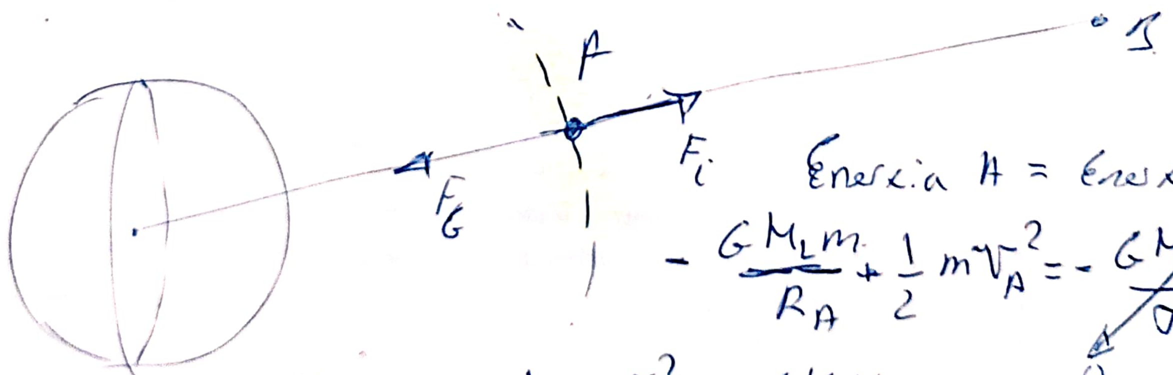
$v^2 = 2'64 \cdot 10^6 \frac{m^2}{s^2}$; $v = 1624 \frac{m}{s}$

$$v = w \cdot R \Rightarrow w = \frac{v}{R} = \frac{1624 m/s}{1'853\ 10^6 m} = 8'76 \cdot 10^{-4} \frac{rad}{s}$$

$$w = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{w} = \frac{2\pi \text{ rad}}{8'76 \cdot 10^{-4} \frac{rad}{s}} = 7165 s$$

$T = 2h$.

c)



Enerxia A = Enerxia B

$$-\frac{G M_L m}{R_A} + \frac{1}{2} m v_A^2 = -\frac{G M_L m}{\infty} + \frac{1}{2} m v_B^2$$

0

$$v_B = 0 \Rightarrow \frac{1}{2} \cancel{m} v_A^2 = + \frac{G M_L \cancel{m}}{R_A}$$

$$v_A = \sqrt{\frac{6'67 \cdot 10^{-11} N \cdot m^2/Kg \cdot 7'36 \cdot 10^{22} Kg \cdot 2}{1'853\ 10^6 m}} = 2302 \frac{m}{s}$$

15.-

$T_c = 4'60$ anos
$M_c = 9'43\,10^{20}$ Kg
$R_c = 477$ Km $= 4'77\,10^5$ m

DATOS
$G = 6'67.10^{-11} \frac{N.m^2}{Kg^2}$

a) g_c??

b) $m_{nave} = 1000$ Kg Enerxia de escape.

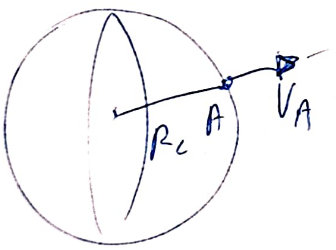
c) $d_{ceres-sol}$? DATO $d_{Terra-Sol} = 1'50\,10^{11}$ m; $T_{TERRA} = 1$ ano

a)

$$g_c = G\frac{M_c}{R_c^2} = 6'67.10^{-11}\frac{N.m^2}{Kg^2}\cdot\frac{9'43\,10^{20}Kg}{(4'77.10^5m)^2} =$$

$$g_c = 0'28\ \frac{m}{s^2}$$

b)



$$\frac{1}{2}m v_A^2 - G\frac{M_c m}{R_c} = \frac{1}{2}m v_B^2 - G\frac{M_c m}{R}$$

0 0

$$E_c = G\frac{Mm}{C^2} = \frac{G M_c m}{R_c} = 6'67.10^{-11}\frac{N.m^2}{Kg^2}\,\frac{9'43\,10^{20}Kg.\,10^3Kg}{4'77.10^5m} =$$

Enerxia $E = 1'32.10^8$ J

c) $$\frac{T_c^2}{R_c^3} = \frac{T_T^2}{R_T^3} \Rightarrow R_c^3 = \frac{T_c^2}{T_T^2}\cdot R_T^3 \Rightarrow R_c = R_T\left(\frac{T_c}{T_T}\right)^{2/3}$$

$$R_c = 1'5\,10^{11}m.\left(\frac{4'6}{1}\right)^{2/3} = 4'11.10^5 \text{ anos}$$

mellor utilizar d. para o radio de xiro dos planetas

16.-

$m_A = m_B = 150\ Kg$ $\quad m_A\ A(0,0)$

$m_B\ B(12,0)$

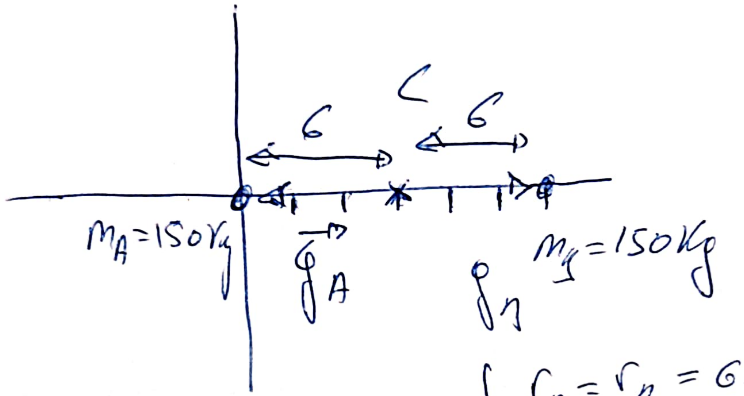
$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

a) g en $C(6,0)$ e D en $(6,8)$

b) $m = 2\ Kg$ $\quad \vec{V}_D = -4 \cdot 10^{-4} \vec{j}\ (m/s)$; V_C?

c) $C \rightarrow D$ mru? mru?

a)



$$|\vec{g}_A| = |\vec{g}_B| = G \frac{m}{r^2} \quad \begin{cases} r_A = r_B = 6m \\ m_A = m_B = 150\ Kg \end{cases}$$

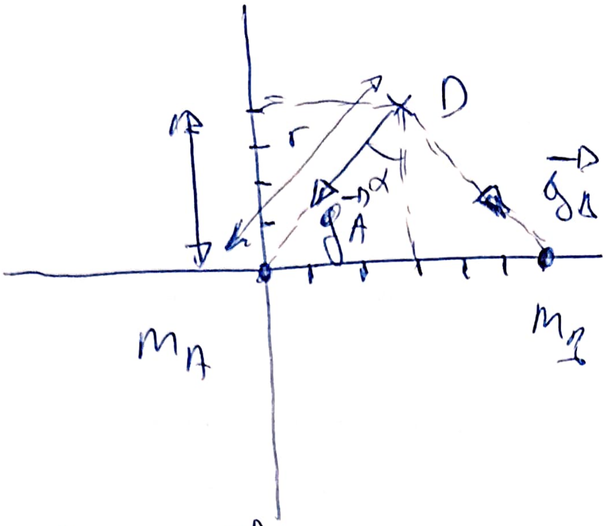
g en C cero

calculemos o seu valor

$$|\vec{g}_A| = |\vec{g}_B| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{150Kg}{(6m)^2} = 2'78\ 10^{-10} \frac{N}{Kg}$$

$$\vec{g}_A = -2'78 \cdot 10^{-10}\ \vec{i}\ N/Kg$$

$$\vec{g}_B = 2'78 \cdot 10^{-10}\ \vec{i}\ N/Kg$$



$$r^2 = (6m)^2 + (8m)^2 = 100\,m^2$$

$$r = 10\,m.$$

$$|\vec{g}_A| = |\vec{g}_B| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2} \cdot \frac{150\,kg}{100\,m^2} = 1 \cdot 10^{-10} \frac{N}{kg}$$

$$sen\,\alpha = \frac{6}{10} = 0'6$$

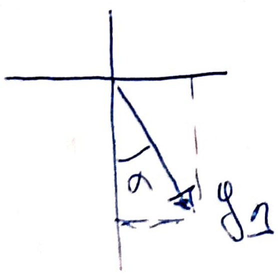
$$cos\,\alpha = \frac{8}{10} = 0'8$$

$$sen\,\alpha = \frac{(g_A)_x}{|g_A|} \Rightarrow (g_A)_x = |g_A| \cdot sen\,\alpha$$

$$cos\,\alpha = \frac{(g_A)_y}{|g_A|} \Rightarrow (g_A)_y = |g_A| \cdot cos\,\alpha$$

$$(g_A)_x = 1 \cdot 10^{-10} \frac{N}{kg} \cdot 0'6 = 6 \cdot 10^{-11}\, N/kg$$

$$(g_A)_y = 1 \cdot 10^{-10} \frac{N}{kg} \cdot 0'8 = 8 \cdot 10^{-11}\, N/kg$$



$$sen\,\alpha = \frac{(g_B)_x}{|g_B|} \Rightarrow (g_B)_x = |g_B| \cdot sen\,\alpha$$

$$cos\,\alpha = \frac{(g_B)_y}{|g_B|} \Rightarrow (g_B)_y = |g_B| \cdot cos\,\alpha$$

$$\vec{g}_A = -(g_A)_x\,\vec{i} - (g_A)_y\,\vec{j}$$

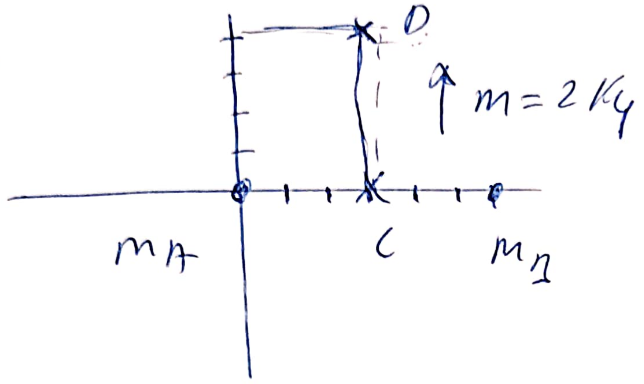
$$\vec{g}_B = (g_B)_x\,\vec{i} - (g_B)_y\,\vec{j}$$

$\vec{g}_A = -6\cdot10^{-11}\,\vec{i} - 8\cdot10^{-11}\,\vec{j}\ \frac{N}{Kg}$

$\vec{g}_B = 6\cdot10^{-11}\,\vec{i} - 8\cdot10^{-11}\,\vec{j}\ \frac{N}{Kg}$

$g_T = 16\cdot10^{-11}\,\vec{j}\ N/Kg$

b) V_C?



enerxia C = Enerxia D

$(E_p)_C + (E_c)_C = (E_p)_D + (E_c)_D$

$(E_p)_C = -G\frac{m_A m}{r_{AC}} - G\frac{m_B m}{r_{BC}}$

enerxia potencial creadas por la masa A e B en el punto C cuando esta la masa m.

