c) $\frac{x-4}{2} - \frac{x+1}{3} \leq \frac{1}{6}$

$3(x-4) - 2(x+1) \leq 1$

$3x - 12 - 2x - 2 \leq 1$

$x \leq 15$

Sol: $(-\infty, 15]$

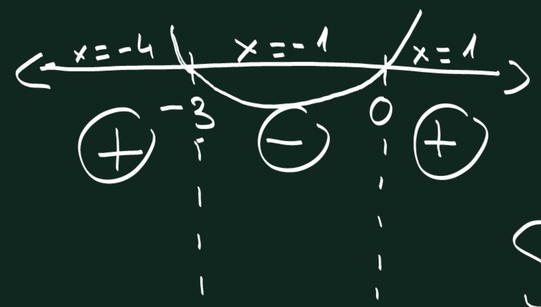
d) $x^2 + 3x \leq 0 \Rightarrow x(x+3) = 0$

$\to x = 0$

$\to x + 3 = 0$

$\Downarrow$

$x = -3$



$(-\infty, -3)(-3, 0)(0, +\infty)$

Sol: $[-3, 0]$

1

a) $5 - x \leq 12$

$\cdot(-1)$ $\quad -x \leq 7$

$x \geq -7$

$-7 \leq x$

$\Rightarrow$ Sol: $[-7, +\infty)$

b) $\frac{x-3}{2} - \frac{2-x}{3} > 3$

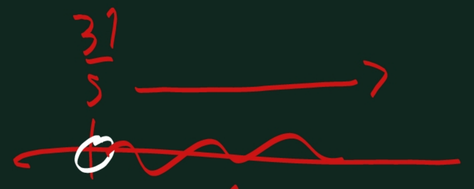
$\frac{3(x-3)}{6} - \frac{2(2-x)}{6} > \frac{18}{6}$

$3x - 9 - 4 + 2x > 18$

$3x + 2x > 18 + 4 + 9$

$5x > 31$

$x > \frac{31}{5}$



Sol $\left[\frac{31}{5}, +\infty\right)$

c) $\frac{5}{6}(3-x) - \frac{1}{2}(x-4) \geq \frac{1}{3}(2x-3) - x$

$$\frac{5\cdot(3-x)}{\cancel{6}} - \frac{3(x-4)}{\cancel{6}} \geq \frac{2(2x-3)}{\cancel{6}} - \frac{6x}{\cancel{6}}$$

$$15-5x-3x+12 \geq 4x-6-6x$$

$$-5x-3x-4x+6x \geq -6-15-12$$

$$-6x \geq -33$$

$$x \leq \frac{33}{6} \rightarrow x \leq \frac{11}{2} \rightarrow \text{Sol}: (-\infty, 11/2]$$

Inecuaciones de 2º Grado

1º - Convertimos la inecuación en una ecuación.

2º - Resolvemos la ecuación $\longrightarrow$ 3 intervalos

3º - Comprobamos el signo en cada intervalo.

$ax^2+bx+c=0$

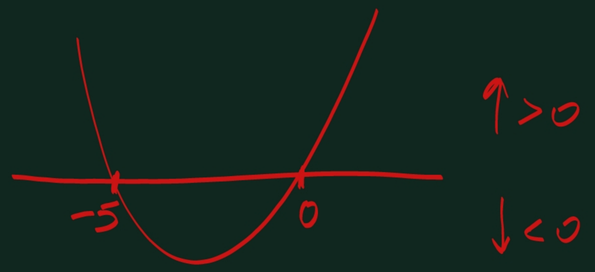
$a>0$

$a<0$

② a) $x^2+5x \leq 0$ → Parábola Pos.

$x^2+5x=0$

$x(x+5)=0$ → $\boxed{x=0}$; $x+5=0$ → $\boxed{x=-5}$



↑ > 0

↓ < 0

Me piden "≤ 0":

Sol: $[-5, 0]$

$(-\infty, -5)$	$(-5, 0)$	$(0, +\infty)$
$x=10$	$x=-2$	
+	−	+

② b) $3(x-5)^2-12\geq 0 \rightarrow (x-5)^2\geq \frac{+12}{3} \rightarrow x^2-10x+25\geq 4$

$x^2-10x+21\geq 0 \rightarrow x^2-10x+21=0$

$$x=\frac{+10\pm\sqrt{(-10)^2-4\cdot 1\cdot 21}}{2\cdot 1}=\frac{10\pm\sqrt{100-84}}{2}=\frac{10\pm\sqrt{16}}{2}=\frac{10\pm 4}{2}\begin{cases}7\\3\end{cases}$$

$-\infty$		0		4		8		$+\infty$
	+	3	−			7	+	

Sol: $(-\infty,+3]\cup[+7,+\infty)$

(2) c) $\frac{x^2-9}{5} - \frac{x^2-4}{45} \leq \frac{1-2x}{3} \rightarrow \frac{3(x^2-9)}{15} - \frac{x^2-4}{15} \leq \frac{5(2x+1)}{15}$

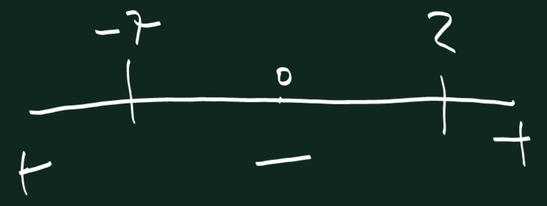
$3x^3-27-x^2+4 \leq -10x+5 \rightarrow 3x^2-x^2+10x-27+4-5 \leq 0$

$2x^2+10x-28 \rightarrow 2(x^2+5x-14)$

$\frac{-5 \pm \sqrt{5^2-4\cdot(-14)}}{2} = \frac{-5 \pm \sqrt{25+56}}{2} = \frac{-5 \pm 9}{2}$

$\rightarrow \frac{-5-9}{2} = -7$

$\rightarrow \frac{-5+9}{2} = 2$



$[-7, 2]$

Inecuacións Polinómicas (grao>2)

