$\boxed{12}$ a) $\displaystyle\int \frac{\sqrt{\ln x}}{x}\,dx = \int \sqrt{t}\,dt = \frac{t^{1/2+1}}{1/2+1} = \frac{2}{3}\sqrt{t^3}+C = \frac{2}{3}\sqrt{(\ln x)^3}+C$

$t=\ln x$
$dt=\frac{1}{x}dx$

b) $\displaystyle\int \frac{e^x-3e^{2x}}{1+e^x}\,dx = \int \frac{\overbrace{e^x(1-3e^x)\,dx}^{dt}}{1+e^x} = \int \frac{1-3t-3+3}{1+t}\,dt =$

$t=e^x \to dt=e^x\,dx$

$\displaystyle = \int -3\frac{1+t}{1+t}+\frac{4}{1+t}\,dt = \int -3+\frac{4}{1+t}\,dt = -3t+4\ln|1+t| =$

$= -3e^x+4\ln(1+e^x)+C$

$t=e^x$

c) $\displaystyle\int \frac{dx}{\sqrt{x}\,(1+x)} = \int \frac{2t\,dt}{t(1+t^2)} = \int \frac{2\,dt}{1+t^2} = 2\,\text{arctg}(t) = 2\,\text{arctg}\sqrt{x}+C$

$x=t^2$
$dx=2t\,dt$

d) $\displaystyle\int \frac{1\cdot(1+\ln x)\,dx}{x(2+\ln x)} = \int \frac{1+t+1-1}{2+t}\,dt = \int \frac{2+t}{2+t}-\frac{1}{2+t}\,dt = \int 1-\frac{1}{2+t}\,dt =$

$t=\ln x$
$dt=\frac{1}{x}dx$

$= t-2\ln|1+t| = \ln x - 2\ln|1+\ln x|+C$

e) $\displaystyle\int \frac{dx}{\sqrt[3]{x}-1} = \int \frac{3t^2\,dt}{t-1} = \int 3t+3+\frac{3}{t-1}\,dt = \frac{3}{2}t^2+3t+3\ln|t-1| = \frac{3}{2}\sqrt[3]{x^2}+3\sqrt[3]{x}+3\ln|\sqrt[3]{x}-1|+C$

$x=t^3 \Rightarrow t=\sqrt[3]{x}$

$x=t^3$
$dx=3t^2\,dt$

$$\begin{array}{r|l} 3t^2 & t-1 \\ \cline{2-2} -3t^2+3t & 3t+3 \\ \hline 3t & \\ -3t+3 & \\ \hline 3 & \end{array}$$

$\Rightarrow 3t^2=(t-1)(3t+3)+3$

$\displaystyle \frac{3t^2}{t-1} = 3t+3+\frac{3}{t-1}$

f) $\displaystyle\int \frac{\sqrt[4]{x}}{1+\sqrt{x}}\,dx = \int \frac{t\cdot 4t^3}{1+t^2}\,dt = 4\int \frac{t^4\,dt}{1+t^2} = 4\int t^2-1+\frac{1}{1+t^2}\,dt = 4\left(\frac{t^3}{3}-t+\text{arctg}\,t\right) = \frac{4}{3}\sqrt[4]{x}-4\sqrt[4]{x}+4\,\text{arctg}\sqrt[4]{x}+C$

$x^4=t \Rightarrow t=\sqrt[4]{x}$

$x=t^4$
$dx=4t^3\,dt$

$$\begin{array}{r|l} t^4 & 1+t^2 \\ \cline{2-2} -t^4-t^2 & t^2-1 \\ \hline -t^2 & \\ t^2+1 & \\ \hline 1 & \end{array}$$

$\Rightarrow t^4=(1+t^2)(t^2-1)+1$

$\displaystyle \frac{t^4}{1+t^2} = t^2-1+\frac{1}{1+t^2}$

g) $\displaystyle\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx = \int \frac{dt}{\sqrt{4-t^2}} = \int \frac{dt}{\sqrt{4\left(1-\frac{t^2}{4}\right)}} = \int \frac{dt}{2\sqrt{1-\left(\frac{t}{2}\right)^2}} = \int \frac{\frac{1}{2}\,dt}{\sqrt{1-\left(\frac{t}{2}\right)^2}} = \text{arcsen}\left(\frac{t}{2}\right) = \text{arcsen}\left(\frac{e^x}{2}\right)+C$