$(E_p)_D = -\frac{G m_A m}{r_{AD}} - G\frac{m_B m}{r_{BD}}$

$\frac{1}{2}v_C^2 - \left(\frac{G m_A}{r_{AC}} + \frac{G m_B}{r_{BC}}\right) = \frac{1}{2}v_D^2 - \left(G\frac{m_A}{r_{AD}} + G\frac{m_B}{r_{BD}}\right)$

$\frac{1}{2}v_C^2 - 2\frac{G m_A}{6m} = \frac{1}{2}v_D^2 - 2\frac{G m_A}{10m}$

$\frac{1}{2}v_C^2 = \frac{1}{2}v_D^2 + 2 G m_A\left(\frac{1}{6m} - \frac{1}{10m}\right)$

$v_C^2 = v_D^2 + 4\ 6'67\cdot10^{-11}\frac{N\cdot m^2}{Kg^2}\cdot 150Kg\cdot 6'66\cdot10^{-2}\,m^{-1}$

$v_C^2 = \left(1\cdot10^{-4}\frac{m}{s}\right)^2 + 2'67\cdot^{-9}\frac{m^2}{s}$; $v_C^2 = 12'67\cdot10^{-9}\frac{m^2}{s^2}$

$V_C = 1'13\cdot10^{-4}\,m/s$

19.-

$\begin{cases} t = 5\,10^2 s \\ \text{sol-terra.} \end{cases}$

$t_{sol-xupiter} = 2'6\,10^3 s$

$T_T = 3'1\,10^7 s$

$c = 3.10^8 m/s$

$G = 6'67.10^{-11} \frac{N.m^2}{Kg^2}$

a) T_X ?

b) $(v_0)_X$?

c) M_s ?

$d = v.t \; (e = v.t)$

a) $d_{S-T} = 3.10^8 \frac{m}{s} . 5\,10^2 s = 1'5.10^{11} m.$

$d_{S-X} = 3.10^8 \frac{m}{s} . 2'6.10^3 s = 7'8.\;10^{11} m.$

$\frac{T^2}{R^3} = cte \quad ; \quad \frac{T_T^2}{d_T^3} = \frac{T_X^2}{d_X^3}$

$T_X^2 = \frac{d_X^3}{d_T^3} . T_T^2 \quad ; \quad T_X = T_T \left(\frac{d_X}{d_T}\right)^{1'5}$

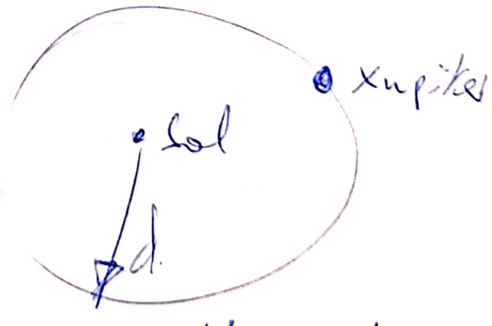
$T_X = 3'1\,10^7 s \left(\frac{7'8.10^{11}}{1'5.\,10^{11}}\right)^{1'5} = 3'68.10^8 s$

b)

$v = \omega . r$

$\downarrow$

d



$v = \frac{2\pi}{T} . d. = \frac{2\pi}{3'68.10^8 s} . 7'8.10^{11} m. = 1'33\,10^4 \frac{m}{s}$

c)

$F_G = F_i$ suponiendo orbitas circulares

$$G \frac{M_s \cancel{m_x}}{d_x^2} = \cancel{m_x} \frac{v_x^2}{\cancel{d_x}}$$

$$M_s = \frac{v_x^2 \cdot d_x}{G} = \frac{(1'33 \cdot 10^4 \, m/s)^2 \cdot 7'8 \cdot 10^{11} \, m}{6'67 \cdot 10^{-11} \, \frac{N \cdot m^2}{Kg^2}}$$

$$M_s = 2'07 \cdot 10^{30} \, Kg$$

PROBLEMAS

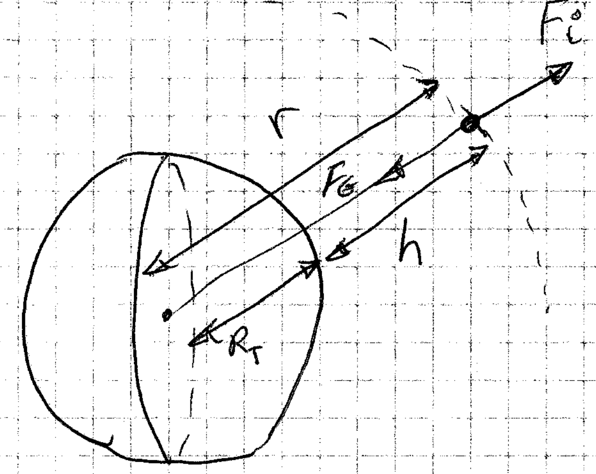
26.- $T = 24h \cdot \frac{3600s}{1h} = 86.400s$

Datos: $R_T = 6'38 \cdot 10^6 m$; $M_s = 8\,10^2 Kg$

$M_T = 5'98\,10^{24} Kg$

$G = 6'67 \cdot 10^{-11} N \cdot m^2 \cdot Kg^{-2}$

a)



$F_G = F_i$

$$G\frac{M_T m}{r^2} = m\frac{v^2}{r}$$

$$\Downarrow$$

$$G\frac{M_T}{r^2} = \frac{v^2}{r}$$

$$G\frac{M_T}{r^2} = \frac{\omega^2 r^2}{r} \Rightarrow G\frac{M_T}{r^2} = \frac{4\pi^2}{T^2}r$$

$$r^3 = \frac{G M_T T^2}{4\pi^2} ; \quad r = \left(\frac{6'67 \cdot 10^{-11} N \cdot m^2 \cdot Kg^{-2} \cdot 5'98\,10^{24} Kg \cdot (86400\,s)^2}{4\pi^2}\right)^{1/3}$$

$r = 4'23\,10^7 m$; $h = r - R_T = 3'59 \cdot 10^7 m$

b) $$F = G\frac{M \cdot m}{r^2} = m\frac{v^2}{r}$$

$$F = G\frac{M \cdot m}{r^2} = 6'67 \cdot 10^{-11}\frac{N\,m^2}{Kg^2}\,\frac{5'98\,10^{24} Kg \cdot 800 Kg}{(4'23\,10^7 m)^2} =$$

$= 178'3 N$

c) $E_m = E_c + E_p$

Non foi pulla calcular E_c e E_p, sempre que se trate dun obxecto que xira nunha orbita podemos facer o seguinte

$$F_G = F_i \quad ; \quad G\frac{M \cdot m}{r^{\cancel{2}}} = m\frac{v^2}{\cancel{r}}$$

$$G\frac{Mm}{r} = m \cdot v^2$$

$$E_c = \frac{1}{2}mv^2 = \frac{1}{2}\,\frac{GM \cdot m}{r}$$

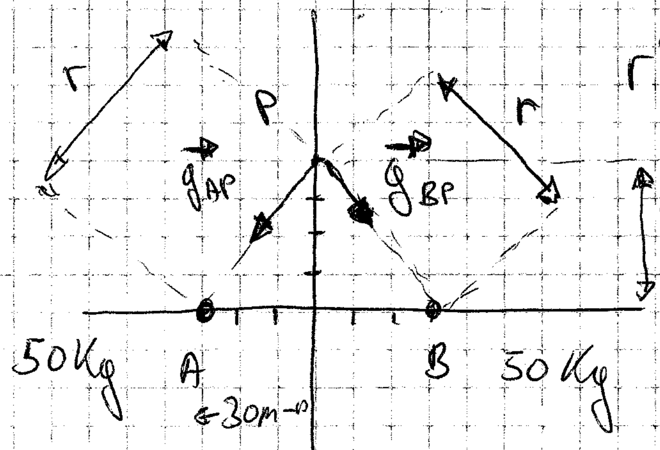
$$E_p = -G\frac{M \cdot m}{r}$$

$$E_M = \frac{1}{2}\,\frac{GM \cdot m}{r} - \frac{GMm}{r} = -\frac{1}{2}\,\frac{GMm}{r}$$

$$E_M = -\frac{1}{2}\,6'67 \cdot 10^{-11}\frac{N \cdot m^2}{kg^2} \cdot \frac{5'98\,10^{24}\,kg \cdot 800\,kg}{4'23\,10^{7}\,m}$$