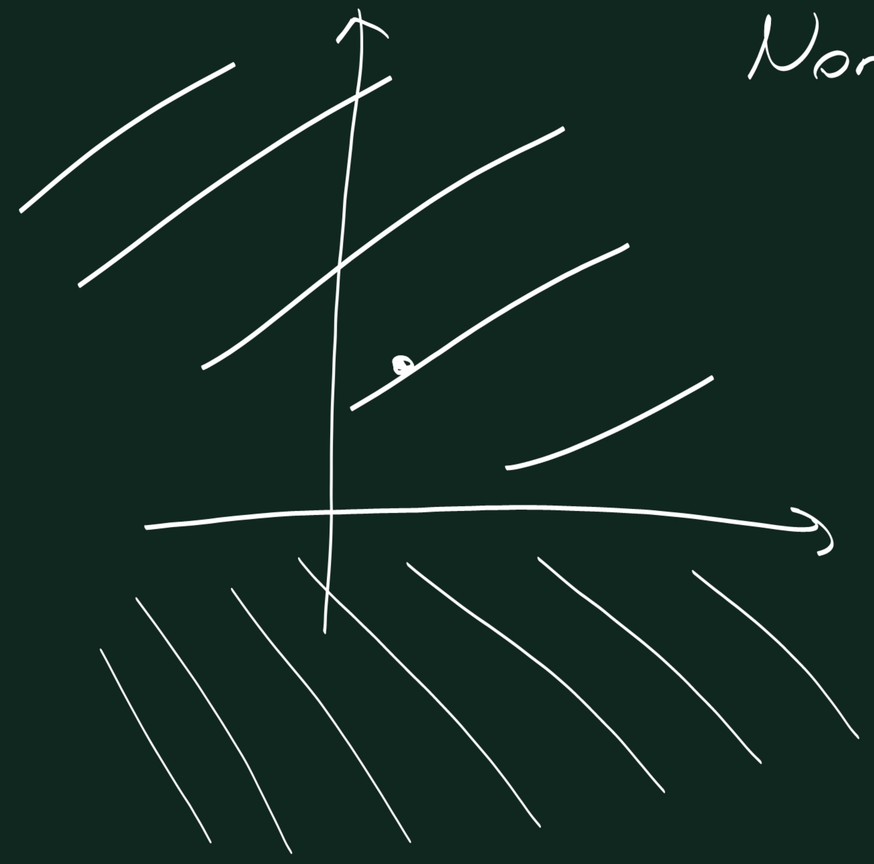
1º Factorizo: Ruffini ata grao 2 e despois ec. 2º grao

2º Resolvemos a ecuación

3º Facemos os intervalos

4º Comprobamos o signo en cada intervalo

③ a) $x^4+4<0$ $\quad \phi$



Non ten solución.

③

b) $x^4 - 3x^2 - 4 \leq 0$

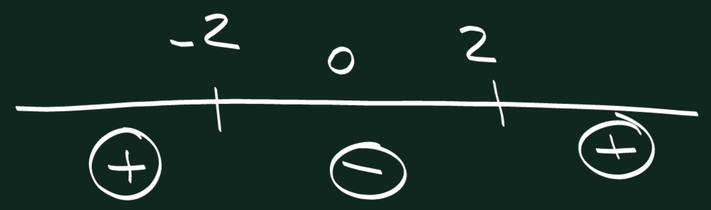
$x^2 = t \rightarrow x^4 = t^2$

$t^2 - 3t - 4 \leq 0$

$t^2 - 3t - 4 = 0 \rightarrow t = \dfrac{3 \pm \sqrt{9+16}}{2} = \dfrac{3 \pm 5}{2} \begin{cases} 4 \\ -1 \end{cases}$

$t = x^2 = 4 \rightarrow x = \pm\sqrt{4} \begin{cases} 2 \\ -2 \end{cases}$

$x = \pm\sqrt{-1} = \nexists$



Sol: $[-2, 2]$

Inecuacións Racionais

1º Pasamos todos os termos ao 1º membro.

2º Factorizamos num. e den.

3º Resolvemos num. e den. por separado.

4º Facemos os intervalos

5º Comprobamos as solucións (signo) en cada intervalo.

④ e) $\frac{x^2-1}{x^2} \geq 0$

$\frac{(x+1)(x-1)}{x^2} \geq 0$

$(x+1)\cdot(x-1)=0$

$\boxed{x=1}$

$\boxed{x=-1}$

$x^2=0 \rightarrow \boxed{x=0}$

	-5	-1	0	+1
x^2-1	+	−	−	+
x^2	+	+	+	+
$\frac{x^2-1}{x^2}$	+	−	−	+

Sol: $(-\infty,-1] \cup [1,+\infty)$

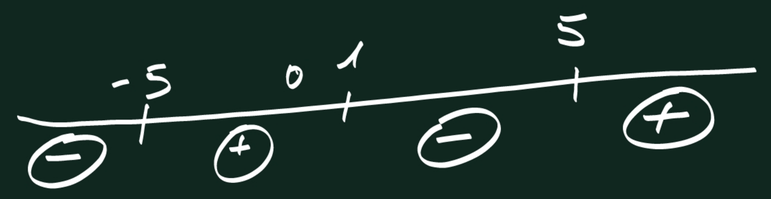
③ c) $x^3 - x^2 - 25x + 25 > 0 \rightarrow (x-1)\cdot(x^2-25) > 0$

$(x-1)\cdot(x+5)(x-5) > 0$

$(x-1)\cdot(x+5)\cdot(x-5) = 0$

$x = 1,\ x = -5,\ x = 5$

	1	-1	-25	25
1		1	0	-25
	1	0	-25	$\boxed{0}$



Sol: $(-5, 1) \cup (5, +\infty)$

4 a) $\frac{1}{x+2} \leq 0$

$x+2=0$

$\boxed{x=-2}$

		-2	
1	$+$		$+$
$x+2$	$-$		$+$
$\frac{1}{x+2}$	$-$		$+$

Sol: $(-\infty,-2)$.

