

1.-

$$m = 0.5 \text{ Kg}$$

$$E = 6.0 \cdot 10^{-3} \text{ J (energia máxima)}$$

$$F_{\text{max}} = 0.3 \text{ N}$$

A equação do movimento harmónico é:

$$y = A \sin(\omega t + \varphi_0)$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $? \quad \quad ? \quad \quad ?$

$$E_{\text{max}} = \frac{1}{2} K A^2 ; E_K = \frac{1}{2} K (A^2 - y^2)$$

$$\omega^2 = \frac{K}{m}$$

$$|F_{\text{max}}| = K \cdot A \Rightarrow F = -K \cdot y$$

$$E_{\text{max}} = \frac{1}{2} K A^2 ; |F_{\text{m}}| = K \cdot A$$

$$K = \frac{F_m}{A}$$

$$\Rightarrow E_m = \frac{1}{2} \frac{F_m}{A} \cdot A^2 \Rightarrow A = \frac{2E_m}{F_m} = \frac{2 \times 6.0 \cdot 10^{-3} \text{ J}}{0.3 \text{ N}}$$

$$A = 0.04 \text{ m}$$

$$F_m = K A \Rightarrow K = \frac{F_m}{A} = \frac{0.3 \text{ N}}{0.04 \text{ m}} = 7.5 \frac{\text{N}}{\text{m}}$$

$$\omega^2 = \frac{F_m}{A} = \frac{0.3 \text{ N}}{0.04 \text{ m}} = \left(7.5 \frac{\text{N}}{\text{m}} / 0.5 \text{ Kg} \right)^{1/2} = 3.87 \frac{\text{rad}}{\text{s}}$$

UNIDADES

$$= \frac{\frac{\text{N}}{\text{m}}}{\text{Kg}} = \frac{\frac{\text{K m}}{\text{m s}^2}}{\text{Kg}} = \frac{1}{\text{s}^2} \Rightarrow \left(\frac{1}{\text{s}} \right)^{1/2} = \frac{\text{rad}}{\text{s}}$$

Para determinar φ_0 , para $t=0 \Rightarrow y = +A$

$$y = +A \Rightarrow \underset{\text{em } t=0}{\sin(\omega t + \varphi_0)} = 1 \Rightarrow \sin \varphi_0 = 1 \Rightarrow$$

$$\Rightarrow \varphi_0 = \frac{\pi}{2}$$

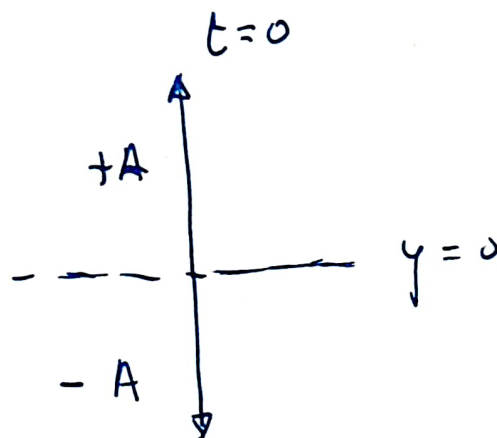
a)

$$y = 4'0 \cdot 10^{-2} \sin\left(3'87t + \frac{\pi}{2}\right) \text{ (S.I.)}$$

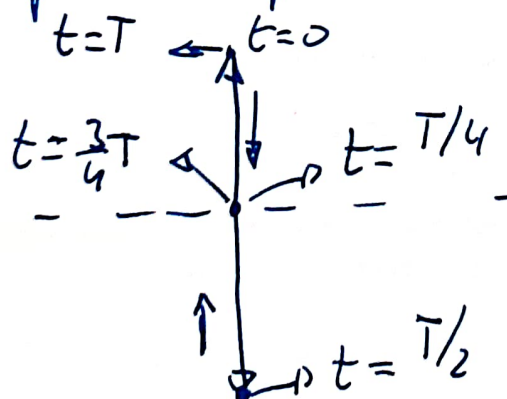
b)

$$E_k? \text{ e } E_p \text{ para } t = \frac{T}{4} \left(t = \frac{T}{4}\right)$$

Movimento



o período é o tempo que tarda em realizar uma oscilação completa pelo tanto o tempo que ~~saíndo~~ saindo da parte superior volta a esse ponto



$$E_c = \frac{1}{2} K (A^2 - y^2)$$

para $t = \frac{T}{4} \Rightarrow y = 0$

$E_c = \frac{1}{2} K A^2$ a energia cinética é máxima
pelo tanto $E_k = 6,0 \cdot 10^{-3} \text{ J}$

Se a energia cinética é máxima a potencial é
mínima $E_p = 0 \Leftrightarrow y = 0$

Também o poderíamos raciocinar, da seguinte
maneira.

$$E_k = \frac{1}{2} m v^2$$

$$E_k = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \phi_0)$$

$$E_k = \frac{1}{2} m A^2 \omega^2 \cos^2\left(\frac{2\pi}{T} \cdot \frac{T}{4} + \frac{\pi}{2}\right)$$

$$E_k = \frac{1}{2} m A^2 \omega^2 \cos^2\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$\downarrow \pi$

$$E_k = \frac{1}{2} K A^2 \cos^2 \pi \Rightarrow E_k = \frac{1}{2} K A^2 (-1)^2$$

$\uparrow$
?

c) $\omega^2 = \frac{K}{m} \Rightarrow 4\pi^2 f^2 = \frac{K}{m} \Rightarrow f^2 = \frac{K}{4\pi^2 m}$

$$f_1^2 = \frac{K}{4\pi^2 m_1} \Rightarrow f_2^2 = \frac{K}{4\pi^2 m_2}$$

$$f_1^2 m_1 = f_2^2 m_2 \Rightarrow f_2^2 = f_1^2 \cdot \frac{m_1}{m_2} ; f_2 = \frac{f_1}{2}$$

