

Nome: _____

- O exame ten que estar feito a bolígrafo, non se corrixe lapis.
- O procedemento de resolución ten que estar indicado e as contas estar escritas.

1. Resolve os seguintes límites:

$$\begin{aligned} \text{a) } \lim_{x \rightarrow +\infty} \left(\frac{x-2}{x+1} \right)^{3x^2} &= 1^\infty = \text{Ind.} \rightarrow \lim \left(1 + \frac{x-2}{x+1} - 1 \right)^{3x^2} = \\ &= \lim \left(1 + \frac{x-2}{x+1} - \frac{x+1}{x+1} \right)^{3x^2} = \lim \left(1 + \frac{-3}{x+1} \right)^{3x^2} = \\ &= \lim \left(\frac{1}{\frac{x+1}{-3}} \right)^{\frac{x+1}{-3} \cdot \frac{-3}{x+1} \cdot 3x^2} = e^{\lim \left(-\frac{9x^2}{x+1} \right)} = e^{-\infty} = \frac{1}{e^\infty} = 0 \end{aligned}$$

$$\begin{aligned} \text{b) } \lim_{x \rightarrow +\infty} (\sqrt{5x+1} - \sqrt{2x-3}) &= \infty - \infty = \\ \text{Ind.} \rightarrow \lim \frac{(\sqrt{5x+1} - \sqrt{2x-3}) \cdot (\sqrt{5x+1} + \sqrt{2x-3})}{(\sqrt{5x+1} + \sqrt{2x-3})} &= \lim \frac{5x+1-2x+3}{(\sqrt{5x+1} + \sqrt{2x-3})} = \\ \lim \frac{3x+4}{\text{grado } 1/2} &= +\infty \end{aligned}$$

$$\begin{aligned} \text{c) } \lim_{x \rightarrow -2} \frac{2x^3+2x^2+8}{3x^2+5x-2} &= \frac{-16+8+8}{12-10-2} = \frac{0}{0} = \text{Ind.} \rightarrow \\ \lim_{x \rightarrow -2} \frac{(x+2)(2x^2-2x+4)}{(x+2)(3x-1)} &= \frac{8+4+4}{-6-1} = -\frac{16}{7} \end{aligned}$$

$$\begin{aligned} \text{d) } \lim_{x \rightarrow 6} \frac{x^3-36x}{x^2-12x+36} &= \frac{216-216}{36-72+36} = \frac{0}{0} = \text{Ind.} \rightarrow \lim_{x \rightarrow 6} \frac{x \cdot (x+6) \cdot (x-6)}{(x-6)^2} = \\ \frac{6 \cdot (6+6)}{(6-6)} &= \frac{72}{0} = \infty? \\ \lim_{x \rightarrow 6^-} \frac{x \cdot (x+6)}{(x-6)} &= \frac{72}{0^-} = -\infty \\ \lim_{x \rightarrow 6^+} \frac{x \cdot (x+6)}{(x-6)} &= \frac{72}{0^+} = +\infty \end{aligned}$$

Non hai límite porque os laterais non coinciden.

2. Calcula as asíntotas da seguinte función e determina a súa posición relativa para facer un esbozo da mesma:

$$a) f(x) = \frac{x^2+3}{2x-1}$$

$$Dom = \mathbb{R} - \left\{\frac{1}{2}\right\} \rightarrow AV \text{ en } x = \frac{1}{2}?$$

$$\lim_{x \rightarrow \frac{1}{2}^-} \frac{x^2+3}{2x-1} = \frac{3,5}{0^-} = -\infty$$

$$\lim_{x \rightarrow \frac{1}{2}^+} \frac{x^2+3}{2x-1} = \frac{3,5}{0^+} = +\infty$$

Asíntota Vertical en $x=1/2$

$$\lim_{x \rightarrow -\infty} \frac{x^2+3}{2x-1} = -\infty$$

$$\lim_{x \rightarrow +\infty} \frac{x^2+3}{2x-1} = +\infty$$

Non hai asíntotas horizontais

$$m = \lim_{x \rightarrow \pm\infty} \frac{x^2+3}{\frac{2x-1}{x}} = \lim_{x \rightarrow \pm\infty} \frac{x^2+3}{2x^2-1} = \lim_{x \rightarrow \pm\infty} \frac{x^2+\dots}{2x^2-\dots} = \frac{1}{2}$$