b) $\frac{x^2-9}{x-1} \leq 0$

$x^2-9=0 \begin{cases} x=+3 \\ x=-3 \end{cases}$

$x-1=0 \longrightarrow x=1$

		-3	1	3	
x^2-9	$+$	$-$	$-$		$+$
$x-1$	$-$	$-$	$+$		$+$
$\frac{x^2-9}{x-1}$	$-$	$+$	$-$		$+$

Sol: $(-\infty,-3] \cup (1,3]$

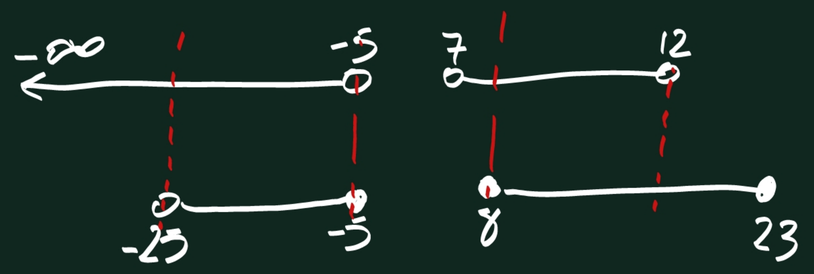
Sistemas de Inecuacións

1º Resolvemos por separado as inecuacións.

2º A solución do sistema serán os intervalos onde coincidan as solucións de todas as inecuacións.

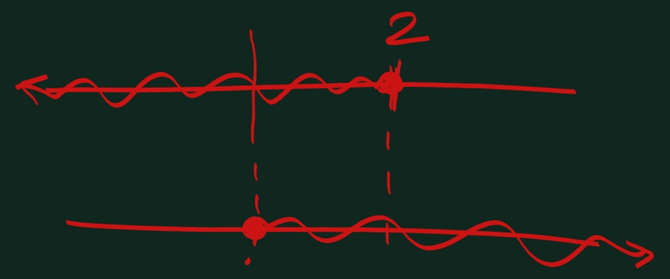
Ex: $Sol_1: (-\infty, -5) \cup (7, 12)$

$Sol_2: (-25, -5] \cup [8, 23]$



$Sol: (-25, -5) \cup [8, 12)$

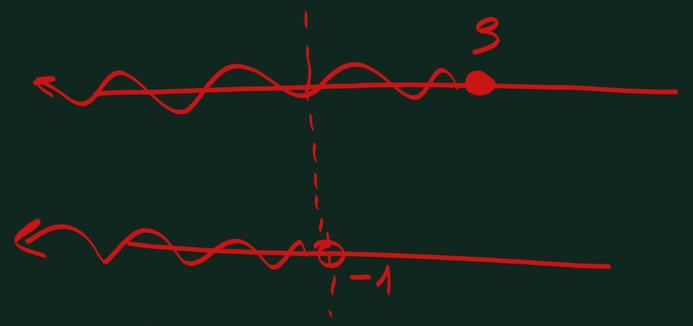
5 a) $\left.\begin{array}{l} x \leq 2 \\ x \geq 0 \end{array}\right\}$



Sol: $[0, 2]$

b) $\left.\begin{array}{l} 2x-3 \leq 6-x \\ 4-2x > 6 \end{array}\right\}$ $\left.\begin{array}{l} 3x \leq 9 \\ -2x > 2 \end{array}\right\}$ $\begin{array}{l} x \leq 3 \\ x < -1 \end{array}$

$-2 > 2x \rightarrow -1 > x$



Sol: $(-\infty, -1)$

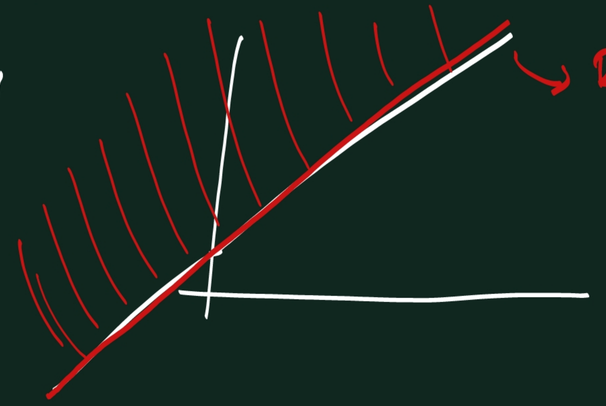
Inecuacións con 2 incógnitas

1º Reescribimos a inecuación deixando a "y" soa: $y \geq \ldots$ ou $y \leq \ldots$

2º Representamos a recta (calculamos 2 puntos)

3º A solución será toda a zona que quede por debaixo ou por enriba da recta.

Ex: $y \geq x+1$



Recta incluída

1 d) $7(3-x) \geq 5$

$21 - 7x \geq 5$

$-7x \geq 5 - 21$

$-7x \geq -16$

$x \leq 16/7$

Sol: $(-\infty, 16/7]$

e) $\dfrac{3-\frac{1}{3}x}{3+\frac{1}{2}} \geq \dfrac{3x-\frac{5}{2}}{1-\frac{2}{3}} \Rightarrow \left(3-\frac{x}{3}\right):\left(3+\frac{1}{2}\right) \geq \left(3x-\frac{5}{2}\right):\left(1-\frac{2}{3}\right)$

$\Rightarrow \left(\frac{18}{6}-\frac{2x}{6}\right):\left(\frac{18}{6}+\frac{3}{6}\right) \geq \left(\frac{18x}{6}-\frac{15}{6}\right):\left(\frac{6}{6}-\frac{4}{6}\right)$

$\Rightarrow (18-2x):(18+3) \geq (18x-15):(6-4) \Rightarrow$

$\Rightarrow \dfrac{18-2x}{21} \geq \dfrac{18x-15}{2} \Rightarrow \dfrac{36-4x}{42} \geq \dfrac{378x-315}{42} \Rightarrow$