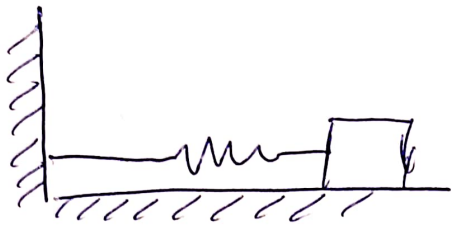
$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{387 \text{ rad/s}} = ? \Rightarrow f_1 = \frac{387 \text{ rad/s}}{2\pi} = 0,616 \text{ s}^{-1}$$

$$f_2 = \frac{0,616 \text{ s}^{-1}}{\sqrt{2}} = 0,435 \text{ s}^{-1}$$

2.-

$$m = 0.5 \text{ kg}$$

$$5 \text{ N} \Rightarrow 4 \text{ m.} ; F = 50 \text{ N} \Rightarrow x = 4 \text{ m.}$$



imoi calcular K e ω

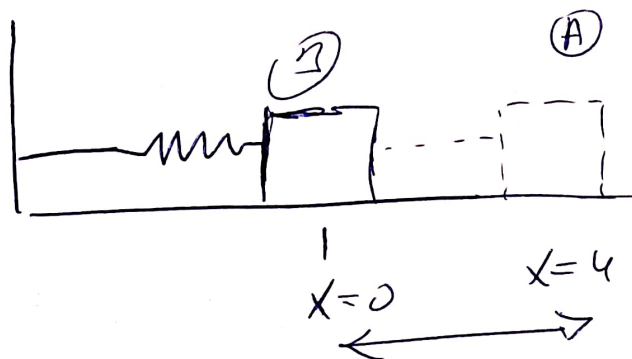
$$F = -Kx \Rightarrow K = \frac{F}{x} = \frac{50 \text{ N}}{0.04 \text{ m}} = 1250 \frac{\text{N}}{\text{m}}$$

$$\omega^2 = \frac{K}{m} \Rightarrow \omega = \left(\frac{1250 \text{ N/m}}{0.5 \text{ kg}} \right)^{\frac{1}{2}} = 50 \frac{\text{rad}}{\text{s}}$$

$$\downarrow$$

$$= \frac{\frac{\text{N}}{\text{m}}}{\frac{\text{kg}}{1}} = \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}}}{\frac{\text{kg}}{1}} = 1 \text{ s}^{-2} \Rightarrow \text{rad/s}$$

a)



Energia (A) - Energia en B. o con outro signo

$$E_A = 0$$

$$E_B = \frac{1}{2} K A^2$$

$\left| \right.$ $\omega =$ a diferença de energias

$$E_A = \frac{1}{2} K A^2 = \frac{1}{2} 125 \frac{\text{N}}{\text{m}} \cdot (0.04 \text{ m})^2 = 0.1 \text{ J}$$

c)

$$v = ?$$

$$x = A \sin(\omega t + \varphi_0)$$

posição inicial $x = 4 \text{ cm}$, do gráfico b)

$$v = \frac{dx}{dt} = A \omega \cos(\omega t + \varphi_0)$$

$$\cos^2(\omega t + \varphi_0) + \sin^2(\omega t + \varphi_0) = 1$$

$$\cos^2(\omega t + \varphi_0) = 1 - \sin^2(\omega t + \varphi_0)$$

$$v = A \omega \sqrt{1 - \sin^2(\omega t + \varphi_0)}$$

$$v = \omega \sqrt{A^2 - A^2 \sin^2(\omega t + \varphi_0)}$$

$$v = \omega \sqrt{A^2 - x^2}$$

$$v = 15.8 \frac{\text{rad}}{\text{s}} \sqrt{(4 \cdot 10^{-2} \text{ m})^2 - (2 \cdot 10^{-2} \text{ m})^2} = 0.55 \frac{\text{m}}{\text{s}}$$

c)

$$\omega^2 = \frac{K}{m} \quad ; \quad m : \text{constante} \quad ; \quad K : \text{constante}$$

o que implica $\omega^2 \propto \omega$ constante

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{15.8 \text{ rad/s}}{2\pi}$$

$$f = 2.52 \text{ s}^{-1}$$

3.- $m = 200 \text{ g}$

$E_T = 8 \text{ J}$

MHS

$A = 40 \text{ cm.}$

$x = -40 \text{ cm}$ para $t = 0$

a) K ? b) equação MHS c) $v_{\max}$ e $a_{\max}$.

a) $\omega^2 = \frac{K}{m} = ?$



$x = A \sin(\omega t + \varphi_0)$

$t = 0 \Rightarrow \sin(\omega t + \varphi_0) = -1$

$\sin \varphi_0 = -1 \Rightarrow \varphi_0 = \frac{3}{2}\pi$

$E_T = \frac{1}{2} K A^2 \Rightarrow K = \frac{2 E_T}{A^2}$

$K = \frac{2 \cdot 8 \text{ J}}{(0.4 \text{ m})^2} = 100 \text{ N/m.}$

$\omega^2 = \frac{K}{m} \Rightarrow \omega^2 = \frac{100 \text{ N/m.}}{0.2 \text{ kg}} = \sqrt{500 \frac{\text{rad}}{\text{s}^2}} =$

$\omega = 22.36 \frac{\text{rad}}{\text{s}}$

$x = 0.4 \sin\left(22.36 \frac{\text{rad}}{\text{s}} \cdot t + \frac{3}{2}\pi\right)$