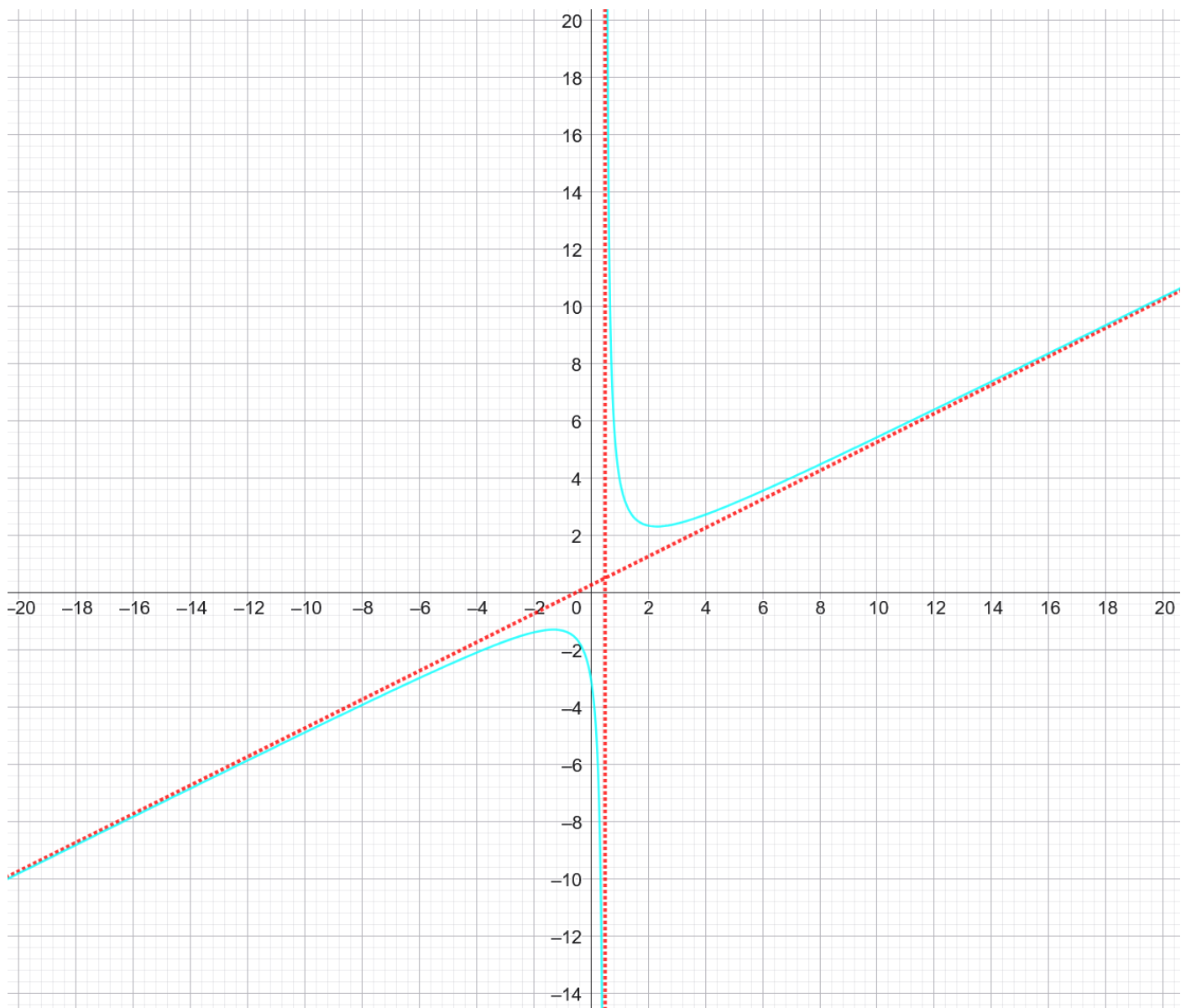
$$\begin{aligned} n &= \lim_{x \rightarrow \pm\infty} \frac{x^2+3}{2x-1} - m \cdot x = \lim_{x \rightarrow \pm\infty} \frac{x^2+3}{2x-1} - \frac{x}{2} = \lim_{x \rightarrow \pm\infty} \frac{x^2+3-x^2+\frac{x}{2}}{2x-1} \\ &= \lim_{x \rightarrow \pm\infty} \frac{\frac{x}{2}+3}{2x-1} = \lim_{x \rightarrow \pm\infty} \frac{\frac{x}{2}+\dots}{2x-\dots} = \lim_{x \rightarrow \pm\infty} \frac{x}{4x} = 1/4 \end{aligned}$$

Asíntota oblicua en $y = \frac{x}{2} + \frac{1}{4}$

$$\begin{aligned}\lim_{x \rightarrow \pm\infty} f(x) - (mx + n) &= \lim_{x \rightarrow \pm\infty} \frac{x^2 + 3}{2x - 1} - \frac{x}{2} - \frac{1}{4} \\ &= \lim_{x \rightarrow \pm\infty} \frac{4x^2 + 12 - 4x^2 + 2x - 2x + 1}{8x - 4} = \lim_{x \rightarrow \pm\infty} \frac{13}{8x - 4}\end{aligned}$$

$$\lim_{x \rightarrow -\infty} \frac{13}{8x - 4} = \frac{13}{-\infty} = 0^- \rightarrow \text{A función queda por abaixo}$$

$$\lim_{x \rightarrow +\infty} \frac{13}{8x - 4} = \frac{13}{+\infty} = 0^+ \rightarrow \text{A función queda por enriba}$$



3. Para a seguinte función:

a. Explica onde hai discontinuidades e de que tipo son. (0.4 ptos.)

En $x = -3 \rightarrow$ Discontinuidade non evitable por salto infinito.

En $x = 5 \rightarrow$ Discontinuidade non evitable por salto finito.

b. Di cales son os seguintes límites: (0.6 ptos.)

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$\lim_{x \rightarrow 5} f(x) = \text{non existe}$$

$$\lim_{x \rightarrow 10} f(x) = 11$$