$\Rightarrow 36-4x \geq 378x-315 \Rightarrow -378x-4x \geq -315-36 \Rightarrow$

$\Rightarrow -382x \geq -351 \Rightarrow x \leq \dfrac{351}{382}$ Sol: $\left(-\infty, \dfrac{351}{382}\right)$

$$\frac{3-\frac{1}{3}x}{3+\frac{1}{2}} \geq \frac{3x-\frac{5}{2}}{1-\frac{2}{3}}$$

$$\frac{3-\frac{1}{3}x}{\frac{7}{2}} \geq \frac{3x-\frac{5}{2}}{\frac{1}{3}} \Rightarrow \frac{1}{3}\left(3-\frac{1}{3}x\right) \geq \frac{7}{2}\left(3x-\frac{5}{2}\right)$$

④ c) $\frac{x}{x+3} + 1 < 0$

$$\frac{x}{x+3} + \frac{x+3}{x+3} < 0$$

$$\frac{2x+3}{x+3} < 0$$

- $2x+3=0 \rightarrow \boxed{x=-3/2}$
- $x+3=0 \rightarrow \boxed{x=-3}$

		-3		$-3/2$	
$2x+3$	$-$		$-$		$+$
$x+3$	$-$		$+$		$+$
$\frac{2x+3}{x+3}$	$+$		$-$		$+$

Sol: $(-3, -3/2)$

d) $\dfrac{x^2-3x+2}{x^2+2x+6} < 3$

$$\frac{x^2-3x+2}{x^2+2x+6} - 3 < 0$$

$$\frac{x^2-3x+2}{x^2+2x+6} - \frac{3x^2+6x+18}{x^2+2x+6} < 0$$

$$\frac{-2x^2-9x-16}{x^2+2x+6} < 0$$

- $-2x^2-9x-16=0$

$$x = \frac{9 \pm \sqrt{81-128}}{-4} = \nexists \rightarrow$$ Non corta ao eixo 'x'. É sempre negativa (<0)

- $x^2+2x+6=0$

$$x = \frac{-2 \pm \sqrt{4-24}}{2} = \nexists \rightarrow$$ É sempre positiva. (>0)