c) v_{max} e a_{max}

$$v = \frac{dx}{dt} = A\omega \cos(\omega t + \varphi_0)$$

$$v_{max} = A\omega$$

$$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \varphi_0)$$

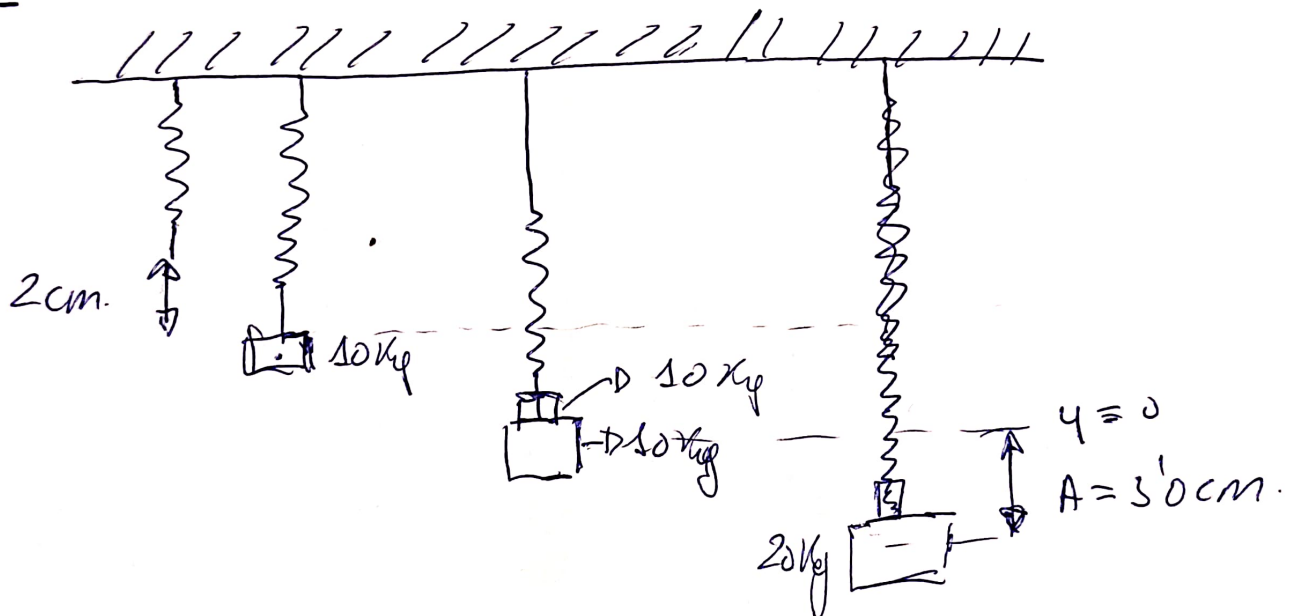
$$a_{max} = A\omega^2$$

sen signo

$$v_{max} = A\omega = 0.4m \cdot 22.36 \frac{rad}{s} = 8.94 m/s$$

$$a_{max} = |A\omega^2| = 0.4m \cdot \left(22.36 \frac{rad}{s}\right)^2 = 200 \frac{m}{s^2}$$

4.-



a) K ? b) y, v, a ? c) E_c e E_p

$$a) F = -ky \Rightarrow y = 0.02 \text{ m}; F = 20 \text{ Kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} =$$

$$F = 98 \text{ N}$$

$$\Rightarrow k = \left| \frac{F}{y} \right| = \frac{98 \text{ N}}{0.02} = 4.900 \frac{\text{N}}{\text{m}}$$

$$b) \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4.900 \text{ N/m}}{20 \text{ Kg}}} = 15.65 \frac{\text{rad}}{\text{s}}$$

$$\text{para } t=0 \Rightarrow y = -3.0 \text{ cm.}$$

$$y = A \sin(\omega t + \varphi_0) \Rightarrow \varphi_0 = \frac{3\pi}{2}$$

$$\left(y = 0.03 \sin\left(15.65t + \frac{3\pi}{2}\right) \text{ (S.I.)} \right.$$

$$\left. \Rightarrow v = \frac{dy}{dt} = A\omega \cos(\omega t + \varphi_0) \right.$$

$$v = 0.03 \text{ m} \cdot 15.65 \frac{\text{rad}}{\text{s}} \cos\left(15.65t + \frac{3\pi}{2}\right)$$

$$v = 0.47 \cos\left(15.65t + \frac{3\pi}{2}\right) \text{ (S.I.)}$$

$$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \varphi_0)$$

$$a = -0.03 \text{ m} \left(15.65 \frac{\text{rad}}{\text{s}}\right)^2 \sin\left(15.65t + \frac{3\pi}{2}\right)$$

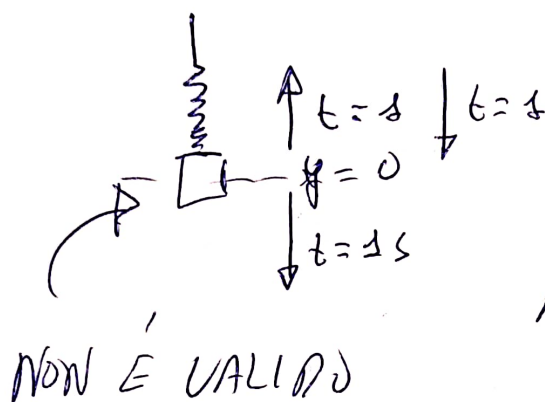
$$a = -7.35 \sin\left(15.65 + \frac{3\pi}{2}\right)$$

$$F = -K \cdot y = 4900 \frac{N}{m} \left(0.03 \sin \left(15.65t + \frac{3\pi}{2} \right) \right)$$

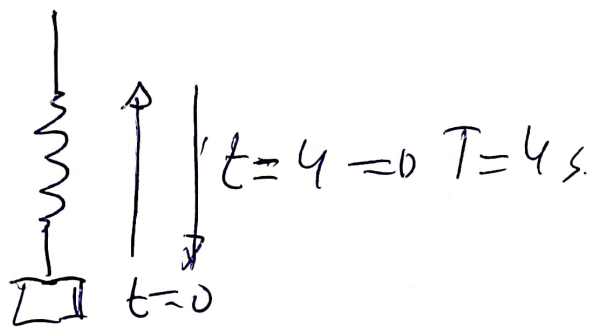
$$F = -147 \sin \left(15.65t - \frac{3\pi}{2} \right) \quad (\text{S.I.})$$

c) E_K e E_P os 2 segundos.

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2 \cdot \pi \text{ rad.}}{15.65 \frac{\text{rad}}{\text{s}}} = \underline{\underline{0.4 \text{ s}}}$$



Começa a oscilar
no ponto inferior



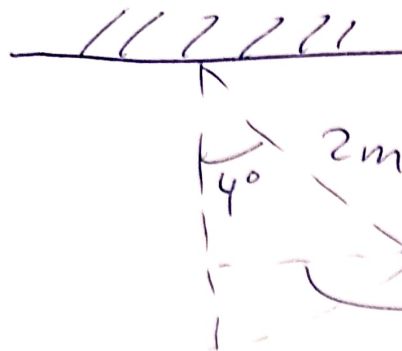
$t=4$ está na posição superior $v=0$

$$E_K = 0$$

$$E_P = E_T = \frac{1}{2} K A^2 = \frac{1}{2} 4900 \frac{N}{m} (0.03m)^2$$

$$E_P = 2.25 \text{ J}$$

5.-



$$A = \sin 4^\circ \cdot 2m = 0.154m$$

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T = 2\pi \sqrt{\frac{2m}{9.81m/s^2}} = 2.84s$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.84s} = 2.21 \frac{rad}{s}$$

$$x = A \sin(\omega t + \varphi_0)$$

$$t=0 \quad x=A \quad \Rightarrow \varphi_0 = \frac{\pi}{2}$$

$$x = 0.154 \sin\left(2.21t + \frac{\pi}{2}\right)$$

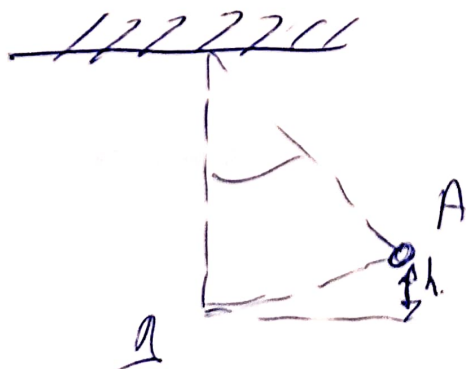
$$v = \frac{dx}{dt} = A \omega \sin(\omega t + \varphi_0)$$

cando pasa pola posición de equilibrio

$$v \Rightarrow v_{max}$$

$$v_{max} = A \omega = 0.154m \cdot 2.21 \frac{rad}{s} = 0.341 \frac{m}{s}$$

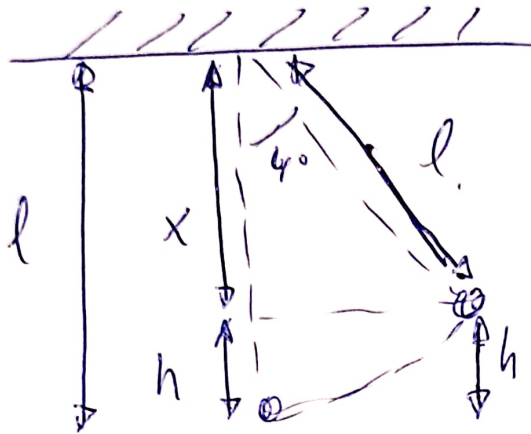
c)