$$E_m = -3'77 \cdot 10^{9}\ J$$

27.-

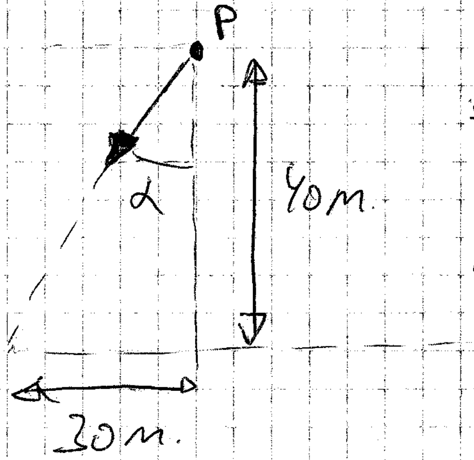


$r^2 = (30m)^2 + (40m)^2$

$r^2 = 2500m^2$

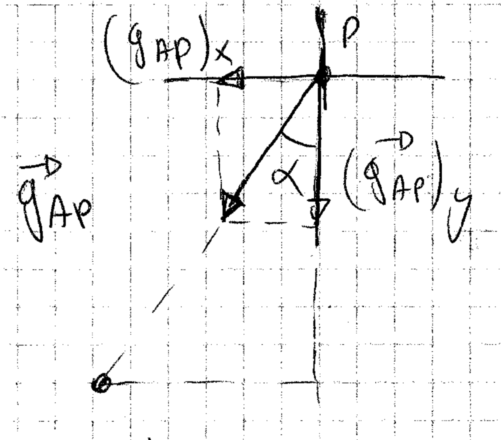
$r = 50m$

$$|\vec{g}_{AP}| = |\vec{g}_{BP}| = G\frac{M}{r^2} = 6'67\,10^{-11}\frac{N.m^2}{Kg^2}\,\frac{50Kg}{2500m^2} = 1'33\,10^{-12}\frac{N}{Kg}$$



$$\operatorname{sen}\alpha = \frac{30m}{50m} = \frac{3}{5} = 0'6$$

$$\cos\alpha = \frac{40m}{50m} = \frac{4}{5} = 0'8$$

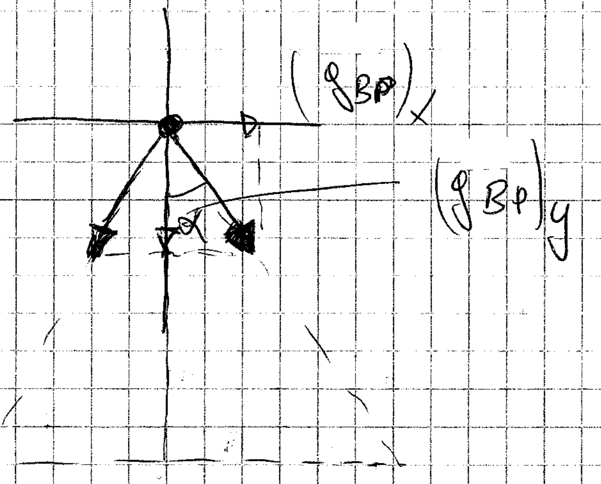


$$\operatorname{sen}\alpha = \frac{(g_{AP})_x}{|\vec{g}_{AP}|}$$

$$\cos\alpha = \frac{(g_{AP})_y}{|\vec{g}_{AP}|}$$

$$\vec{g}_{AP} = -1'33\,10^{-12}\cdot 0'6\,\vec{i} - 1'33\,10^{-12}\cdot 0'8\,\vec{j}$$

$$\vec{g}_{AP} = -7'98.10^{-13}\,\vec{i} - 1'06\,10^{-12}\,\vec{j}\ (N/Kg)$$



$$\vec{g}_{BP} = |\vec{g}_{BP}| \cdot \text{sen}\alpha \, \vec{i} - |\vec{g}_{BP}| \cdot \cos\alpha \, \vec{j}$$

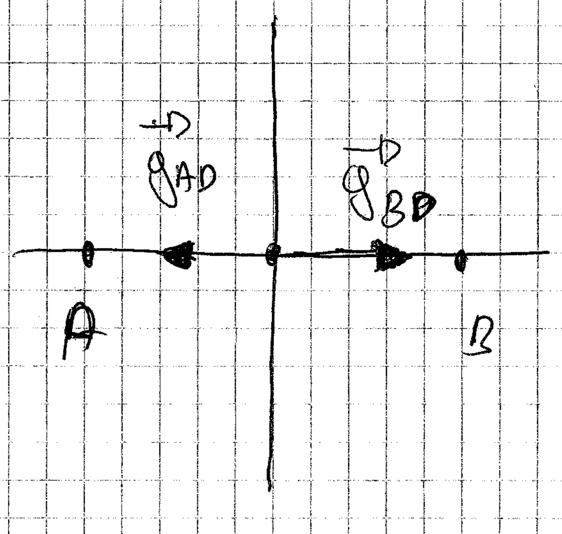
$$\vec{g}_{BP} = 1'33 \cdot 10^{-12} \, 0'6 \, \vec{i} - 1'33 \cdot 10^{-12} \, 0'8 \, \vec{j} \; (N/Kg)$$

$$\vec{g}_{BP} = 7'98 \cdot 10^{-13} \, \vec{i} - 1'06 \, 10^{-12} \, \vec{j} \; (N/Kg)$$

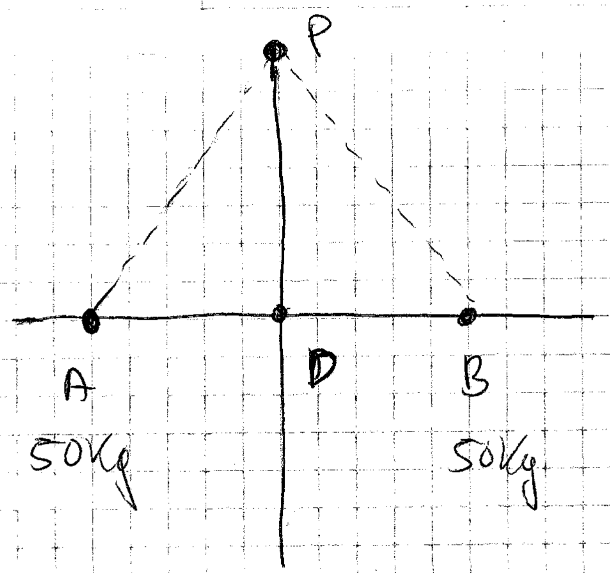
$$\vec{g}_P = \vec{g}_{AP} + \vec{g}_{AP} = 2 \left(-1'06 \, 10^{-12} \, \vec{j} \right) (N/Kg)$$

$$\vec{g}_P = -2'12 \cdot 10^{-12} \; (N/Kg)$$

No punto O(0,0)



c)



$$V = -G\frac{m}{r}$$

$V_P = V_{AP} + V_{BP} \Rightarrow$ Distancia a mesma r, a masa tamen e igual

$$V_{PA} = V_{PB} = -6'67\cdot10^{-11}\frac{N\cdot m^2}{Kg^2}\frac{50Kg}{50m} = -6'67\cdot10^{-11}\frac{N\cdot m}{Kg}$$

$$V_P = 2\left(-6'67\cdot10^{-11}\frac{N\cdot m}{Kg}\right) = -1'33\cdot10^{-10}\frac{J}{Kg}$$

$$V_D = V_{AD} + V_{BD}$$

$$V_D = 2\left(-6'67\cdot10^{-11}\frac{N\cdot m^2}{Kg^2}\frac{50Kg}{30m}\right) = -2'22\cdot10^{-10}\, J/Kg$$

28.-

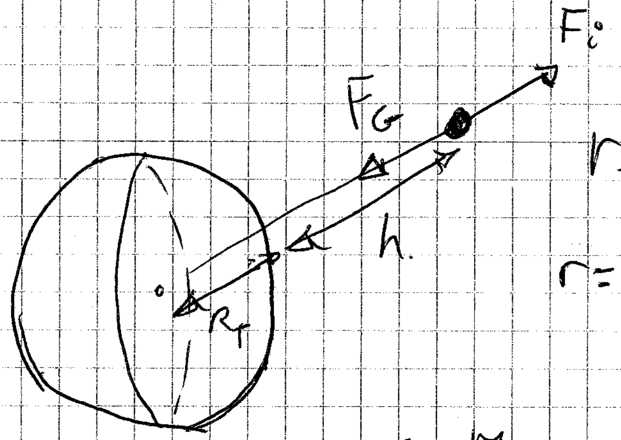
$m = 100\ Kg$

$h = 6000\ Km = 6\,10^6\ m.$

$g_0 = 9'80\ m/s^2$

$R_T = 6.400\ Km = 6'4\,10^6\ m.$

a) T ? b) Peso ?



$r = R_T + h.$

$r = 12'4\,10^6\ m.$

$$G\frac{M_T m}{r^2} = m\frac{v^2}{r}$$

$$G\frac{M_T}{r^{\cancel{2}}} = \frac{v^2}{\cancel{r}}\ ;\quad G\frac{M_T}{r} = w^2 r^2\ ;\quad G\frac{M_T}{r} = \frac{4\pi^2}{T^2}r^2$$

$$T^2 = \frac{4\pi^2 r^3}{G M_T}$$

$$g_0 = \frac{G M_T}{r_T^2} \Rightarrow G M_T = g_0 r_T^2$$

$$T^2 = \frac{4\pi^2 r^3}{g_0 r_T^2}$$

$$T = \left[\frac{4\pi^2(12'4\,10^6\ m.)^3}{9'80\ \frac{m}{s^2}\ (6'4\,10^6\ m)^2}\right]^{1/2} = 1'37\,10^4\ s$$

b)

$$g = \frac{GM_T}{r^2}$$

poñendo esta expresión en función de g_0

$$g_0 = \frac{GM_T}{r_T^2} \Rightarrow g_0 r_T^2 = GM_T$$

$$g = \frac{g_0 r_T^2}{r^2} ; \quad g = \frac{9'80\,m/s^2 \cdot (6'4\,10^6\,m)^2}{(12'4\,10^6\,m)^2} = 2'61\,m/s^2$$

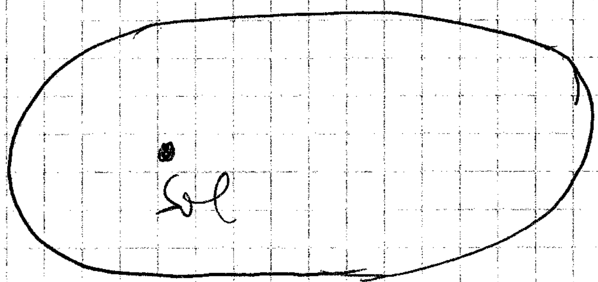
$$P = mg = 100\,Kg \cdot 2'61\,m/s^2 = 261\,N$$

