Unidade 5 — Límites e Continuidade

O límite dunha función cando 'x' tende ao infinito é o valor ao que se aproxima a función ao tomar valores cada vez maiores.

$$f(x) = \frac{200x^2 - 30x + 12}{x^2 + 6x + 9}$$

x	1	10	100	1000	10000
f(x)	11,38	116,64	188,23	198,78	199,89

$$\lim_{x \to +\infty} \frac{200x^2 - 30x + 12}{x^2 + 6x + 9} = 200$$

$$\lim_{x\to+\infty}\left(f(x)\pm g(x)\right)=\lim_{x\to+\infty}f(x)\pm\lim_{x\to+\infty}g(x)$$

$$\lim_{x\to+\infty}\left(f(x)\cdot g(x)\right)=\lim_{x\to+\infty}f(x)\cdot\lim_{x\to+\infty}g(x)$$

$$\lim_{x\to+\infty}\left(f(x)/g(x)\right)=\lim_{x\to+\infty}f(x)/\lim_{x\to+\infty}g(x)$$

$$\lim_{x\to+\infty}f(x)^{g(x)}=\left(\lim_{x\to+\infty}f(x)\right)^{\lim_{x\to+\infty}g(x)}$$

Suma ou Resta

$$+\infty + k = +\infty \qquad \lim (x+3) = +\infty$$
$$+\infty - k = +\infty \qquad \lim (x-3) = +\infty$$
$$+\infty + \infty = +\infty \qquad \lim_{x \to +\infty} (x+3x) = +\infty$$

Multiplicación

$$+\infty \cdot k = +\infty \ (k \neq 0) \qquad \lim_{x \to +\infty} (-2x) = -\infty$$
$$\infty \cdot \infty = +\infty \qquad \lim_{x \to +\infty} x(x+1) = +\infty$$

División

$$\frac{\infty}{k} = +\infty \ (k \neq 0) \qquad \lim_{x \to +\infty} \frac{x-1}{2} = +\infty$$
$$\frac{k}{\infty} = 0 \qquad \lim_{x \to +\infty} \frac{-3}{x+1} = 0$$
$$\frac{k}{0} = +\infty \ (k \neq 0) \qquad \lim_{x \to +\infty} \frac{2}{1/x} = +\infty$$

Potencias

Se $k > 0$ $\begin{cases} \infty^k = +\infty \\ \infty^{-k} = \dfrac{1}{\infty^k} = 0 \end{cases}$

$$\lim_{x \to +\infty} (x^3+5) = +\infty$$
$$\lim_{x \to +\infty} (5x-2)^{-3} = \lim_{x \to +\infty} \frac{1}{(5x-2)^3} = 0$$

Se $k > 1$ $\left\{ k^{+\infty} = \infty \right.$ $\qquad \lim_{x \to +\infty} e^{2x-1} = +\infty$

Se $0 < k < 1$ $\left\{ k^{+\infty} = 0 \right.$ $\qquad \lim_{x \to +\infty} (1/2)^x = 0$

$$\infty^\infty = \infty \qquad \lim_{x \to +\infty} (x-2)^{2x} = +\infty$$

Nalgúns casos, ao realizar operacións con límites obtemos un valor indeterminado. Estas expresións chámanse indeterminacións: $\frac{\infty}{\infty}$, $\infty - \infty$, 1^{∞}

Indeterminacións tipo: ∞/∞

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)}$$

- Se o grao de $P >$ grao de $Q \to \lim_{x\to\infty} = \infty$
- Se o grao de $P <$ grao de $Q \to \lim_{x\to\infty} = 0$
- Se o grao de $P =$ grao de $Q \to \lim_{x\to\infty} = \frac{a}{b}$

$$\lim_{x\to\infty} \frac{3x^3+x}{5x^2+x} = \lim_{x\to\infty} \frac{\frac{3x^3}{x^3}+\frac{x}{x^3}}{\frac{5x^2}{x^3}+\frac{x}{x^3}} = \lim_{x\to\infty} \frac{3+\frac{1}{x^2}}{\frac{5}{x}+\frac{1}{x^2}} = \frac{3+0}{0+0} = \frac{3}{0} = \infty$$