Enerxia A = Enerxia B

$$mgh = \frac{1}{2} mv^2$$

$h?$



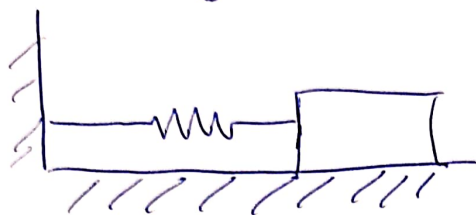
$$h = l - x ; x = l \cos 40^\circ$$

$$h = l - l \cos 40^\circ = l(1 - \cos 40^\circ)$$

$$h = 2m(1 - \cos 40^\circ) = 4.87 \times 10^{-3} \text{ m.}$$

$$v = \sqrt{2gh} \Rightarrow v = \sqrt{2 \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot 4.87 \times 10^{-3} \text{ m}} = 0.31 \frac{\text{m}}{\text{s}}$$

6.- $m = 0.5 \text{ kg}$



MHS

$A = 0.20 \text{ m}$

$f = 5 \text{ s}^{-1}$

$t = 0 \quad x = A$

$$x = A \sin(\omega t + \varphi_0)$$

$$\varphi_0 = 0 \Rightarrow \sin 0^\circ = 0.$$

$$\omega = 2\pi f \Rightarrow \omega = 10\pi \frac{\text{rad}}{\text{s}}$$

$$x = 0.2 \sin(10\pi t). \text{ (S.I.)}$$

Para $t = T/8$ ¿e a?, b) Energía mecánica

$$\omega = \frac{2\pi}{T}; \quad \omega t = \frac{2\pi}{T} \frac{T}{8} = \frac{\pi}{4} \quad (45^\circ)$$

$$t = \frac{T}{8} = \frac{1}{f \cdot 8} = \frac{1}{40 \text{ s}^{-1}} = 0.025 \text{ s}$$

$$\sin(10\pi \cdot 0.025 \text{ s}) = 0.707 \quad // \quad \cos(10\pi \cdot 0.025 \text{ s}) = 0.707$$

↓
3.1416...

$$v = A\omega \cos(\omega t)$$

$$v = 0.2 \text{ m} \cdot 10\pi \frac{\text{rad}}{\text{s}} \cdot 0.707 = 4.42 \text{ m/s}$$

$$a = -A\omega^2 \sin(\omega t)$$

$$a = -0.2 \cdot 100\pi^2 \cdot 0.707 = -39.6 \text{ m/s}^2$$

Páx 12

$$b) E_m = \frac{1}{2} K A^2$$

$$K ? ; \omega^2 = \frac{K}{m} \Rightarrow K = \omega^2 \cdot m = \left(10 \pi \frac{\text{rad}}{\text{s}} \right)^2 \cdot 0.1 \text{ kg}$$

$$K = 10 \pi^2 \text{ N/m} \quad \left(\frac{\text{kg}}{\text{s}^2} \cdot \frac{\text{m}}{\text{m}} = \frac{\text{N}}{\text{m}} \right)$$

$$E_m = \frac{1}{2} 10 \pi^2 \frac{\text{N}}{\text{m}} \cdot (0.2 \text{ m})^2 = 0.2 \pi^2 \frac{\text{N}}{\text{m}} \cdot \text{m}^2 = 1.97 \text{ J}$$

c)

$$\omega = 2\pi f \Rightarrow \frac{K^{1/2}}{m^{1/2}} = 2\pi f$$

$$f = \sqrt{\frac{K}{m}} \cdot \frac{1}{2\pi} \Rightarrow f = \sqrt{\frac{K}{4\pi^2 m}}$$

$$f_1 = \sqrt{\frac{K}{4\pi^2 m_1}}$$

$$f_2 = \sqrt{\frac{K}{4\pi^2 m_2}} ; m_2 = 2m_1$$

$$f_2 = \sqrt{\frac{K}{4\pi^2 2m_1}} ; \Rightarrow f_2 = \frac{1}{\sqrt{2}} \cdot f_1$$

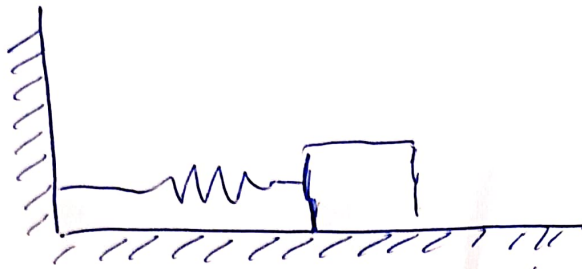
7. -

$$m = 50g$$

$$A = 20cm = 0.2m$$

$$x = -20cm \quad t = 0$$

$$E_m = 0.5J$$



a) k ? b) equação do movimento; c) E_c $x = 15cm$

$$a) \quad E_m = \frac{1}{2} k A^2 \Rightarrow \frac{2Em}{A^2} = k$$

$$k = \frac{2 \cdot 0.5J}{(0.2m)^2} = \frac{2.5/0^{-1}J}{4 \cdot 10^{-2}m^2} = 2.5 \cdot 10^2 N/m$$

$$b) \quad x = A \sin(\omega t + \varphi_0)$$

$$\text{para } t = 0 \Rightarrow \sin \varphi_0 = -1 \Rightarrow \varphi_0 = \frac{3}{2}\pi$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \left(\frac{25 \cdot 10^2 N/m}{0.05 kg} \right)^{1/2} = 250 \frac{rad}{s}$$