29.-

$T_T = 1$ ano $\quad R_T = 1'5\,10^{11}\,m$

$T_X = 12$ anos $\quad R_X = ?$

$T_N = ?$ $\quad R_N = 4'5\,10^{12}\,m$



$\frac{T^2}{R^3} = cte$ $\quad$ 3ª lei de Kepler

$$\frac{T_T^2}{R_T^3} = \frac{T_X^2}{R_X^3} \rightarrow R_X^3 = \frac{T_X^2}{T_T^2} \cdot R_T^3$$

$$R_X^3 = \frac{T_X^2}{T_T^2} \cdot R_T^3$$

$$R_X = R_T \left(\frac{T_X}{T_T}\right)^{2/3}$$

$$R_X = 1'5\,10^{11}\,m \left(\frac{12\text{ años}}{1\text{ año}}\right)^{2/3} = 7'86 \cdot 10^{11}$$

b)
$$\frac{T_T^2}{R_T^3} = \frac{T_N^2}{R_N^3} \Rightarrow T_N^2 = \frac{R_N^3}{R_T^3} \cdot T_T^2$$

$$T_N = T_T \left(\frac{R_N}{R_T}\right)^{3/2}$$

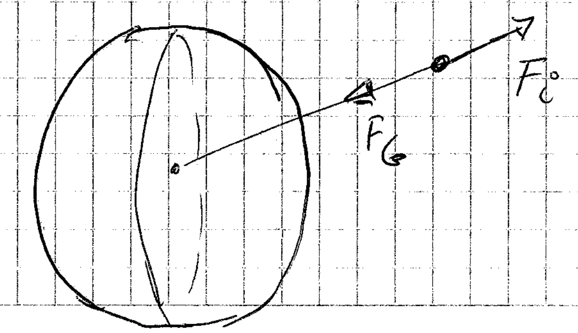
$$T_N = 1\text{ año} \left(\frac{4'5\,10^{12}\,m}{1'5\,10^{11}\,m}\right)^{3/2}$$

$$T_N = 164'3\text{ años}$$

30.-

$m = 64'5\ Kg$ | $R_T = 6370\ Km = 6'370\,10^6\,m$

$R = 2'32\,R_T$ | $g_0 = 9'80\ m/s^2$



a)

$$G\frac{M_T \cancel{m}}{r^2} = \cancel{m}\frac{v^2}{r} \ ; \quad G\frac{M_T}{r^2} = \frac{4\pi^2 r^2}{T^2 \cdot r}$$

$$T^2 = \frac{4\pi^2 r^3}{G M_T}$$

poñer GM_T en función de g_0

$$g_0 = G\frac{M_T}{R_T^2}$$

$$\rightarrow T^2 = \frac{4\pi r^3}{g_0 R_T^2}$$

$$T^2 = \frac{4\pi^2 (2'32 R_T)^3}{g_0 R_T^2} \ ; \quad T^2 = \frac{4\pi^2 (2'32)^3 R_T^{\cancel{3}}}{g_0 \cancel{R_T^2}}$$

$$T = \left(\frac{4\pi^2 (2'32)^3 \; 6'370 \cdot 10^6}{9'80 \, m/s^2}\right)^{1/2} = 17.900 \, s$$

b) $\text{Peso} = m \cdot g$

$$g = G\frac{M_T}{r^2} \ ; \quad g = \frac{g_0 R_T^2}{r^2}$$

$$g = g_0 \frac{\cancel{R_T^2}}{(2'32)^2 \cancel{R_T^2}} \Rightarrow g = 1'82 \, m/s^2$$

$$\text{Peso} = m \cdot g = 64'5 \, kg \cdot 1'82 \frac{m}{s^2} = 117'4 \, N$$

31.-

$M_L = 0'011 M_T$

$R_L = R_T/4$

$(Peso)_T = 980N \quad (g_0 = 9'80\ m.s^{-2})$

a) a masa non depende do campo gravitatorio, é a mesma en todolos puntos do Universo

$$M_T = M_L = \frac{980N}{9'80 m.s^{-2}} = 100\ Kg$$

$$g_L = G\frac{M_L}{R_L^2} \quad ; \quad g_0 = G\frac{M_T}{R_T^2} \Rightarrow G = \frac{g_0 R_T^2}{M_T}$$

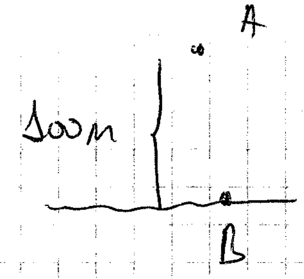
$$g_L = \frac{g_0 R_T^2}{M_T} \cdot \frac{M_L}{R_L^2} \quad ; \quad g_L = \frac{g_0 R_T^2\ 0'011 M_T}{M_T (R_T/4)^2}$$

$$g_L = \frac{g_0 \cancel{R_T^2}\ 0'011 \cancel{M_T}\ 16}{\cancel{M_T} \cancel{R_T^2}} \Rightarrow g_L = 16 \cdot 0'011\ g_0$$

$$g_L = 1'72\ m/s^2$$

$$P_L = 100\ Kg \cdot 1'72\ m/s^2 = 172N$$

b) Dado que a distancia é moi pequena sobre a superficie da Lua podemos utilizar a seguinte expresión para a enerxía potencial.



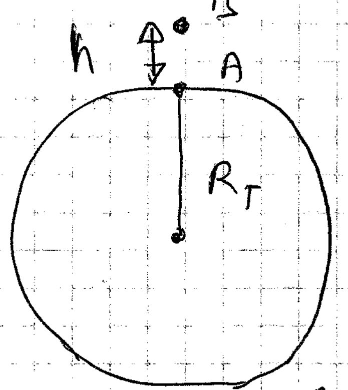
Enerxia A = Enerxia B.

$$(E_p)_A = (E_c)_B$$

$$m g_L h_A = \frac{1}{2} m v_B^{\,} \Rightarrow v_B = \sqrt{2 g_L h_A}$$

$$v_B = (2 \cdot 1'72\ m/s^2 \cdot 100m)^{1/2} = 18'55\ m/s$$

Por que é posible facer esta aproximación



$$R_A = R_T$$

$$R_B = R_T + h.$$

$$(E_p)_B = -G \frac{M_T m}{(R_T + h)} \quad ; \quad (E_p)_A = -G \frac{M_T m}{R_T}$$

$$\Delta E_p = (E_p)_B - (E_p)_A$$

$$\Delta E_p = -G \frac{M_T m}{(R_T+h)} - \left(-G \frac{M_T m}{r_T}\right) = G M_T m \left(\frac{1}{R_T} - \frac{1}{R_T + h}\right)$$

$$= G\, M_T m \left(\frac{\cancel{R_T} + h - \cancel{R_T}}{R_T (R_T + h)}\right) = G \frac{M_T m h}{R_T (R_T + h)}$$

$$R_T (R_T + h) \simeq R_T^2 \quad \text{si } h << R_T$$

$$\Delta E_p = (E_p)_B - (E_p)_A = \frac{G M_T m \cdot h}{R_T^2}$$

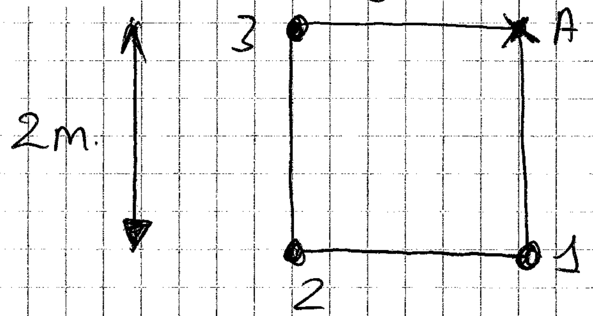
$$g = G\frac{M_T}{R_T^2} \quad ; \quad \Delta E_p = m g h$$

$$(R_T + h) R_T \simeq R_T^2$$

exemplo

$$(6'370 \cdot 10^6 m + 100 m) \cdot 6'370 \cdot 10^6 m = (6'370 \cdot 10^6 m)^2$$

32.- $m = 10 \, Kg$



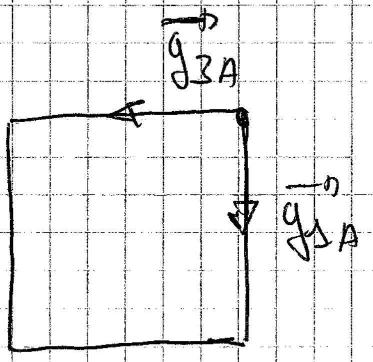
A vértice onde colocalo $\vec{g}$ e V

$$\vec{g} = -G\frac{M}{r^2}\vec{u}_r$$

A distancia de 1, e 3 a A e a mesma.