Num.	$\ominus$
Den.	$\oplus$
In.	$\ominus$

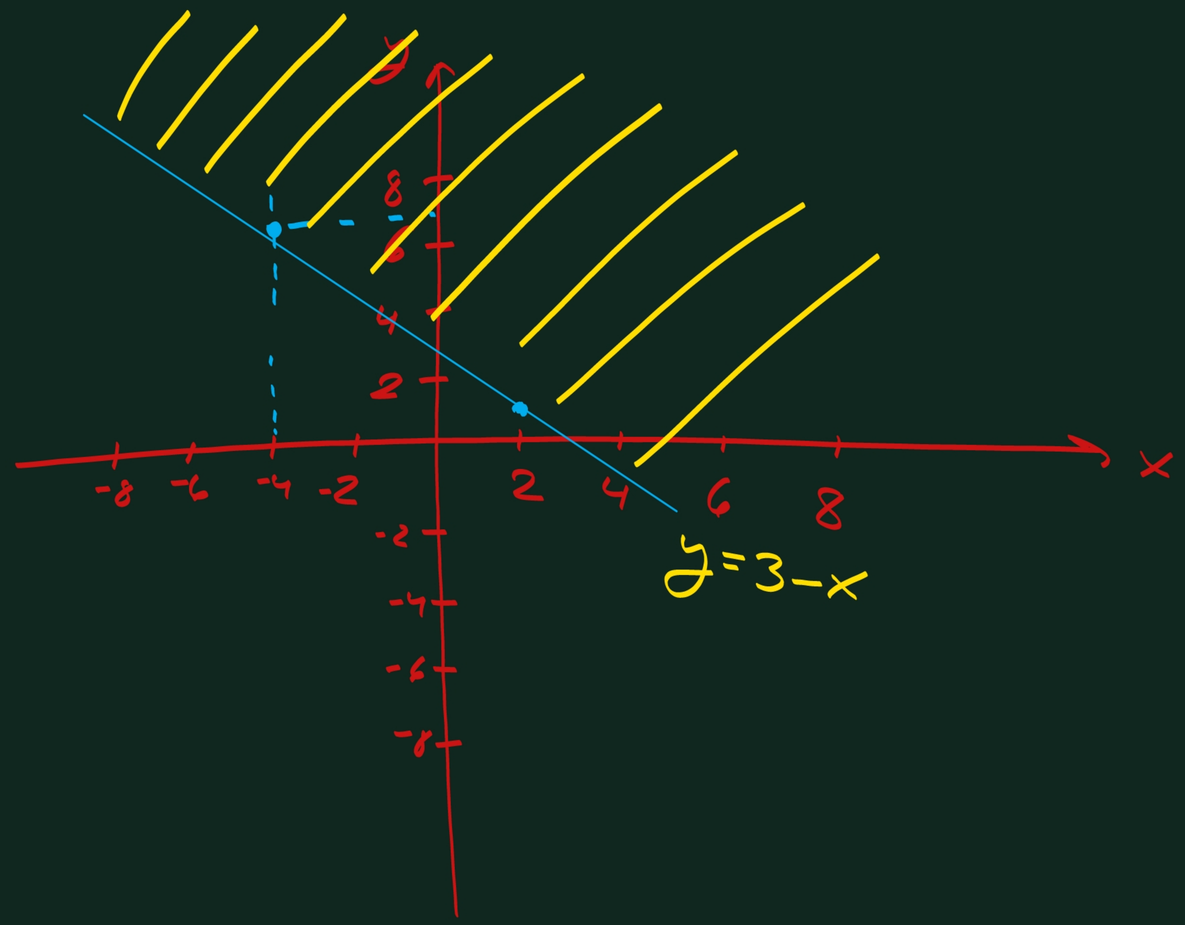
Sol: $\mathbb{R}$

$(-\infty, +\infty)$

6 a) $x + y > 3$

$y > 3 - x$

x	y
2	1
-4	7

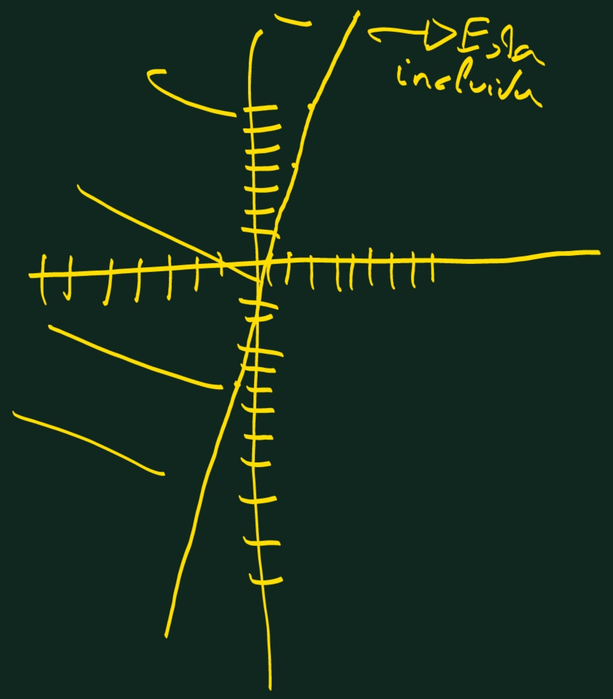


6 b) $3x - y \leq 2$

$-y \leq 2 - 3x$

$y \geq 3x - 2$

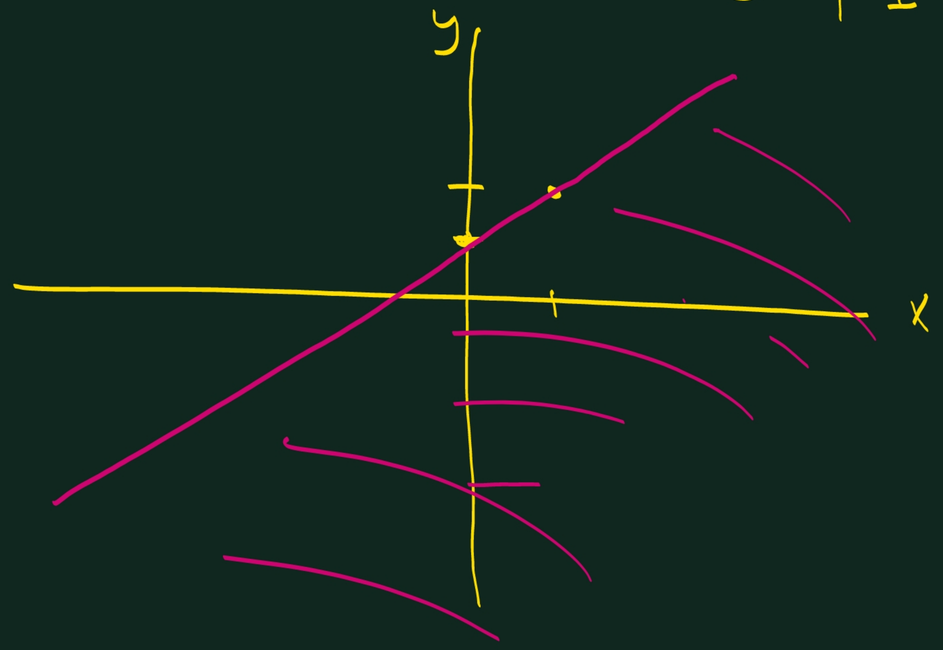
x	y
1	1
2	4
-1	-5
3	7



c) $x + 1 \geq y$

$y \leq x + 1$

x	y
1	2
0	1



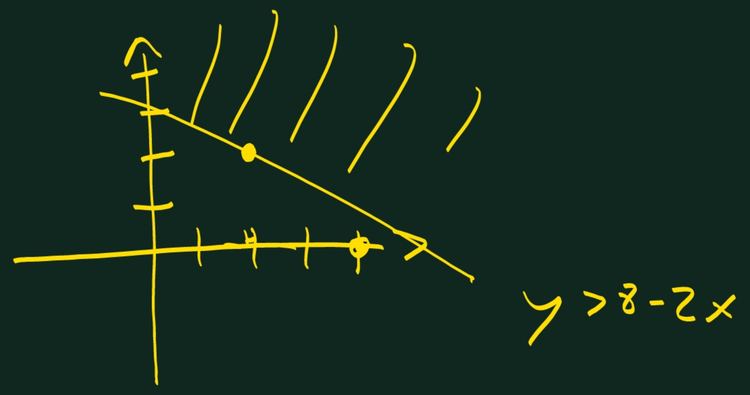
6 d) $x + \frac{y}{2} > 4$

$$\frac{2x}{2} + \frac{y}{2} > \frac{8}{2}$$

$$2x + y > 8$$

$$y > 8 - 2x$$

x	y
2	2
4	0



La recta no se incluye.