$$x = 0.2 \sin(250t + \frac{3}{2}\pi)$$

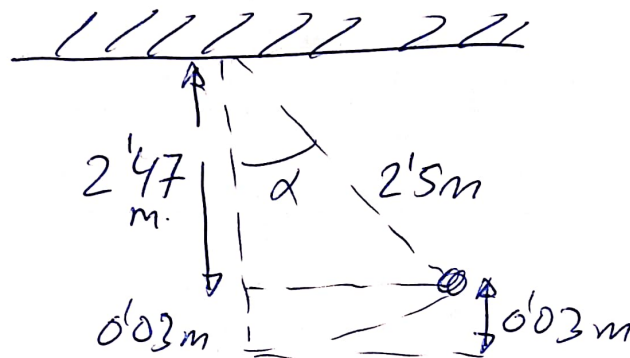
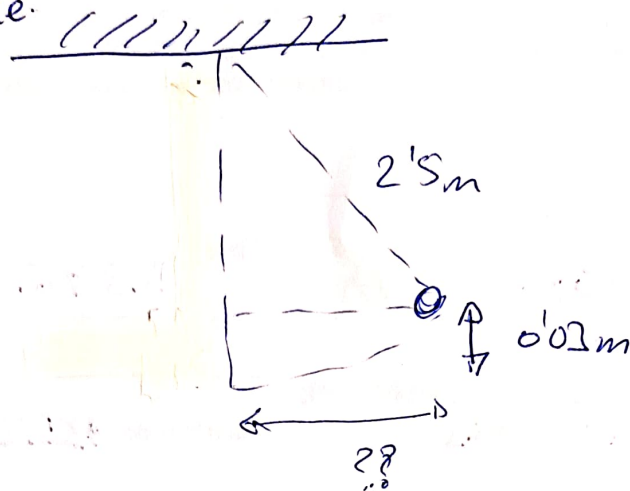
c) E para $x = 15$

$$E_k = \frac{1}{2} k \sqrt{A^2 - x^2}$$

$$E_k = \frac{1}{2} \cdot 25 \frac{N}{m} \sqrt{(0.2m)^2 - (0.15m)^2} = 1.65J$$

8. - $l = 2.5 \text{ m}$ $g = 9.8 \frac{\text{m}}{\text{s}^2}$ $T = 2\pi \sqrt{\frac{l}{g}}$
 $h = 0.03 \text{ m}$

c) Amplitude.



$$\cos \alpha = \frac{2.47 \text{ m}}{2.5 \text{ m}} \Rightarrow \cos \alpha = 0.988$$

$$\arccos 0.988 = 8.89^\circ$$

$$\sin 8.89^\circ = 0.154$$

$$\sin 8.89^\circ = \frac{x}{2.5 \text{ m}} \Rightarrow x = 0.39 \text{ m}$$

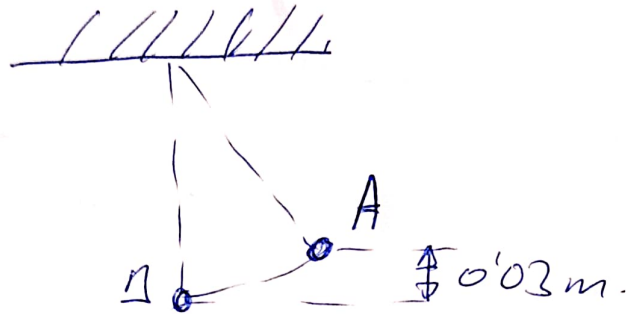
$$A = 0.39 \text{ m}$$

b) período

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 2\pi \sqrt{\frac{2.5m}{9.8m/s^2}} = 3.17s$$

a)

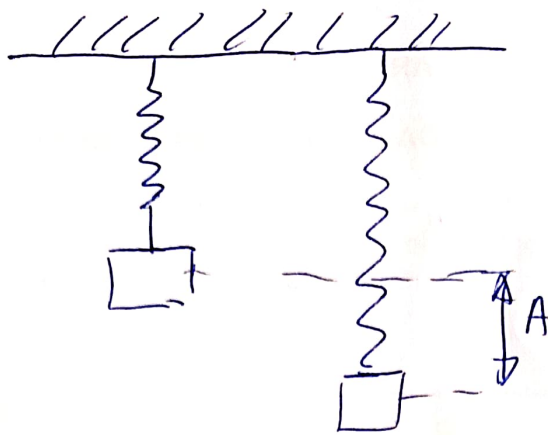


Energia A = Energia 1

$$mgh_A = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{2gh}$$

$$v_1 = \sqrt{2 \cdot 9.8 \frac{m}{s^2} \cdot 0.03m} = 0.77 m/s$$

9.-



$$K = 500 \frac{N}{m}$$

$$M = 100g$$

$$E_T = 5J$$

a) A?

b) v_{max} , f

c) gráfico da energia

$$a) E = \frac{1}{2} K A^2 \Rightarrow A = \left(\frac{2 E_T}{K} \right)^{1/2} = \left(\frac{2 \cdot 5J}{500 \frac{N}{m}} \right)^{1/2} = 0.14m.$$

b) v_{max} ?

$$y = A \sin(\omega t + \phi_0)$$

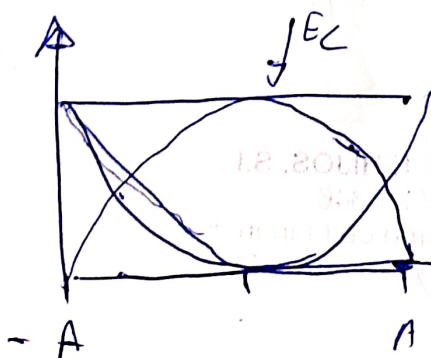
$$v = \frac{dy}{dt} = A \omega \cdot \cos(\omega t + \phi_0)$$

$$v_{max} = A \omega$$

$$\omega^2 = \frac{K}{m} \Rightarrow \omega = \left(\frac{500 N/m}{0.1 kg} \right)^{1/2} = 70.71 \frac{rad}{s}$$

$$\Rightarrow v_{max} = 0.14m \cdot 70.71 \frac{rad}{s} = 9.9 \frac{m}{s}$$

c) Energia



$$m = 5 \text{ g}$$

$$f = 1 \text{ Hz}$$

$$A = 10 \text{ cm} = 0.1 \text{ m}$$

$$\text{para } t=0 \quad \phi = A/2$$

a) Equação do movimento

b) E_m

c) E_c máxima? e potencial

$$a) \quad \omega = 2\pi \cdot f = 2\pi \cdot 1 \text{ s}^{-1} = 2\pi \frac{\text{rad}}{\text{s}}$$