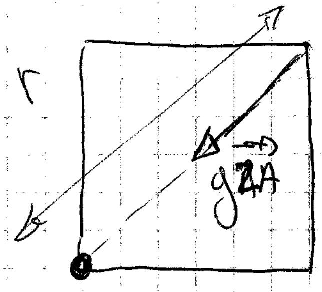
$$|\vec{g}_{3A}| = |\vec{g}_{1A}| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \frac{10 Kg}{(2m)^2}$$

$$= 1'67 \cdot 10^{-10} \frac{N}{Kg}$$



$$\vec{g}_{1A} = -1'67 \cdot 10^{-10} \, \vec{j} \; N/Kg$$

$$\vec{g}_{3A} = -1'67 \cdot 10^{-10} \, \vec{i} \; N/Kg$$

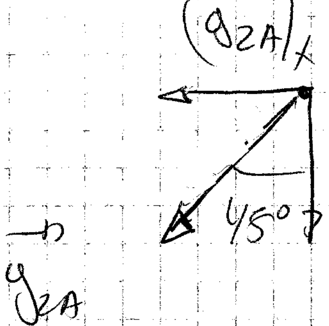


m = 10 Kg

$$r^2 = (2m)^2 + (2m)^2$$

$$r^2 = 8\,m^2 \Rightarrow r = 2'83\,m$$

$$|\vec{g}_{2A}| = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{10\,Kg}{8\,m^2} = 8'34 \cdot 10^{-11} \frac{N}{Kg}$$



$$(g_{2A})_x = (g_{2A})_y = |\vec{g}_{2A}| = 5'90 \cdot 10^{-11}\, N/Kg$$

$$\text{sen}\, 45^\circ = \cos 45^\circ = 0'71$$

$$\vec{g}_{2A} = -5'90 \cdot 10^{-11}\, \vec{i} - 5'90 \cdot 10^{-11}\, \vec{j}\ (N/Kg)$$

$$\vec{g}_2 = \vec{g}_{1A} + \vec{g}_{2A} + \vec{g}_{3A}$$

$$\vec{g}_A = -2'26 \cdot 10^{-10}\, \vec{i} - 2'26 \cdot 10^{-10}\, \vec{j}\ (N/Kg)$$

potencial.

$$V_{3A} = V_{1A} = -G\frac{m}{r} = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{10\,Kg}{2\,m} = -3'34 \cdot 10^{-10}\, J/Kg$$

$$V_{2A} = -G\frac{m}{r'} = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg} \cdot \frac{10\,Kg}{2'83\,m} = -2'36 \cdot 10^{-10}\, J/Kg$$

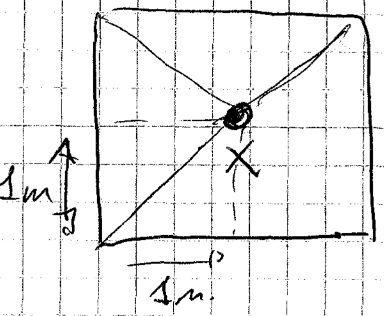
$$V_A = 2 \times (-3'34 \cdot 10^{-10}\, J/Kg) + (-2'36 \cdot 10^{-10}\, J/Kg)$$

$$V_A = -9'04 \cdot 10^{-10}\, J/Kg$$

b) $W_{\infty \rightarrow}$ CENTRO DO CUADRADO

$W_{\infty}^{\cancel{X}} = -(E_{p\cancel{X}} - E_{p\infty})$ $\qquad E_{p\infty} = 0$

$E_{pX} = m' V_X$



$$V_X = 3\left(-G\,\frac{m}{\sqrt{2}\,m}\right)$$

$$V_X = -3 \cdot 6'67 \cdot 10^{-11}\,\frac{N \cdot m^2}{Kg^2} \cdot \frac{10\,Kg}{\sqrt{2}\,m} = -1'41\,10^{-9}\,J/Kg$$

$$W_{\infty}^{X} = -1\,Kg\,(-1'41\,10^{-9}\,J/Kg) = 1'41\,10^{-9}\,J$$

33.-

$$G\frac{M_T}{r^2}=\frac{v^2}{r}\ ;\quad G\frac{M_T}{r}=v^2$$

$$g_0=\frac{G M_T}{r_T^2}\Rightarrow G M_T=g_0 r_T^2$$

$$v^2=\frac{g_0 r_T^2}{r}\ ;\quad v=\left(\frac{9'80\,m/s^2\cdot(6'378\,10^6 m)^2}{3'64\,10^7 m}\right)^{1/2}$$

$$v=3309\ m/s$$

b)

$$E_m=-\frac{1}{2}\,G\frac{M_T m}{r}$$

$$E_m=-\frac{1}{2}\,\frac{g_0 r_T^2 m}{r}\ ,\quad E_m=-\frac{1}{2}\,\frac{9'80\,m/s^2\cdot(6'378\,10^6 m)^2\cdot 300kg}{3'64\,10^7 m}$$

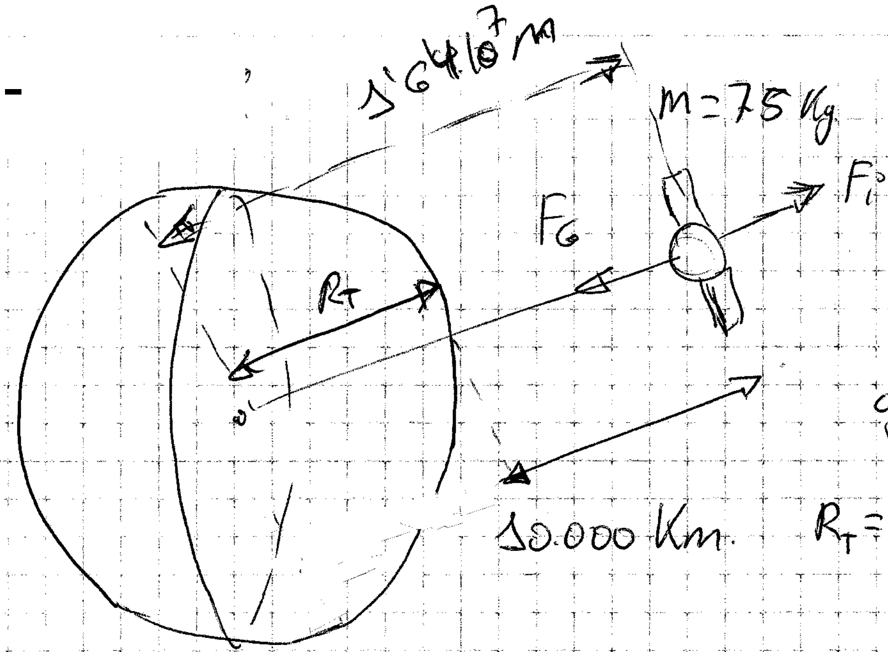
$$E_m=-1'64\,10^9\ J$$

$$E_m=\frac{1}{2}mv^2-G\frac{M\cdot m}{r}$$

$$G\frac{Mm}{r^2}=m\frac{v^2}{r}\Rightarrow m v^2=G\frac{Mm}{r}$$

$$E_m=\frac{1}{2}\,G\frac{Mm}{r}-G\frac{Mm}{r}=-\frac{1}{2}\,G\frac{Mm}{r}$$

34.-



$g_T = 9'80\ m/s^2$

$R_T = 6.400\ Km = 6'4\cdot 10^6\ m$

a) v? ; T? ; b) Peso

$$F_G = F_i$$

$$G\frac{M_T m}{r^2} = m\frac{v^2}{r} \; ; \; G\frac{M_T}{r^2} = \frac{v^2}{r}$$

$$v^2 = \frac{G M_T}{r}$$

$$g_0 = G\frac{M_T}{r_T^2} \Rightarrow g_0 r_T^2 = G M_T$$

$$v^2 = \frac{g_0 r_T^2}{r} \; ; \; v = \left(\frac{9'80\ m/s^2 \cdot (6'4\cdot 10^6 m)^2}{1'64\cdot 10^7}\right)^{1/2}$$

$$v = 4947\ m/s$$

$$v = \omega \cdot r \Rightarrow v = \frac{2\pi}{T}\cdot r \Rightarrow T = \frac{2\pi \cdot r}{v}$$

$$T = \frac{2\cdot\pi\cdot 1'64\cdot 10^7 m}{4947\ m/s} = 20.829\ m$$

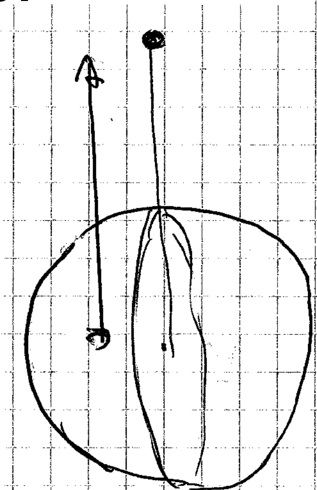
b)

$$Peso = m \cdot g$$

$$Peso = m \, G \frac{M_T}{r^2} \; ; \; Peso = m \cdot \frac{g_0 R_T^2}{r_T^2}$$

$$Peso = 75 Kg \cdot 9'80 m/s^2 \left(\frac{6'4 \cdot 10^6 m}{1'64 \cdot 10^7 m} \right)^2 = 112 N$$

35.-



$$F_G = F_i$$

$$G \frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{r}}$$

$$v^2 = G \frac{M_T}{r}$$

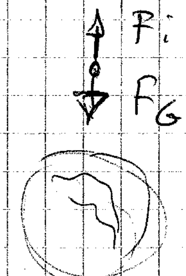
$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$R_T = 6400 \, Km.$

$g_0 = 9'80 \, m/s^2$

$$v^2 = \frac{g_0 R_T^2}{r} \; ; \; v^2 = \frac{9'80 m/s^2 \cdot (6'4 \cdot 10^6 m)^{\cancel{2}}}{2 \cdot \cancel{(6'4 \cdot 10^6 m)}}$$

$$v = 5.600 \, m/s$$



b)

$$Peso = m\,g$$

$$g = G\frac{M_T}{(2R_T)^2}\;;\quad m = \frac{P_T}{g_T}$$

$$g_T = G\frac{M_T}{R_T^2}$$

$$Peso = \frac{P_T}{\cancel{G\frac{M_T}{R_T^2}}}\cdot \cancel{G}\frac{\cancel{M_T}}{4\cancel{R_T^2}} = \frac{5000\,N}{4} = 1250\,N$$