5

c) $\left.\begin{array}{l} 3x-1 \geq 7-x \\ 1-x \leq 1-2x \end{array}\right\}$ $\left.\begin{array}{l} 4x \geq 8 \\ x \leq 0 \end{array}\right\}$ $\left.\begin{array}{l} x \geq 2 \rightarrow [2, +\infty) \\ x \leq 0 \rightarrow (-\infty, 0] \end{array}\right\}$

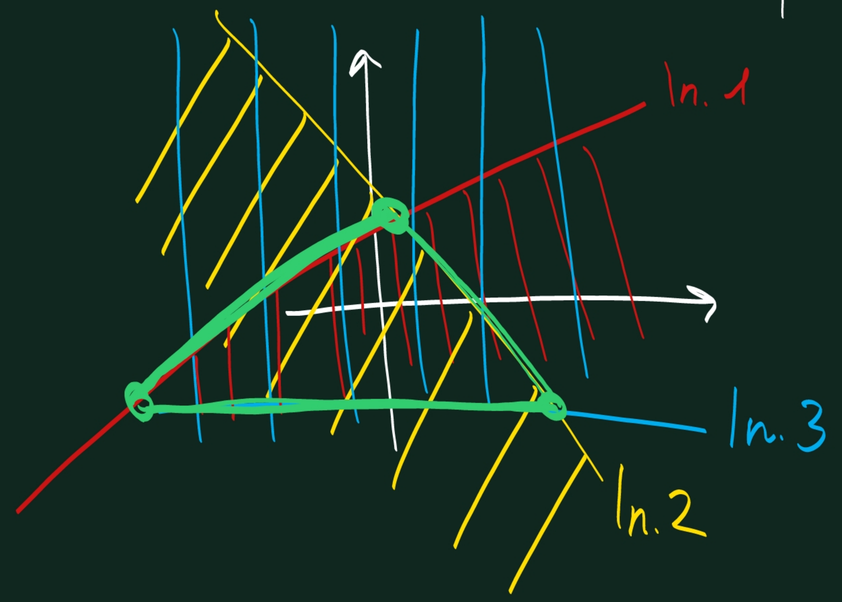
Sol: $\nexists$ $\emptyset$ non existe

Sistemas de Inecuacións con 2 incógnitas

1º Resolver por separado cada inecuación e representamos as rectas.

2º Sombreo a solución de cada inecuación.

3º A solución do sistema será a rexión sombreada por todas as solucións.



7 c)