$$\sin(\omega t + \phi_0) \text{ si } t=0 \quad \sin \phi_0 = \frac{1}{2}$$

$$\phi = 30^\circ$$

$$y = 0.1 \sin\left(2\pi t + \frac{\pi}{6}\right) \quad (\text{S.I.})$$

$$b) \quad E_m ? \quad E_m = \frac{1}{2} k A^2$$

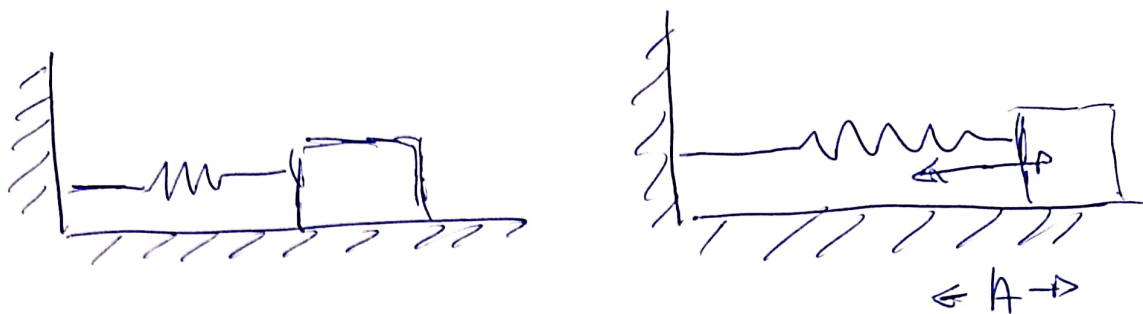
$$\omega^2 = \frac{k}{m} \Rightarrow k = \omega^2 \cdot m = \left(2\pi \frac{\text{rad}}{\text{s}}\right)^2 0.005 \text{ kg}$$

$$k = 0.197 \text{ N/m}$$

$$E_m = \frac{1}{2} 0.197 \frac{\text{N}}{\text{m}} \cdot (0.1 \text{ m})^2 = 10^{-3} \text{ J}$$

c)

11.-



$$A = 10 \text{ cm.} \quad m = 500 \text{ g}$$

$$T = 2 \text{ s}$$

a) v para $x = 5 \text{ cm.}$ b) aceleración para $x = 5 \text{ cm.}$

c) $E_m.$

$$x = A \sin(\omega t + \varphi_0)$$

estirase e soltase, para $t=0$, $A = +A$.

$$\sin(\omega t + \varphi_0) = 1 \Rightarrow \sin \varphi_0 = 1 \Rightarrow \varphi_0 = \frac{\pi}{2}$$

cambio en polo cambiar

$$x = A \sin(\omega t + \pi)$$

$$x = A \cos(\omega t)$$

$$\omega = \frac{2\pi \text{ rad}}{2 \text{ s}} = \pi \frac{\text{rad}}{\text{s}}$$

$$x = 0.1 \cos(\pi t)$$

$$v = \omega \sqrt{A^2 - x^2} \Rightarrow v = \pi \frac{\text{rad}}{\text{s}} \left((0.1 \text{ m})^2 - (0.05 \text{ m})^2 \right)$$

$$v = \pi \frac{\text{rad}}{\text{s}} (0.01 \text{ m}^2 - 0.0025 \text{ m}^2)$$

$$v = \pi \frac{\text{rad}}{\text{s}} (7.5 \cdot 10^{-3} \text{ m}^2)^{1/2} = 0.27 \frac{\text{m}}{\text{s}}$$

$$b) a = -\omega^2 \cdot x$$

$$a = -\pi^2 \frac{\text{rad}^2}{\text{s}^2} \cdot 0.05 \text{ m} = -0.49 \frac{\text{m}}{\text{s}^2}$$

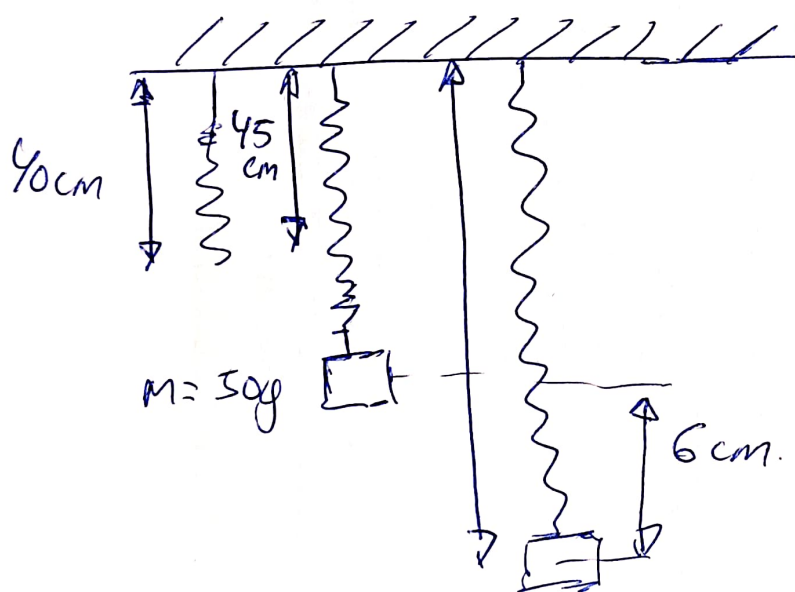
$$c) E_m = \frac{1}{2} K A^2$$

$$\omega^2 = \frac{K}{m} \Rightarrow K = \omega^2 \cdot m = \left(\pi \frac{\text{rad}}{\text{s}} \right)^2 \cdot 0.1 \text{ kg}$$

$$\cancel{\omega^2} K = 0.99 \frac{\text{N}}{\text{m}}$$

$$E_m = \frac{1}{2} 0.99 \frac{\text{N}}{\text{m}} \cdot (0.1 \text{ m})^2 = 5 \cdot 10^{-3} \text{ J}$$

12-



a) k ? b) equação de M.H.S. c) E_p ?

$$F = -Kx \Rightarrow$$

$$k = \frac{F}{x} = \frac{0.050 \text{ kg} \cdot 9.8 \text{ m/s}^2}{0.05 \text{ m}} = 9.8 \frac{\text{N}}{\text{m}}$$

b)

$$A = 0.06 \text{ m.}$$

para $t=0$ $y=-A$

$$\underset{t=0}{\text{sen}}(\omega t + \varphi_0) = -1 \Rightarrow \varphi_0 = \frac{3}{2}\pi$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} = \left(\frac{9.8 \text{ N/m.}}{0.05 \text{ kg}} \right)^{1/2} = 14 \frac{\text{rad}}{\text{s}}$$

$$y = A \text{ sen} \left(14 \frac{\text{rad}}{\text{s}} \cdot t + \frac{3}{2}\pi \right)$$

c)

$$W = -E_p$$

$$W = \underset{\substack{\downarrow \\ dy}}{\vec{F}} \cdot \underset{\substack{\downarrow \\ dy}}{d\vec{r}} : |\vec{F}| \cdot |d\vec{y}| \cdot \underset{\substack{\uparrow \\ \theta^0 = 180}}{\cos \theta}$$

$$W = |\vec{F}| |d\vec{y}| ; W = \int_0^y k \cdot y \, dy$$

$$W = \left[\frac{1}{2} k y^2 \right]_0^y \Rightarrow W = \frac{1}{2} k y^2$$

13.-

0.01 kg MMS $y = 5 \cos(2t + \pi/6)$ SI.