36.-

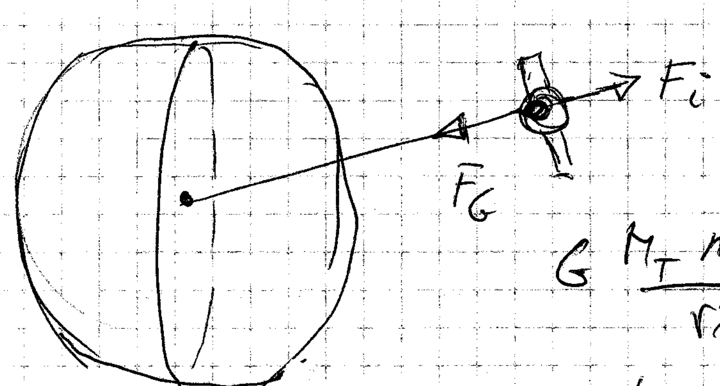
$M = 200\,Kg.$

$v = 10.800\,\frac{Km}{h}\,\frac{1000\,m}{1\,Km}\,\frac{1\,h}{3600\,s} =$

$= 3.000\,m/s$

$g_0 = 9'80\,m/s^2$

$R_T = 6\,370\,Km.$



$$G\frac{M_T\,m}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$g_0 = G\frac{M_T}{R_T^2} \Rightarrow G M_T = g_0 R_T^2$$

$$r = \frac{G\,M_T}{v^2}\;;\quad r = \frac{g_0 R_T^2}{v^2}\;;$$

$$r = \frac{9'80\,m/s^2\cdot(6'37\,10^6\,m)^2}{(3000\,m/s)^2} = 4'42\cdot 10^7\,m.$$

$r = 4'42 \cdot 10^7 m$

$r = R_T + h \Rightarrow h = r - R_T = 4'42 \cdot 10^7 m - 6'370 \cdot 10^6 m$

$h = 3'78 \cdot 10^6 m$

b)

$$E_m = -\frac{1}{2} G \frac{M_T m}{r} \quad ; \quad E_m = E_c + E_p$$

$$E_m = \frac{1}{2} m v^2 - G \frac{M_T m}{r}$$

$$F_G = F_c \quad ; \quad G \frac{M_T m}{r^2} = m \frac{v^2}{r}$$

$$G \frac{M_T}{r} = m v^2 \quad ;$$

$$E_m = \frac{1}{2} G \frac{M_T}{r} - G \frac{M_T m}{r} = -\frac{1}{2} G \frac{M m}{r}$$

$$E_m = E_{TOTAL} = -\frac{1}{2} G \frac{M m}{r} = -\frac{1}{2} \frac{g_0 R_T^2 \cdot m}{r}$$

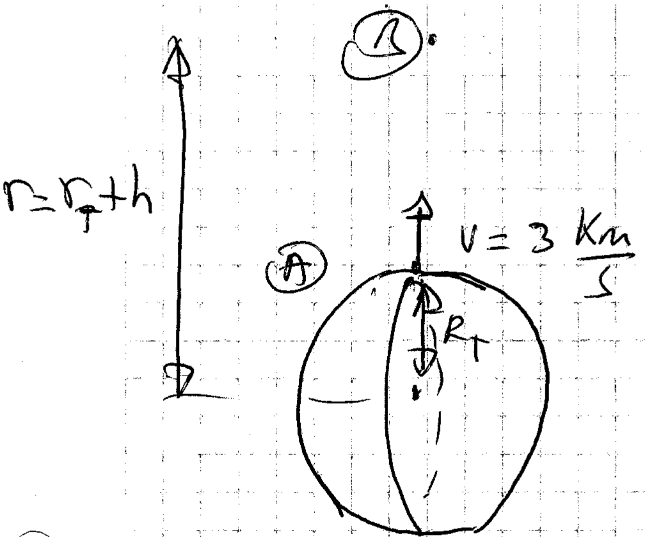
$$E_m = -\frac{1}{2} \frac{9'80 m/s^2 \cdot (6'370 \cdot 10^6 m)^2 \cdot 200 kg}{4'42 \cdot 10^7 m}$$

$$E_m = -9'0 \cdot 10^8 J$$

UNIDADES $\frac{\frac{m}{s^2} \cdot m^2 \; kg}{\cancel{m}} = kg \frac{m}{s^2} \cdot m$

N

37.-



$$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$$

$$R_T = 6'378 \cdot 10^6 m$$

$$M_T = 5'98 \cdot 10^{24} Kg$$

a) h? b) v?

Enerxía (A) = Enerxía (B)

$$(E_p)_A + (E_c)_A = (E_p)_B$$

$$-G \frac{M_T \cancel{m}}{R_T} + \frac{1}{2} \cancel{m} v^2 = -G \frac{M_T \cancel{m}}{r}$$

$$r = \frac{-G \, M_T}{-G \frac{M_T}{R_T} + \frac{1}{2} v^2}$$

$$r = \frac{-6'67 \cdot 10^{-11} N \cdot m^2/Kg^2 \cdot 5'98 \cdot 10^{24} Kg}{\frac{-6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot 5'98 \cdot 10^{24} Kg}{6'378 \cdot 10^6 m} + \frac{1}{2} (3 \cdot 10^3 m/s)^2}$$

$$r = 6'87 \cdot 10^6 m \quad // \quad h = 4'94 \cdot 10^5 m$$

para facelo na calculadora, probar a utilizar as memorias

$$r = \frac{A}{(B+C)}$$

$$A = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot 5'98\, 10^{24}\, Kg$$

$$B = \frac{-6'67\, 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot 5'98 \cdot 10^{24}\, Kg}{6'378 \cdot 10^6\, m}$$

$$C = \frac{1}{3}\left(3\, 10^3\, m/s\right)^2$$

b)

$$F_G = F_c$$

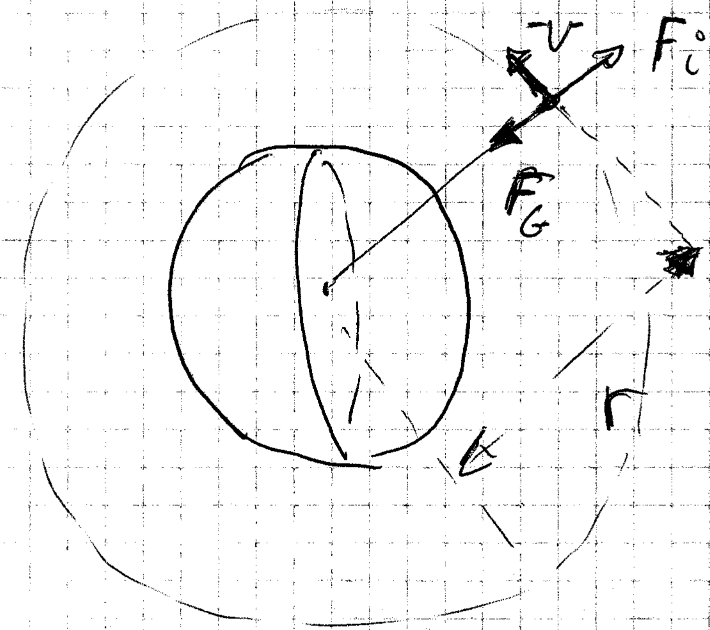
$$G \frac{M_T m}{r^2} = m \frac{v^2}{r}$$

$$v = \left(2 \frac{G M_T}{r}\right)^{1/2}$$

$$v = \left(\frac{2 \cdot 6'67 \cdot 10^{-11}\, N \cdot m^2/Kg^2 \cdot 5'98 \cdot 10^{24}\, Kg}{6'87\, 10^6\, m}\right)^{1/2}$$

$$v = 10.775\ m/s$$

38.-



$m = 25\,Kg$

$T = 24h = 86400s$

$G = 6'67\,10^{-11} \frac{N.m^2}{Kg^2}$

$M_T = 5'98.10^{24} Kg$

a) r? b) E_c, E_p ; E_m

a)

$$G\frac{M_T \cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}} \quad ; \quad G\frac{M_T}{r} = v^2$$

$$G\frac{M_T}{r} = w^2 r^2 \Rightarrow G\frac{M_T}{r} = \frac{4\pi^2}{T^2} r^2$$

$$r^3 = \frac{G M_T T^2}{4\pi^2} \quad ; \quad r = \left(\frac{G M_T T^2}{4\pi^2}\right)^{1/3}$$

$$r = \left(\frac{6'67.10^{-11} N.m^2/Kg^2 . 5'98.10^{24} Kg . (86400s)^2}{4\pi^2}\right)^{1/3}$$

$r = 4'225\,10^7 m.$

b)

$$E_c = \frac{1}{2} m v^2$$

para non calcular a velocidade

$$E_c = \frac{1}{2} m v^2 = \frac{1}{2} G \frac{M_T m}{r}$$

$$E_m = -\frac{1}{2} G \frac{M_T m}{r}$$

$$E_p = -\frac{G M_T m}{r}$$

$$E_p = -6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2} \cdot \frac{5'98 \cdot 10^{24} Kg \cdot 25 Kg}{4'225 \cdot 10^6 m} =$$

$$E_p = -2'36 \cdot 10^8 J$$

$$E_m = \frac{1}{2} E_p =$$

$$E_c = -\left(\frac{1}{2} E_p\right) =$$

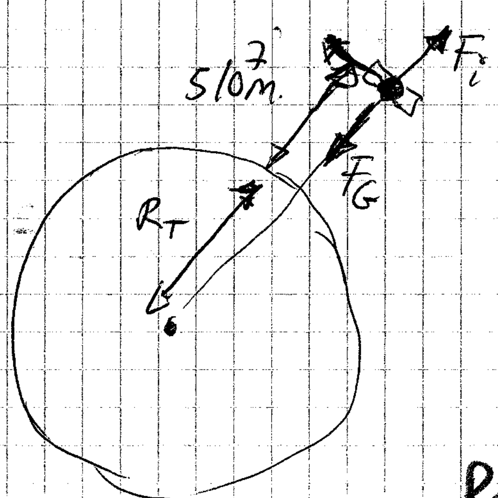
39.-

$m = 200 Kg$ | $g_0 = 9'81 m/s^2$

$r = 5 \cdot 10^7 m$ | $R_T = 6370 Km$

$F_G = ?$

$T ?$



$$F_G = F_i$$

$$G\frac{M_T m}{r^2} = m\frac{v^2}{r} \quad ; \quad g_0 = G\frac{M_T}{R_T^2}$$

$$\frac{g_0 R_T^2}{r^2} = \frac{v^2}{r} \Rightarrow$$

$$F_G = \frac{g_0 R_T^2 m}{r^2}$$

$$F_G = 9'81\, m/s^2 \cdot 200kg \left(\frac{6'37\,10^6 m}{(6'37\,10^6 + 5\,10^7)m}\right)^2$$

$$F_G = 9'81\,\frac{m}{s^2} \cdot 200kg \left(\frac{6'37\,10^6 m}{5'64\,10^7 m}\right)^2 = 25'1\, N$$

b/

$$\frac{g_0 R_T^2}{r^{\cancel{2}}} = \frac{v^2}{\cancel{r}} \quad ; \quad \frac{g_0 R_T^2}{r} = w^2 \cdot r^2$$