$\downarrow$

$$\lim_{x\to\infty} \frac{3x^3\cancel{+x}}{5x^2\cancel{+x}} = +\infty$$

$$\lim_{x\to\infty} \frac{3x^2+x}{5x^3+x} = 0$$

$$\lim_{x\to\infty} \frac{3x^2+x}{5x^2+x} = \frac{3+0}{5+0} = \frac{3}{5}$$

$\downarrow$

$$\lim_{x\to\infty} \frac{3x^2\cancel{+x}}{5x^2\cancel{+x}} = \lim_{x\to\infty} \frac{3\cancel{x^2}}{5\cancel{x^2}} = \frac{3}{5}$$

Indeterminacións tipo : $\infty - \infty$

$f(x) = P(x) - Q(x)$

$$\lim_{x\to\infty} 5x^2 - 7x = \lim_{x\to\infty} x\cdot(x-7) = \infty\cdot(\infty) = \infty$$

En moitos casos quereremos transformalas noutra función do estilo: $\frac{A(x)}{B(x)}$

$$\lim_{x\to\infty}\left(\frac{3x^2+1}{x+2} - \frac{3x^2-x}{x-1}\right) = \lim_{x\to\infty}\left(\frac{(3x^2+1)\cdot(x-1)-(3x^2-x)\cdot(x+2)}{(x+2)\cdot(x-1)}\right) =$$

$$= \lim_{x\to\infty}\left(\frac{3x^3-3x^2+x-1-3x^3-6x^2+x^2+2x}{x^2-x+2x-2}\right) = \lim_{x\to\infty}\left(\frac{-8x^2+3x-1}{x^2+x-2}\right) =$$

$$= -8$$

$$\lim_{x\to\infty}\left(\sqrt{3x+1} - \sqrt{3x-4}\right) = \lim_{x\to\infty}\frac{(\sqrt{3x+1}-\sqrt{3x-4})\cdot(\sqrt{3x-1}+\sqrt{3x-4})}{\sqrt{3x-1}+\sqrt{3x-4}} =$$

$$= \lim_{x\to\infty}\frac{(3x+1)-(3x-4)}{\sqrt{3x+1}+\sqrt{3x-4}} = \lim_{x\to\infty}\frac{5}{\sqrt{3x+1}+\sqrt{3x-4}} = 0$$

Indeterminacións do tipo 1^{∞}

Buscaremos transformar límites do tipo:

$$\lim_{x\to\infty} f(x)^{g(x)} \quad \text{en} \quad \lim_{x\to\infty}\left(1+\frac{1}{A(x)}\right)^{A(x)}.$$

Se $\lim_{x\to\infty} A(x) = +\infty$ o $\lim_{x\to\infty}\left(1+\frac{1}{A(x)}\right)^{A(x)} = e$

1º- Sumamos e restamos 1. Operamos a resta e deixamos indicada a suma.

2º - Dividimos entre o numerador.

3º - Multiplicamos e dividimos o expoñente polo denominador.

4º - Expresar o expoñente como potencia doutra potencia.

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+4}\right)^{2x-1} = \lim\left(1+\frac{3x-2}{3x+4}-1\right)^{2x-1} =$$

$$= \lim\left(1+\frac{3x-2}{3x+4}-\frac{3x+4}{3x+4}\right)^{2x-1} = \lim\left(1+\frac{-6}{3x+4}\right)^{2x-1} =$$

$$= \lim\left(1+\frac{1}{\frac{3x+4}{-6}}\right)^{2x-1} =$$

$$= \lim\left(1+\frac{1}{\frac{3x+4}{-6}}\right)^{\frac{3x+4}{-6}\cdot\frac{-6}{3x+4}\cdot(2x-1)} =$$

$$= \lim\left(\left(1+\frac{1}{\underbrace{\frac{3x+4}{-6}}_{A(x)}}\right)^{\overbrace{\frac{3x+4}{-6}}^{A(x)}}\right)^{\frac{-6}{3x+4}\cdot(2x-1)} =$$