$$\begin{cases} x-y\geq 0 \\ y-2\leq 0 \\ 2x+y\leq 10 \\ y\geq 0 \end{cases}$$

$-y\geq -x \rightarrow y\leq x$ ①

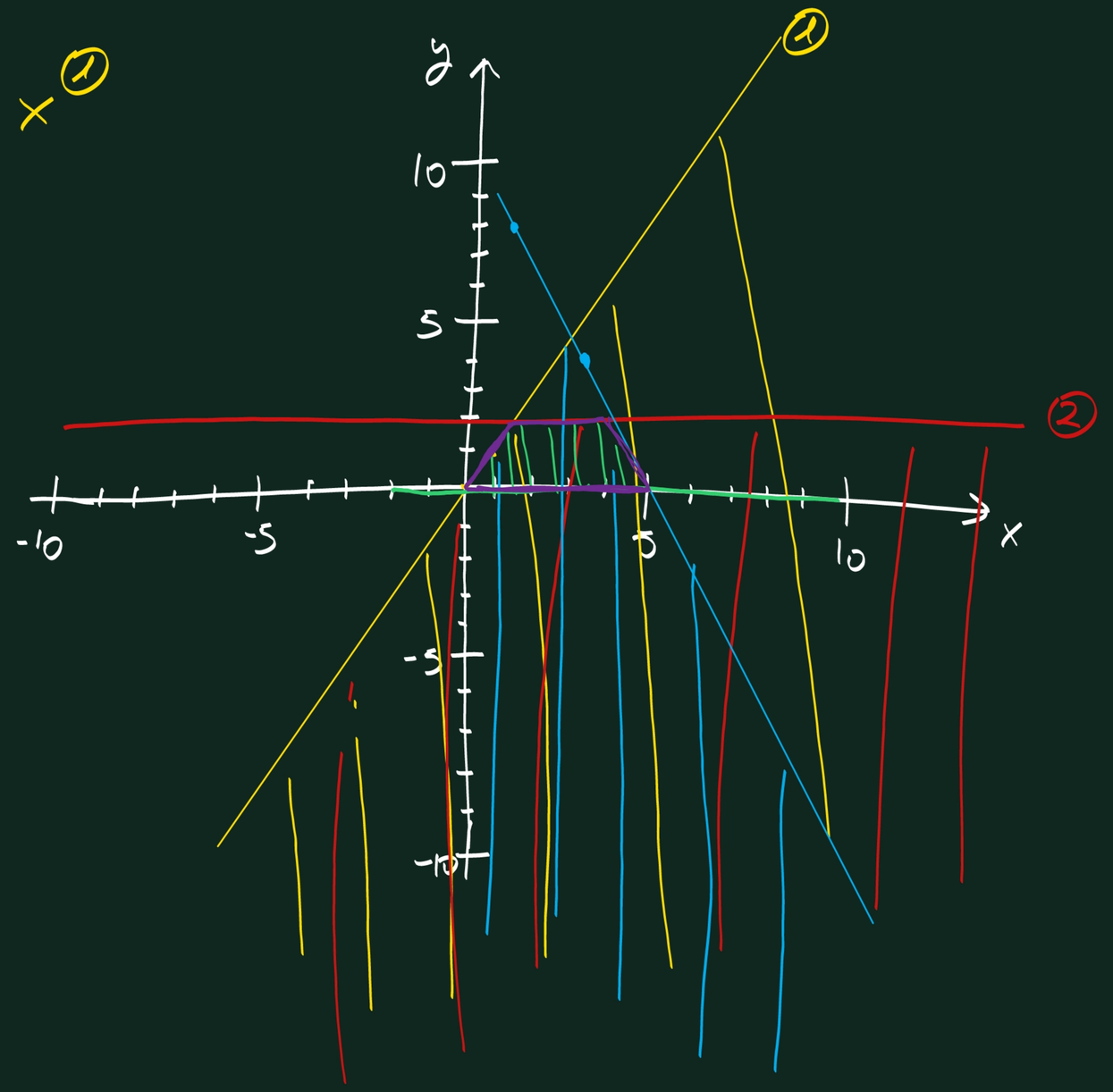
$y\leq 2$ ②

$y\leq 10-2x$ ③

$y\geq 0$ ④

③

x	y
1	8
3	4



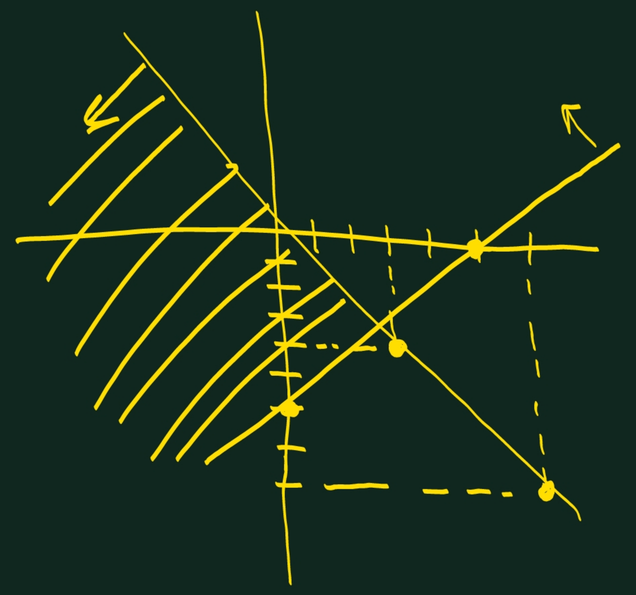
7 a) $\left.\begin{array}{l} y < -2x + 4 \\ y \geq x \end{array}\right\}$

X	Y
0	4
1	2

b) $\left.\begin{array}{l} 6x - 5y \le 30 \\ 4x + 3y \le 0 \end{array}\right\}$ $\left.\begin{array}{l} -5y \le 30 - 6x \\ 3y \le 0 - 4x \end{array}\right\}$ $\left.\begin{array}{l} y \ge \dfrac{30-6x}{-5} \\ y \le \dfrac{-4x}{3} \end{array}\right\}$

x	y
5	0
0	-6

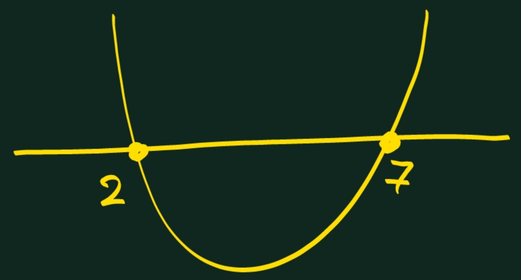
x	y
3	-4
6	-8



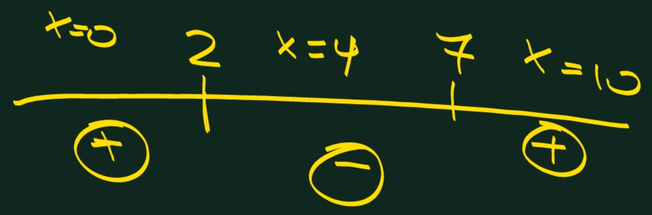
② d) $x^2 - 9x + 14 < 0$

$$x^2 - 9x + 14 = 0$$

$$x = \frac{9 \pm \sqrt{81 - 56}}{2} = \frac{9 \pm \sqrt{25}}{2} = \frac{9 \pm 5}{2} \begin{cases} 7 \\ 2 \end{cases}$$



Sol: (2, 7)



② e) $(x+1)^2-(x-1)^2+12\geq 0$

$x^2+2x+1-(x^2-2x+1)+12\geq 0$

$x^2+2x+1-x^2+2x-1+12\geq 0$

$4x+12\geq 0$

$4x\geq -12$

$x\geq -3$

Sol: $[-3,+\infty)$

② f) $\frac{3x+1}{4} - \frac{1}{3} \leq \frac{2}{15}(3x+2) + \frac{4(1-x)}{3}$

$$\frac{3x+1}{4} - \frac{1}{3} \leq \frac{6x}{15} + \frac{4}{15} + \frac{4-4x}{3}$$

$$\frac{45x+15}{\cancel{60}} - \frac{20}{\cancel{60}} \leq \frac{24x}{\cancel{60}} + \frac{16}{\cancel{60}} + \frac{80-80x}{\cancel{60}}$$

$$45x - 24x + 80x \leq -15 + 20 + 16 + 80$$

$$101x \leq 101$$

$$x \leq 1 \longrightarrow \text{Sol}: (-\infty, 1]$$

② f) $\frac{(x+2)^2}{9} - \frac{x^2-9}{4} \leq \frac{(x+3)^2}{2} + \frac{1}{5}$

$$\frac{20(x+2)^2}{180} - \frac{45(x^2-9)}{180} \leq \frac{90(x+3)^2}{180} + \frac{36}{180}$$

$$20\cdot(x^2+4x+4) - 45(x^2-9) \leq 90(x^2+6x+9)+36$$

$$20x^2+80x+80-45x^2+405 \leq 90x^2+540x+810+36$$

$$+115x^2+460x+361 \geq 0$$

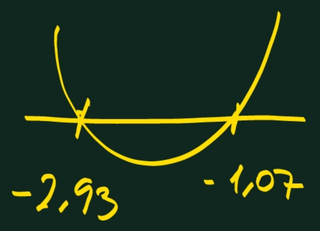
$$x = \frac{-460 \pm \sqrt{211600-116060}}{230} = \frac{-460 \pm 213{,}4}{230} \begin{cases} -1{,}07 \\ -2{,}93 \end{cases}$$

Deberes:

2: f)

3: d) e)