a) y, v, a para $t = 1s$

b) E_p para $y = 2m$

c) E_p negativo.?

a)

$$y = 5 \cos\left(2 \frac{\text{rad}}{s} \cdot 1s + \frac{\pi}{6}\right) = -4.08 \text{ m.}$$

$$y = A \cos(\omega t + \varphi_0)$$

$$v = \frac{dy}{dt} = -A\omega \sin(\omega t + \varphi_0)$$

$$v = -5m \cdot 2 \frac{\text{rad}}{s} \sin\left(2 \frac{\text{rad}}{s} \cdot 1s + \frac{\pi}{6}\right) = -5.9 \frac{m}{s}$$

$$a = \frac{dv}{dt} = -A\omega^2 \cos(\omega t + \varphi_0)$$

$$a = -5m \cdot \left(2 \frac{\text{rad}}{s}\right)^2 \cdot \cos\left(2 \frac{\text{rad}}{s} \cdot 1s + \frac{\pi}{6}\right)$$

$$a = -16.3 \frac{m}{s^2}$$

Se se considera eixe $y \rightarrow \frac{1}{s}$ para todos resultados

b) $E_p = \frac{1}{2} k y^2$; $y = 2\text{cm}$; $y = 0.02\text{m}$.

~~K~~ $\omega^2 = \frac{k}{m} \Rightarrow k = \omega^2 \cdot m$.

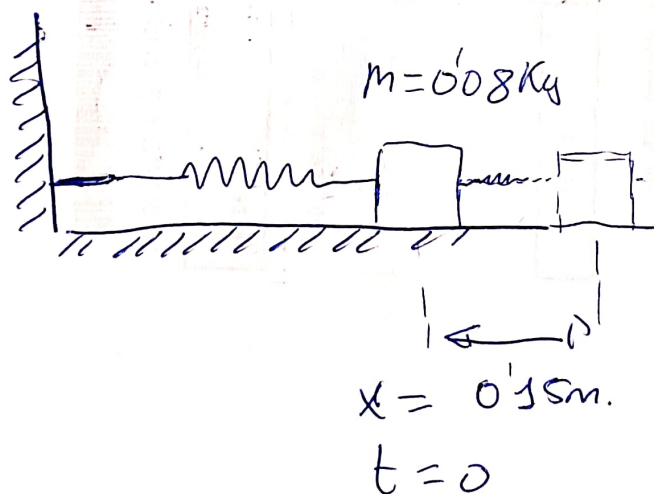
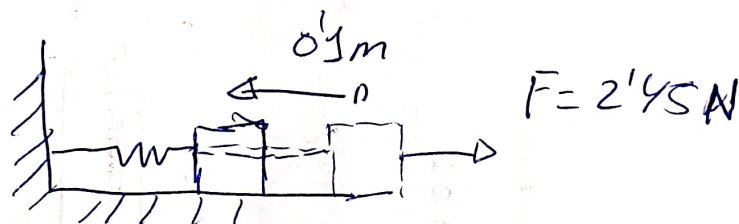
$k = \left(2 \frac{\text{rad}}{\text{s}}\right)^2 \cdot 0.01\text{Kg} = 0.04 \frac{\text{N}}{\text{m}}$

$E_p = \frac{1}{2} 0.04 \frac{\text{N}}{\text{m}} \cdot (2\text{m})^2 = 0.08\text{J}$

c) $E_p = \frac{1}{2} k y^2$

$\hookrightarrow y^2$ sempre positivo $\Rightarrow E_p$
 $\Rightarrow E_p$ sempre positiva.

14.-



a) $k?$; $T?$ b) E_T, E_p, E_c $x = 0.075\text{m}$.

$$F = k \cdot x \Rightarrow x = \frac{F}{k} = \frac{245 \text{ N}}{10 \text{ N/m}} = 24.5 \frac{\text{N}}{\text{m}}$$

$$A = 0.15 \text{ m}$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega^2 = \left(\frac{245 \text{ N/m}}{0.08 \text{ kg}} \right)^{1/2} = 17.5 \frac{\text{rad}}{\text{s}}$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{17.5 \frac{\text{rad}}{\text{s}}} = 0.36 \text{ s}$$

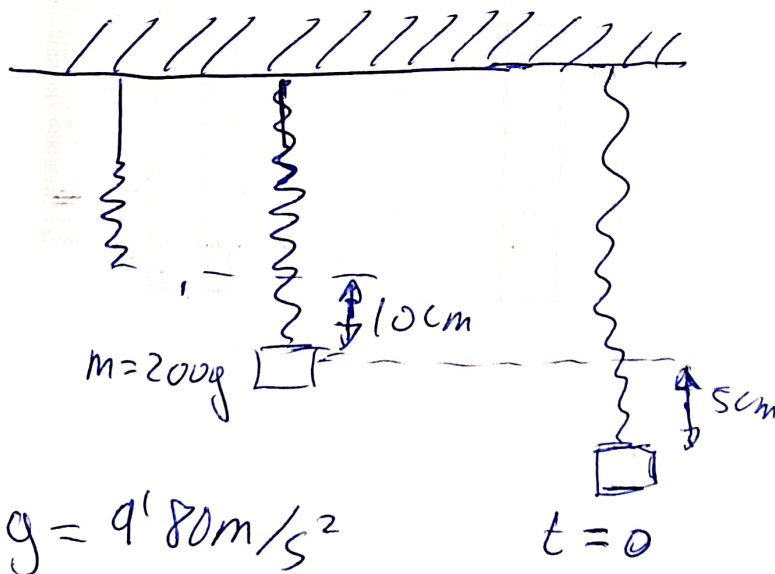
b)

$$E_T = \frac{1}{2} k A^2 = \frac{1}{2} 245 \frac{\text{N}}{\text{m}} \cdot (0.15 \text{ m})^2 = 0.28 \text{ J}$$

$$E_P = \frac{1}{2} k x^2 = \frac{1}{2} 245 \frac{\text{N}}{\text{m}} \cdot (0.075 \text{ m})^2 = 0.069 \text{ J}$$

$$E_K = 0.28 \text{ J} - 0.069 \text{ J} = 0.211 \text{ J}$$

15.-



a) y ? b) E_c e E_p $y = 3 \text{ cm}$.