$$= e^{\lim \frac{-12x+6}{3x+4}} = e^{-4}$$

5) $\lim_{x\to+\infty}\left(\frac{x-2}{x+1}\right)^{3x^2} = (1)^{\infty} = \text{Indet.}$ $\left(\left(1+\frac{1}{A(x)}\right)^{A(x)}\right)^{B(x)}$

$$\lim_{x\to+\infty}\left(1+\frac{x-2}{x+1}-1\right)^{3x^2} = \lim\left(1+\frac{x-2}{x+1}-\frac{x+1}{x+1}\right)^{3x^2} =$$

$$= \lim\left(1+\frac{-3}{x+1}\right)^{3x^2} = \lim\left(1+\frac{1}{\frac{x+1}{-3}}\right)^{3x^2} = \lim\left(1+\frac{1}{\frac{x+1}{-3}}\right)^{\frac{x+1}{-3}\cdot\frac{-3}{x+1}\cdot 3x^2} =$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{\frac{x+1}{-3}}\right)^{\frac{x+1}{-3}}\right)^{\frac{-3}{x+1}\cdot 3x^2} = e^{\lim\frac{-9x^2}{x+1}} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$\lim_{x\to+\infty}\frac{-9x^2}{x+1} = -\infty$$

a) $\lim\limits_{x \to +\infty} 3x^2 + 2x = +\infty$

b) $\lim\limits_{x \to -\infty} \frac{e^x}{3} = \frac{1}{3 \cdot e^{\infty}} = 0$

c) $\lim\limits_{x \to -\infty} \left(2 + \frac{1}{x}\right)^{-x^2} = (0)$

d) $\lim\limits_{x \to +\infty} \frac{3}{\log 2x} - 1 = 0 - 1 \doteq -1$

$$t)\ \lim_{x\to+\infty}\left(\frac{x^2-x}{x^2-2x+1}\right)^{\frac{x^2-1}{3x+2}} = 1^{\infty} = \text{Indet}$$

$$\lim_{x\to+\infty}\left(1+\frac{x^2-x}{x^2-2x+1}-1\right)^{\frac{x^2-1}{3x+2}} = \lim\left(1+\frac{x^2-x}{x^2-2x+1}-\frac{x^2-2x+1}{x^2-2x+1}\right)^{\frac{x^2-1}{3x+2}} =$$

$$\lim\left(1+\frac{x-1}{x^2-2x+1}\right)^{\frac{x^2-1}{3x+2}} = \lim\left(1+\frac{1}{\frac{x^2-2x+1}{x-1}}\right)^{\frac{x^2-1}{3x+2}} = .$$

$$= \lim_{x\to+\infty}\left(\left(1+\frac{1}{\frac{x^2-2x+1}{x-1}}\right)^{\frac{x^2-2x+1}{x-1}}\right)^{\frac{x-1}{x^2-2x+1}\cdot\frac{x^2-1}{3x+2}} = e^{\lim_{x\to\infty}\frac{x^3+\cdots}{3x^3+\cdots}} = e^{1/3}$$

m) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{3x^2-5x}+\sqrt{x^2-7}}{\sqrt{3x^2+9x}-x} = \dfrac{\infty+\infty}{\infty+\infty} = \dfrac{\infty}{\infty} = \text{Ind.}$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2}+\sqrt{x^2}}{\sqrt{3x^2}-x} = \lim_{x\to-\infty} \frac{\frac{\sqrt{3x^2}+\sqrt{x^2}}{x}}{\frac{\sqrt{3x^2}-x}{x}} = \lim_{x\to-\infty} \frac{\sqrt{3\frac{x^2}{x^2}}+\sqrt{\frac{x^2}{x^2}}}{\sqrt{3\frac{x^2}{x^2}}-\frac{x}{x}} =$$