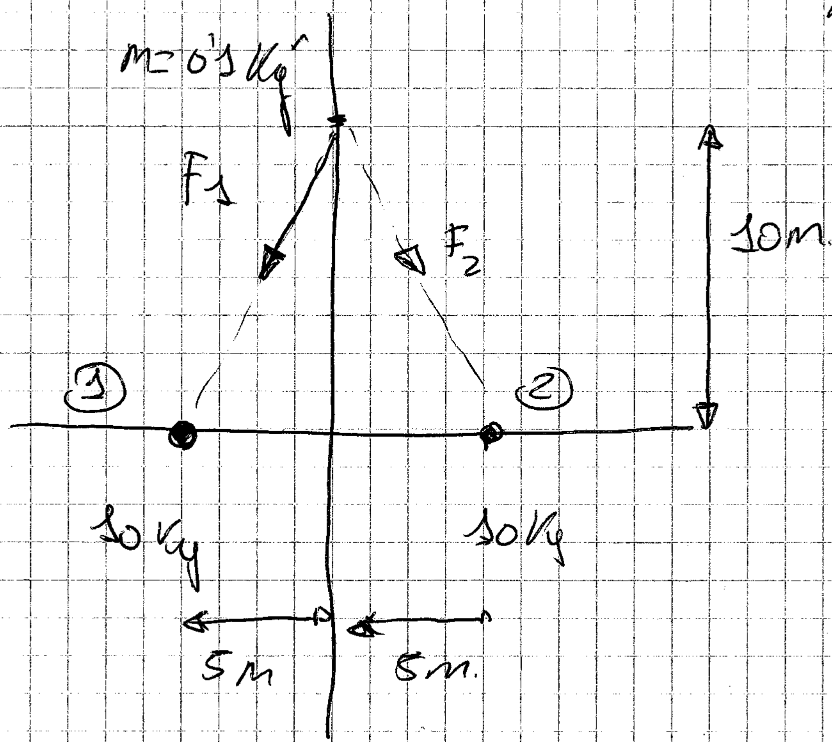
$$\frac{g_0 R_T^2}{r^3} = \frac{4\pi^2}{T^2} \Rightarrow T^2 = \frac{4\pi^2}{\frac{g_0 R_T^2}{r^3}}$$

$$T^2 = \frac{4\pi^2 \cdot r^3}{g_0 R_T^2} \; ; \; T = \left(\frac{4\pi^2 \cdot (5'64\,10^7 m)^3}{9'81\,\frac{m}{s^2} \cdot (6'37\,10^6 m)^2}\right)^{1/2}$$

$$T = 1'33\,10^5 s$$

40.-

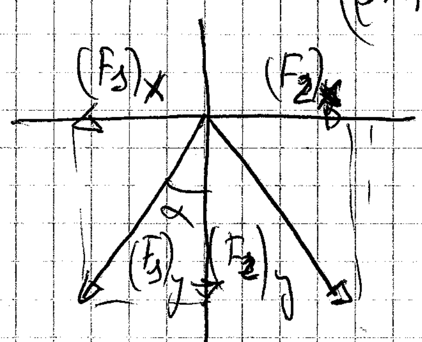
$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$



a? (0, 10) // a? (0,0)

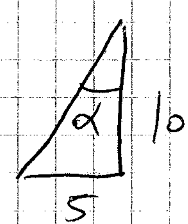
$F = m a \Rightarrow a = \frac{F}{m}$

$|F_1| = |\vec{F_2}| = G \frac{10\,Kg \cdot 0'1\,Kg}{(5m)^2 + (10m)^2} = 5'34 \cdot 10^{-13}\,N$



$(F_1)_x = (F_2)_x$

$(F_1)_y = |F_1| \cos\alpha$

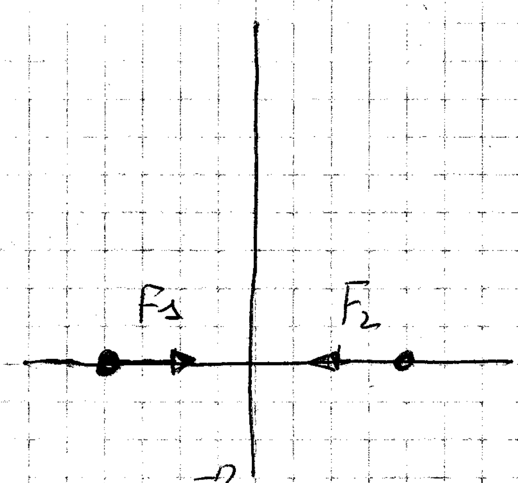


$\cos\alpha = \frac{10m}{\sqrt{125 m^2}} = 0'89$

$\vec{F}_{(0,10)} = -2 \times 0'89 \cdot 5'34 \cdot 10^{-13}\ \vec{j}\ (N)$

$\vec{F}_{(0,10)} = -9'5 \cdot 10^{-13}\ \vec{j}\ N$

Pax25

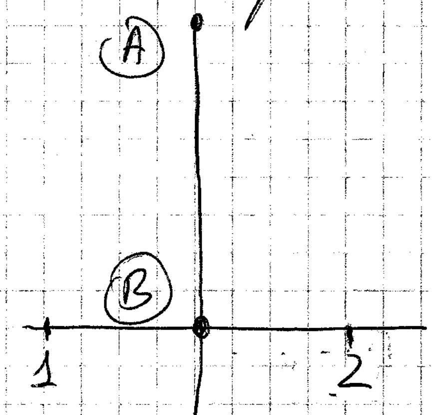


$\vec{F_1}(0,0) = 0$

$$\vec{a}(0,10) = \frac{\vec{F}(0,10)}{m} = \frac{-9'5\,10^{-13}\,\vec{j}\,(N)}{1\,kg} = -9'5\,10^{-13}\,\vec{j}\ m/s^2$$

$$\vec{a}(0,0) = \frac{\vec{F}(0,0)}{m} = 0$$

b) A aceleración non é constante na traxectoria dende o punto (0, 10) o punto (0,0) polo que é necesario facelo mediante enerxías



Enerxía A = Enerxía B

$$(E_p)_A = (E_p)_B + (E_c)_B$$

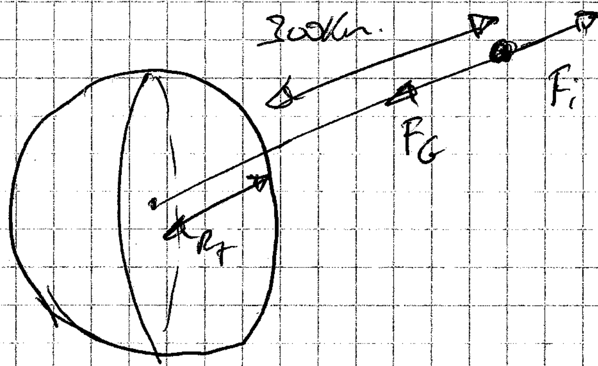
$$(E_p)_A = (E_p)_{A1} + (E_p)_{A2} = -2\,\frac{G\,M.m.}{\sqrt{125 m^2}}$$

$$(E_p)_B = (E_p)_{B1} + (E_p)_{B2} = -2\,\frac{G\,Mm.}{5m}$$

$$-2\,G\,\frac{M\cancel{m}}{\sqrt{125m^2}} = \frac{1}{2}\cancel{m}v_B^2 - 2\,G\,\frac{M.\cancel{m}}{5m}$$

$$v = \left(4.G.M.\left(\frac{1}{5m} - \frac{1}{\sqrt{125m^2}}\right)\right)^{1/2} = 1'7\,10^{-5}\ m/s$$

41.-



$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$

$R_T = 6378\ Km$

$M_T = 5'98 \cdot 10^{24}\ Kg$

a) v velocidade orbital ?
b) T ?

$$F_G = F_i$$

$$G \frac{M_T \cancel{m}}{r^2} = \cancel{m} \frac{v^2}{r} \Rightarrow v = \left(\frac{G\, M_T}{r} \right)^{1/2}$$

$$v = \left(\frac{6'67 \cdot 10^{-11}\ N m^2/Kg^2 \cdot 5'98 \cdot 10^{24}\ Kg}{6'678 \cdot 10^6\ m} \right)^{1/2} = 7728\ m/s$$

b)