$e^x=t$
$e^x dx=dt$

$t=e^x$

h) $\displaystyle\int \frac{dx}{\sqrt{x}\,(4-9x)} = \int \frac{2t\,dt}{t(4-9t^2)} = \int \frac{2\,dt}{4-9t^2} = \int \frac{1/2}{2-3t}\,dt + \int \frac{1/2}{2+3t}\,dt = \frac{1}{2}\left(-\frac{1}{3}\right)\int \frac{-3}{2-3t}\,dt + \frac{1}{2}\cdot\frac{1}{3}\int \frac{3}{2+3t}\,dt =$

$x=t^2$
$dx=2t\,dt$

$\displaystyle \frac{2}{4-9t^2} = \frac{A}{2-3t}+\frac{B}{2+3t} \Rightarrow 2=A(2+3t)+B(2-3t)$

$t=-\frac{2}{3} \Rightarrow 2=4B \Rightarrow B=\frac{1}{2}$

$t=\frac{2}{3} \Rightarrow 2=4A \Rightarrow A=1/2$

$\displaystyle = -\frac{1}{6}\ln|2-3t|+\frac{1}{6}\ln|2+3t| = -\frac{1}{6}\ln|2-3\sqrt{x}|+\frac{1}{6}\ln|2+3\sqrt{x}|+C$

$t=\sqrt{x}$

i) $\displaystyle\int \frac{dx}{x\sqrt{x+1}} = \int \frac{2t\,dt}{(t^2-1)\,t} = \int \frac{2\,dt}{t^2-1} = \int \frac{dt}{t-1}-\frac{dt}{t+1} =$

$x+1=t^2 \Rightarrow dx=2t\,dt$

$\displaystyle \frac{2}{t^2-1} = \frac{A}{t-1}+\frac{B}{t+1} \Rightarrow 2=A(t+1)+B(t-1)$

$t=-1 \Rightarrow 2=-2B \Rightarrow B=-1$

$t=1 \Rightarrow 2=2A \Rightarrow A=1$

$\ln|t-1|-\ln|t+1| = \ln\left|\sqrt{x+1}-1\right|-\ln\left|\sqrt{x+1}+1\right|+C$

j) $\displaystyle\int \sqrt{16-x^2}\,dx = \int \sqrt{16-16\,\text{sen}^2 t}\cdot 4\cos t\,dt = \int 4\sqrt{1-\text{sen}^2 t}\cdot 4\cos t\,dt = 16\int \cos^2 t\,dt =$

$x=4\,\text{sen}\,t$
$dx=4\cos t\,dt$

$1-\text{sen}^2 t=\cos^2 t$

$\displaystyle = 16\int \frac{1+\cos(2t)}{2}\,dt = 8\int 1+\cos(2t)\,dt = 8\left(t+\frac{\text{sen}(2t)}{2}\right) = 8t+8\,\text{sen}\,t\cos t =$

$\left.\begin{array}{l}\cos^2 t+\text{sen}^2 t=1\\ \cos^2 t-\text{sen}^2 t=\cos(2t)\end{array}\right\} \Rightarrow 2\cos^2 t = 1+\cos(2t)$

$\text{sen}(2t)=2\,\text{sen}\,t\cos t$

$\displaystyle = 8\,\text{arcsen}\left(\frac{x}{4}\right)+2x\sqrt{1-\frac{x^2}{16}}+C$

$\text{sen}\,t=\frac{x}{4}$
$\cos t=\sqrt{1-\text{sen}^2 t}$

k) $\displaystyle\int \sqrt{9-2x^2}\,dx = \int \sqrt{9-9\,\text{sen}^2 t}\cdot\frac{3}{\sqrt{2}}\cos t\,dt = \frac{9}{\sqrt{2}}\int \sqrt{1-\text{sen}^2 t}\cdot\cos t\,dt = \frac{9}{\sqrt{2}}\int \cos^2 t\,dt =$

$2x^2=9\,\text{sen}^2 t \Rightarrow x=\frac{3}{\sqrt{2}}\,\text{sen}\,t$

$dx=\frac{3}{\sqrt{2}}\cos t$

$\displaystyle = \frac{9}{\sqrt{2}}\int \frac{1+\cos(2t)}{2}\,dt = \frac{9}{2\sqrt{3}}\left(t+\frac{\text{sen}(2t)}{2}\right) = \frac{9}{2\sqrt{2}}\,\text{arcsen}\left(\frac{\sqrt{2}x}{3}\right)+\frac{9}{2\sqrt{2}}\cdot\frac{\sqrt{2}\cdot x}{3}\sqrt{1-\frac{2}{9}x^2} =$

$\displaystyle = \frac{9\sqrt{2}}{4}\,\text{arcsen}\left(\frac{\sqrt{2}x}{3}\right)+\frac{1}{2}x\sqrt{9-2x^2}+C$