$+\infty$

$$= \lim_{x\to-\infty} \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(-x)$$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2-5x}+\sqrt{x^2-7}}{\sqrt{3x^2+9x}-x} = \lim_{x\to+\infty} \frac{\sqrt{3x^2+\cancel{5x}}+\sqrt{x^2\cancel{-7}}}{\sqrt{3x^2\cancel{-9x}}+x} = \frac{\infty}{\infty} = \text{Ind.}$$

$$= \frac{\sqrt{3x^2}+\sqrt{x^2}}{\sqrt{3x^2}+x} = \frac{x\sqrt{3}+x}{x\sqrt{3}+x} = 1$$

$$\lim_{x\to+\infty} \frac{\frac{\sqrt{3x^2+5x}+\sqrt{x^2-7}}{x}}{\frac{\sqrt{3x^2-9x}+x}{x}} = \lim_{x\to+\infty} \frac{\sqrt{\frac{3x^2+5x}{x^2}}+\sqrt{\frac{x^2-7}{x^2}}}{\sqrt{\frac{3x^2-9x}{x^2}}+\frac{x}{x}} = \frac{\sqrt{3+0}+\sqrt{1-0}}{\sqrt{3-0}+1} = \frac{\sqrt{3}+1}{\sqrt{3}+1} = 1.$$

o) $\lim_{x \to +\infty} \left(\sqrt{5x+1} - \sqrt{2x-3}\right) = \infty - \infty = \text{Ind.}$

$$\lim_{x \to +\infty} \frac{\left(\sqrt{5x+1} - \sqrt{2x-3}\right)\left(\sqrt{5x+1} + \sqrt{2x-3}\right)}{\sqrt{5x+1} + \sqrt{2x-3}} = \lim_{x \to +\infty} \frac{5x+1-2x+3}{\sqrt{5x+1}+\sqrt{2x-3}} =$$

$$= \lim_{x \to +\infty} \frac{\overbrace{3x+4}^{\text{Grao: } 1}}{\underbrace{\sqrt{\cdots} + \sqrt{\cdots}}_{\text{Grao: } 1/2}} = +\infty$$

p) $\lim\limits_{x\to-\infty} \sqrt{3x^2+2x} - \sqrt{3x^2-3} = \lim\limits_{x\to+\infty} \sqrt{3x^2-2x} - \sqrt{3x^2-3} =$

$$= \lim_{x\to+\infty} \sqrt{x(3x-2)} - \sqrt{3x^2-3} = \sqrt{\infty} - \sqrt{\infty} = \infty - \infty = \text{Ind}$$

$$\lim_{x\to+\infty} \left(\sqrt{x(3x-2)} - \sqrt{3x^2-3}\right) \cdot \frac{\sqrt{x(3x-2)}+\sqrt{3x^2-3}}{\sqrt{x(3x-2)}+\sqrt{3x^2-3}} =$$

$$= \lim_{x\to+\infty} \frac{x(3x-2)-(3x^2-3)}{\sqrt{x(3x-2)}+\sqrt{3x^2-3}} = \lim \frac{3x^2-2x-3x^2+3}{\sqrt{x(3x-2)}+\sqrt{3x^2-3}} = \lim_{x\to+\infty} \frac{-2x+3}{\sqrt{3x^2-2x}+\sqrt{3x^2-3}} =$$

$$= \lim_{x\to+\infty} \frac{-2x}{\sqrt{3x^2}+\sqrt{3x^2}} = \lim_{x\to+\infty} \frac{-\cancel{2}x}{\cancel{2}\cdot\sqrt{3x^2}} = \lim \frac{-\cancel{x}}{\cancel{x}\sqrt{3}} = \boxed{\frac{-1}{\sqrt{3}}} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$$

u) $\lim_{x\to+\infty}\left(\dfrac{\sqrt{x}}{\sqrt{x}+1}\right)^{\sqrt{x}} = 1^{\infty} = \text{Ind.}$

$$\lim_{x\to+\infty}\left(1+\underbrace{\frac{\sqrt{x}}{\sqrt{x}+1}-1}\right)^{\sqrt{x}} = \lim_{x\to+\infty}\left(1+\frac{\sqrt{x}-\sqrt{x}-1}{\sqrt{x}+1}\right)^{\sqrt{x}} = \lim_{x\to+\infty}\left(1+\frac{(-1)}{\sqrt{x}+1}\right)^{\sqrt{x}} =$$

$$= \lim_{x\to+\infty}\left(1+\frac{1}{(-\sqrt{x}-1)}\right)^{\sqrt{x}\cdot\frac{(-\sqrt{x}-1)}{(-\sqrt{x}-1)}} = \lim_{x\to+\infty}\left(\left(1+\frac{1}{(-\sqrt{x}-1)}\right)^{(-\sqrt{x}-1)}\right)^{\frac{\sqrt{x}}{(-\sqrt{x}-1)}} =$$