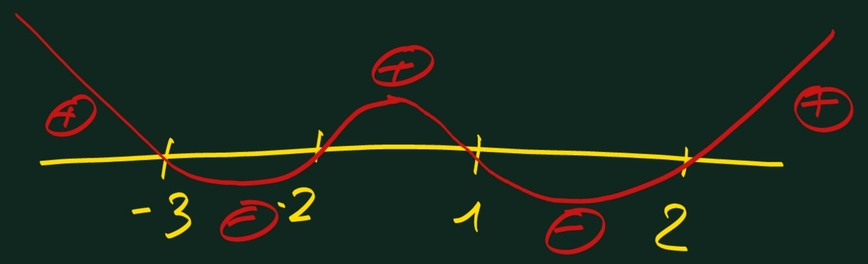
→ Rufini a ta grao 2



Sol: $(-\infty, -2{,}93] \cup [-1{,}07, +\infty)$

③ d) $x^4+2x^3-7x^2-8x+12<0$

	1	2	-7	-8	12
-3	↓	-3	3	12	-12
	1	-1	-4	4	0
-2	↓	-2	6	-4	
	1	-3	2	0	

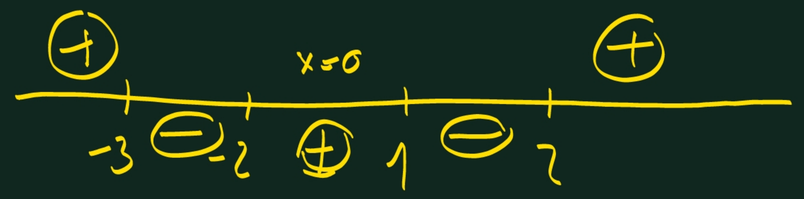


$x^2-3x+2<0$

$x^2-3x+2=0$

$$x=\frac{3\pm\sqrt{(-3)^2-4\cdot1\cdot2}}{2\cdot1}=\frac{3\pm\sqrt{1}}{2}=\frac{3\pm1}{2}=\begin{cases}2\\1\end{cases}$$

sol: $(-3,-2)\cup(1,2)$



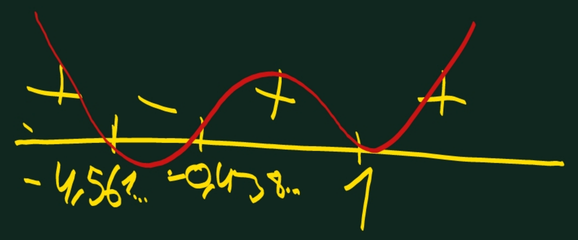
③ e) $x^4+3x^3-7x^2+x+2\le 0$

	1	3	-7	1	2
1		1	4	-3	-2
	1	4	-3	-2	$\boxed{0}$
1		1	5	2	
	1	5	2	$\boxed{0}$	

$x^2+5x+2=0$

$$x=\frac{-5\pm\sqrt{(-5)^2-4\cdot 2}}{2}=\frac{-5\pm\sqrt{17}}{2}=$$

$$\frac{-5\pm 4{,}123}{2}=\begin{cases}\frac{-5+4{,}123\ldots}{2}=-0{,}438\ldots\\ \frac{-5-4{,}123\ldots}{2}=-4{,}561\ldots\end{cases}$$



Sol: $[-4{,}561\ldots,-0{,}438\ldots]\cup\{1\}$

Sistemas de Inecuacións con 2 incógnitas e grao 2

$$\left.\begin{array}{l} y > x^2 - 1 \\ y \leq 2x + 1 \end{array}\right\}$$

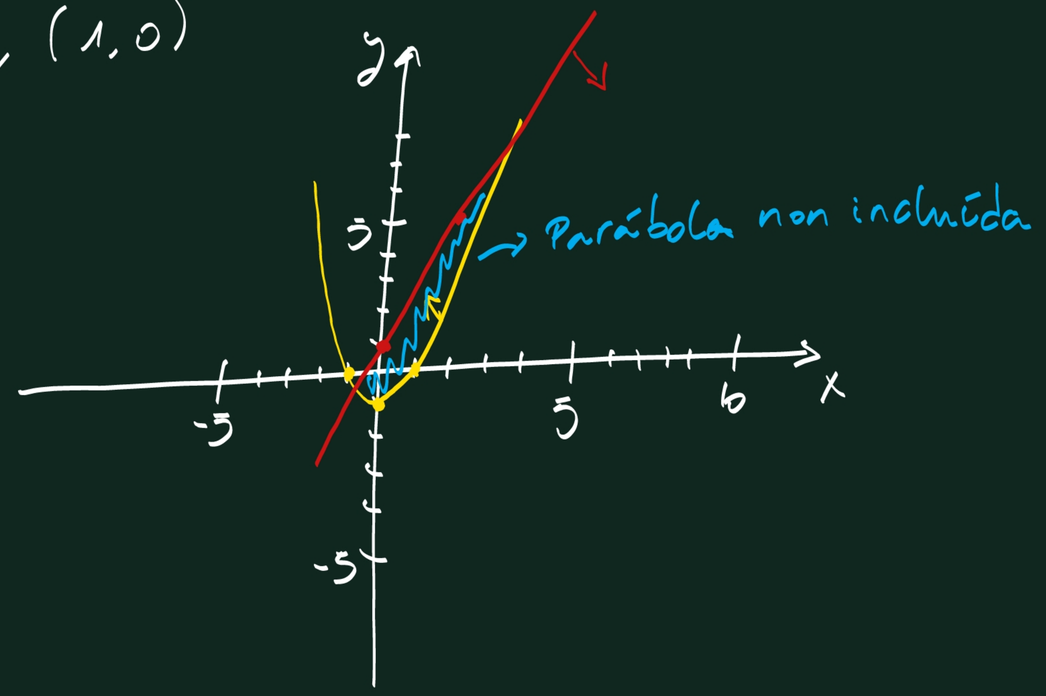
x	y
0	1
2	5

$y > x^2 - 1 \longrightarrow$ Vértice: $(x_0, y_0) = (0, -1)$

$$x_0 = \frac{-b}{2a} = \frac{-0}{2} = 0 \rightarrow y_0 = y(x_0) = -1$$

Ptos. Corte eixo x: $\rightarrow (-1,0), (1,0)$

$y = 0 \longrightarrow x$

$0 = x^2 - 1$

$x = \pm\sqrt{1} = \pm 1$