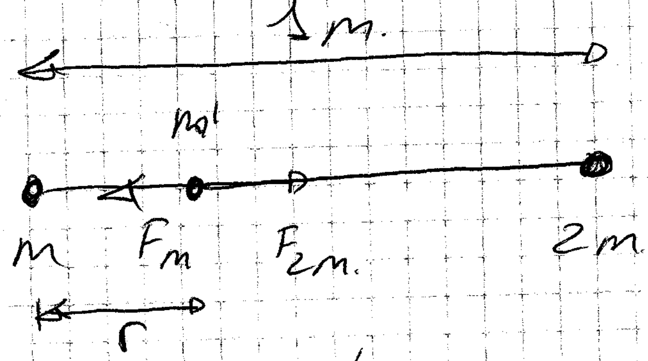
$$v = \omega \cdot r \Rightarrow v = \frac{2\pi}{T} \cdot r$$

$$T = \frac{2\pi \cdot r}{v}\ ; \quad T = \frac{2\pi\ 6'678 \cdot 10^6\ m}{7728\ m/s}$$

$$T = 5429\ s$$

42.-

a)



$$|\vec{F}_m| = |\vec{F}_{2m}|$$

$$F_m = G\frac{mm'}{r^2}$$

$$F_{2m} = G\frac{2mm'}{(1-r)^2}$$

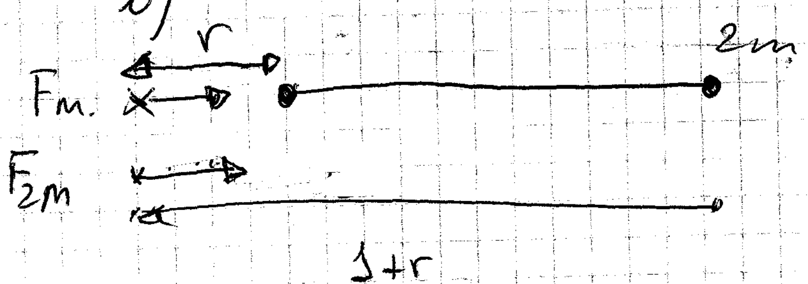
$$F_m = F_{2m} \; ; \quad G\frac{mm'}{r^2} = G\frac{2mm'}{(1-r)^2}$$

$$(1-r)^2 = 2r^2 \; ; \quad 1-2r+r^2 = 2r^2$$

$$-r^2-2r+1=0$$

$$\frac{2 \pm \sqrt{4+4}}{-2} \to \underline{0'41}$$
$$\to - \; \underline{non}$$

b)



$$F_m = F_{2m} \; ; \quad \cancel{G}\frac{\cancel{m}m'}{r^2} = \cancel{G}\frac{\cancel{m}2m}{(1+r)^2}$$

$$(1+r)^2 = r^2 \Rightarrow 1+2r+r^2 = 2r^2$$

Páx 28

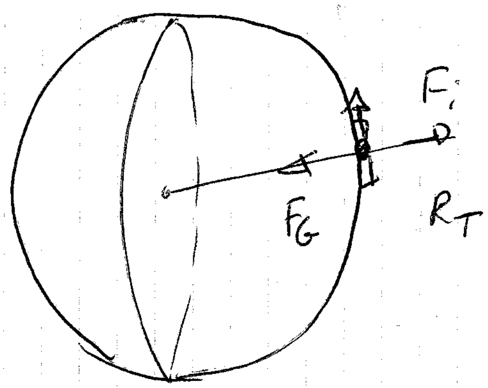
$$(1+r)^2 = 2r^2 \Rightarrow 1+2r+r^2 = 2r^2$$

$$1+2r-r^2 = 0 \Rightarrow r = \frac{-2 \pm \sqrt{4+2}}{-2} \begin{cases} 2'22\,m. \\ \ominus \text{ NON} \end{cases}$$

43.-

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{Kg^2}$; $R_T = 6'38 \cdot 10^6\,m.$

$M_T = 5'98 \cdot 10^{24}\,Kg$



$$G \frac{M_T m}{r^2} = m \frac{v^2}{r}$$

a)

$$v = \sqrt{G \frac{M}{R_T}} \; ; \; v = \left(\frac{6'67 \cdot 10^{-11} N m^2/Kg \cdot 5'98\ 10^{24} Kg}{6'38\ 10^6 m} \right)^{1/2}$$

$$v = 7.90\cancel{7}\ m/s$$

b)

$$v = \omega \cdot r \Rightarrow v = \frac{2\pi}{T} \cdot r$$

$$T = \frac{2\pi \cdot r}{v} = \frac{2 \cdot \pi \cdot 6'38\ 10^6 m}{7.907 m/s} = 5.0\cancel{7}0\ s$$

44.-

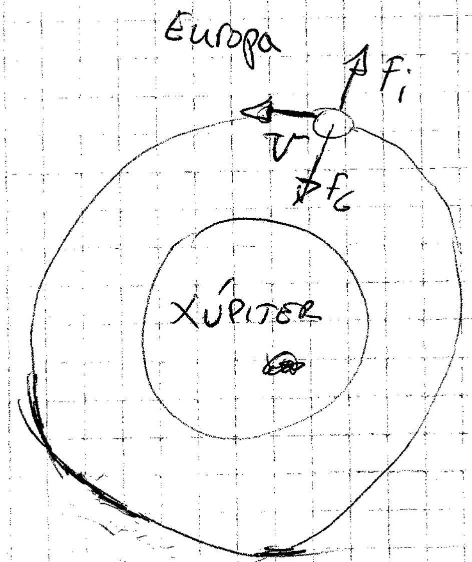
$r = 6'7 \cdot 10^5$ km

$T = 3$ días, 13 horas e 13 minutos

a) v?

b) M_X?

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$



$3 \text{ días} \cdot \frac{24 h}{1 \text{ día}} \cdot \frac{3600 s}{1 h} = 2'592 \cdot 10^5 s$

$13 h \cdot \frac{3600}{1 h} = 4'68 \cdot 10^4 s$

$13 \text{ minutos} \cdot \frac{60 s}{1 min} = 780 s$

$T = 3'068 \cdot 10^5 s$

a) $v = \omega \cdot r = \frac{2\pi}{T} \cdot r$

$v = \frac{2\pi}{3'068 \cdot 10^5 s} \cdot 6'7 \cdot 10^8 m = 13.721 \; m/s$

b) $G \frac{M_X \cancel{m}}{r^2} = \cancel{m} \frac{v^2}{r}$; $M_X = \frac{v^2 \cdot r^2}{G r}$;

$M_X = \frac{(13.721 \; m/s)^2 \cdot 6'7 \cdot 10^8 m}{6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}} = 1'89 \cdot 10^{27} kg$

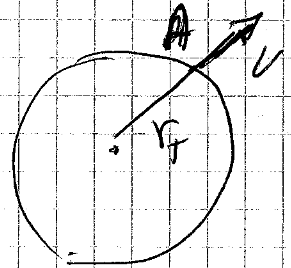
45.-

$G = 6'67 \cdot 10^{-11}\ N \cdot m^2/Kg^2$

$R_T = 6'38 \cdot 10^{6}\ m$

$M_T = 5'98 \cdot 10^{24}\ Kg$

B

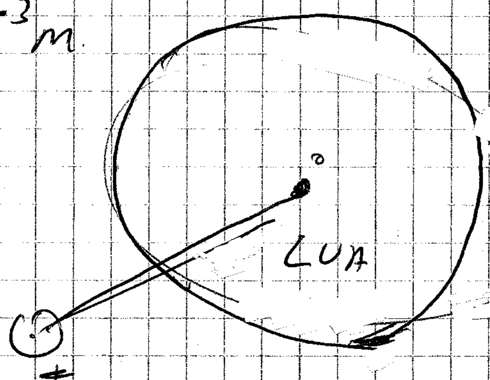


$$\frac{1}{2} m v_A^2 - G \frac{M_T m}{r} = 0$$

$$\frac{1}{2} v_A^2 = \frac{G M_T}{r}$$

$$r = \frac{2 G M_T}{v_A^2} \ ; \ r = \frac{2 \cdot 6'67 \cdot 10^{-11}\ N m^2/Kg^2 \cdot 5'98 \cdot 10^{24}\ Kg}{(3 \cdot 10^{8}\ m/s)^2}$$

$r = 8'9 \cdot 10^{-3}\ m$



$$G \frac{M_T \cancel{M_L}}{r^2} = \cancel{M_L} \frac{v^2}{r}$$

r igual $\Rightarrow$ v igual

46.-

$d_{T-L} = 60 R_T$

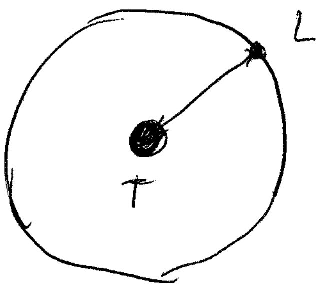
$R_T = 6400\, km.$

$G = 6'67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$

$M_T = 5'98 \cdot 10^{24} kg$

a) A Lúa describe unha órbita circular arredor da Terra.

$$G \frac{M_T m_L}{d_{T-L}^2} = m_L \frac{v_L^2}{d_{T-L}}$$



$$v_L = \left(\frac{G M_T}{d_{T-L}} \right)^{1/2}$$

$$v_L = \left(\frac{6'67 \cdot 10^{-11} N \cdot m^2/kg^2 \cdot 5'98 \cdot 10^{24} kg}{60 \cdot 6'4 \cdot 10^6 m} \right)^{1/2} = 1'02 \cdot 10^3 m/s$$

b)

$$v = \omega \cdot r \Rightarrow v_L = \omega r$$

$$v_L = \frac{2\pi}{T} r \Rightarrow T = \frac{2\pi}{v_L} \cdot r$$

$$T = \frac{2\pi}{1'02 \cdot 10^3 m/s} \cdot 60 \cdot 6'4 \cdot 10^6 m = 2'37 \cdot 10^6 s$$

$$2'37 \cdot 10^6 s \; \frac{1h}{3600 s} \; \frac{1 \text{ día}}{24h} = 27'4 \text{ días}$$

47.-

DATOS

$r_P = \frac{r_T}{2}$

$g_P = 5\ m/s^2$

$g_T = 10\ m/s^2$

a) M_T/M_P ?

b) $\underline{h}$? h_P $h_T = 100\ m$ a mesma velocidade

$$g_P = G\frac{M_P}{r_P^2} \; ; \; g_T = G\frac{M_T}{r_T^2}$$

$$G = \frac{g_P r_P^2}{M_P} \; ; \; G = \frac{g_T r_T^2}{M_T}$$

$$\frac{g_P r_P^2}{M_P} = \frac{g_T r_T^2}{M_T} \Rightarrow \frac{M_P}{M_T} = \frac{g_P r_P^2}{g_T r_T^2} = \frac{5\ m/s^2 \; \frac{r_T^2}{4}}{10\ m/s^2 \; r_T^2}$$

$$\frac{M_P}{M_T} = \frac{1}{2 \cdot 4} = \frac{1}{8}$$

b)

$$\cancel{m} g_T h_T = \frac{1}{2}\cancel{m} v$$

$$g_T h_T = g_P h_P$$

So nas proximidades da superficie para que podamos utilizar $E_p = mgh$.