4)

$$A = 0.05 \text{ m}$$

$$F = -ky \Rightarrow k = \left| -\frac{F}{y} \right| = \frac{F}{y} = \frac{0.2 \text{ kg} \cdot 9.80 \text{ m/s}^2}{0.1 \text{ m}} = 19.6 \frac{\text{N}}{\text{m}}$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \left(\frac{19.6 \text{ N/m}}{0.2 \text{ kg}} \right)^{1/2} = \left(98 \frac{\text{rad}}{\text{s}^2} \right)^{1/2} = 9.9 \frac{\text{rad}}{\text{s}}$$

$$y = 0.05 \text{ m} \sin \left(9.9t + \frac{3}{2}\pi \right)$$

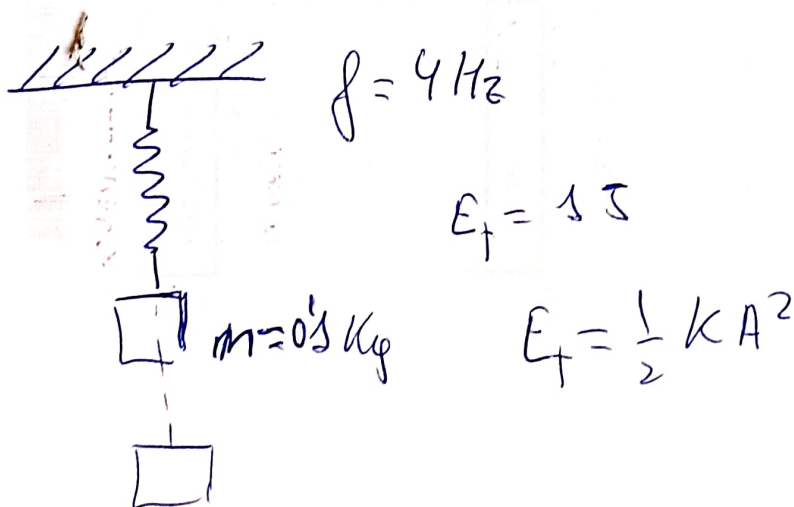
6)

$$E_p = \frac{1}{2} k y^2 = \frac{1}{2} 19.6 \frac{\text{N}}{\text{m}} (0.03 \text{ m})^2 = 8.82 \cdot 10^{-3} \text{ J}$$

$$E_T = \frac{1}{2} k A^2 = \frac{1}{2} 19.6 \frac{\text{N}}{\text{m}} (0.05 \text{ m})^2 = 0.0245 \text{ J} (2.45 \cdot 10^{-2} \text{ J})$$

$$\Rightarrow E_k = E_T - E_p = 0.0245 - 8.82 \cdot 10^{-3} \text{ J} = 0.0157 \text{ J} (1.57 \cdot 10^{-2} \text{ J})$$

16.-



a) k ? A? b) E_k E_p on $y = A/4$

$$a) \omega^2 = \frac{k}{m} \Rightarrow k = \omega^2 \cdot m$$

$$k = (2\pi f)^2 \cdot m = (2\pi \text{ rad } 4.5 \cdot 10^{-4})^2 \cdot 0.1 \text{ kg} = 63.17 \frac{\text{N}}{\text{m}}$$

$$\frac{\text{kg}}{\text{s}^2} \cdot \frac{\text{m}}{\text{m}} = \frac{\text{N}}{\text{m}}$$

$$E_T = \frac{1}{2} k A^2 \Rightarrow A = \left(\frac{2E_T}{k} \right)^{1/2} = \left(\frac{2 \cdot 1 \text{ J}}{63.17 \text{ N/m}} \right)^{1/2} = 0.18 \text{ m}$$

$$b) E_c \text{ e } E_p \text{ en } y = \frac{A}{4}$$

$$E_p = \frac{1}{2} k y^2; E_p = \frac{1}{2} k \left(\frac{A}{4} \right)^2 = \frac{1}{2} k \frac{A^2}{16}$$

$$E_p = \frac{E_T}{16}$$

$$E_c = E_T - E_p$$

$$E_p = \frac{1 \text{ J}}{16} = 0.0625 \text{ J} (6.25 \cdot 10^{-2} \text{ J})$$

$$E_c = 1 - 0.0625 \text{ J} = 0.9375 \text{ J}$$

17.- MMS

$$m = 3 \cdot 10^{-3} \text{ kg}, \quad f = 0.5 \text{ Hz}; \quad A = 0.05 \text{ m}; \quad t = 0 \Rightarrow x = 0$$

a) V e a para $t = 3 \text{ s}$ b) E_c e E_p para $t = 3 \text{ s}$

$$\omega = 2\pi \cdot f = 0.2\pi \frac{\text{rad}}{\text{s}}$$

$$x = A \sin(\omega t + \varphi_0) \quad x = 0 \Rightarrow t = 0$$

$$x = A \sin \varphi_0 \Rightarrow \sin \varphi_0 = 0; \quad \varphi_0 = 0$$

$$x = A \sin \omega t$$

$$v = A\omega \cos \omega t; \quad a = -A\omega^2 \sin \omega t$$

$$v|_{t=3\text{s}} = 0.05 \text{ m} \cdot 0.2\pi \frac{\text{rad}}{\text{s}} \cos(0.6\pi) = -0.097 \text{ m/s}$$

$$a|_{t=3\text{s}} = -0.05 \left(0.2\pi \frac{\text{rad}}{\text{s}}\right)^2 \cdot \sin(0.6\pi) = -0.019 \text{ m/s}^2$$

b)

$$\omega^2 = \frac{k}{m} \Rightarrow k = \omega^2 \cdot m = \left(0.2\pi \frac{\text{rad}}{\text{s}}\right)^2 \cdot 3 \cdot 10^{-3} \text{ kg} = 1.18 \cdot 10^{-3} \text{ N/m}$$

$$E_c = \frac{1}{2} m v^2$$

$$E_t = E_m = \frac{1}{2} k A^2$$

$$E_c = \frac{1}{2} 3 \cdot 10^{-3} \text{ kg} \cdot (-0.097 \text{ m/s})^2 = 1.41 \cdot 10^{-7} \text{ J}$$

$$E_t = \frac{1}{2} 1.18 \cdot 10^{-3} (0.05 \text{ m})^2 = 1.48 \cdot 10^{-6} \text{ J}$$

$$E_p = E_t - E_c = 1.33 \cdot 10^{-6} \text{ J} \quad (1.33 \cdot 10^{-6} \text{ J})$$