$$\lim_{x\to+\infty}\left(1+\frac{1}{f(x)}\right)^{f(x)} = e$$

$$= e^{\lim\limits_{x\to+\infty}\frac{\sqrt{x}}{-\sqrt{x}-1}} = e^{-1} = 1/e$$

n) $\lim_{x \to +\infty} \left(\frac{x^3 - 5x}{x^2 - 1} - \frac{x^2+1}{x+1} \right) = (\infty - \infty) = \text{Ind.}$

$(x+1)(x-1)$

$$\lim_{x \to +\infty} \left(\frac{x^3 - 5x - (x-1)\cdot(x^2+1)}{(x+1)(x-1)} \right) = \lim_{x \to +\infty} \left(\frac{x^3 - 5x - x^3 - x + x^2 + 1}{(x+1)(x-1)} \right) =$$

$$= \lim_{x \to +\infty} \left(\frac{x^2 \cancel{- 6x + 1}}{x^2 \cancel{- 1}} \right) = 1$$

e) $\lim_{x \to +\infty} \left(\frac{3}{2\sqrt{x}} \right)^{\ln(x^2)} = 0^{\infty} = 0$

f) $\lim_{x \to +\infty} \frac{3x^3 - 3x + 5}{x^4 - 2x^2} = \frac{\infty}{\infty} = \text{Ind.}$

$$\lim_{x \to +\infty} = \frac{3x^3 - 3x + 5}{x^4 - 2x^2} = \lim_{x \to +\infty} \frac{\frac{3\cancel{x^3}}{x^{\cancel{4}}} - \frac{3\cancel{x}}{x^{\cancel{4}3}} + \frac{5}{x^4}}{1\frac{\cancel{x^4}}{\cancel{x^4}} - \frac{2\cancel{x^2}}{x^{\cancel{4}2}}} = \frac{\frac{3}{\infty} - \frac{3}{\infty} + \frac{5}{\infty}}{1 - \frac{2}{\infty}} = \frac{0 - 0 + 0}{1 - 0} = \frac{0}{1} = 0$$

v) $\lim\limits_{x \to -\infty} \left(\dfrac{3x}{3x-1}\right)^{1-x} = \lim\limits_{x \to +\infty} \left(\dfrac{-3x}{-3x-1}\right)^{x+1} = 1^{\infty} =$ Indeterminación

$$\lim_{x \to +\infty} \left(1 + \frac{-3x}{-3x-1} - 1\right)^{x+1} = \lim_{x \to +\infty} \left(1 + \frac{-3x}{-3x-1} - \frac{-3x-1}{-3x-1}\right)^{x+1} = \lim_{x \to +\infty} \left(1 + \frac{1}{-3x-1}\right)^{x+1} =$$

$$\lim_{x \to +\infty} \left(1 + \frac{1}{\underset{A(x)}{-3x-1}}\right)^{\overset{A(x)}{(-3x-1)} \cdot \frac{x+1}{(-3x-1)}} = e^{\lim\limits_{x \to +\infty} \frac{x+1}{-3x-1}} = e^{-1/3}$$

$$g)\ \lim_{x\to+\infty} \frac{\sqrt{3x^3-2x+1}}{-2x^2+3x-1} = \frac{\infty}{-\infty} = \text{Ind.}$$

$$\lim_{x\to+\infty} \frac{\sqrt{3x^3\cancel{-2x+1}}}{-2x^2\cancel{+3x-1}} = \lim_{x\to+\infty} \frac{\sqrt{3}\cdot x^{3/2}}{-2\cdot x^2} = 0$$

Límites de funcións nun punto

"O límite dunha función cando x tende a un punto a, é o valor que toma a función ao estudar o seu comportamento para valores próximos ao punto a, tanto maiores como menores".

- Límites Laterais:

"Son os límites da función cando x toma valores próximos ao punto a".

- Límite pola esquerda: $\lim\limits_{x \to a^-} f(x)$

- Límite pola dereita: $\lim\limits_{x \to a^+} f(x)$

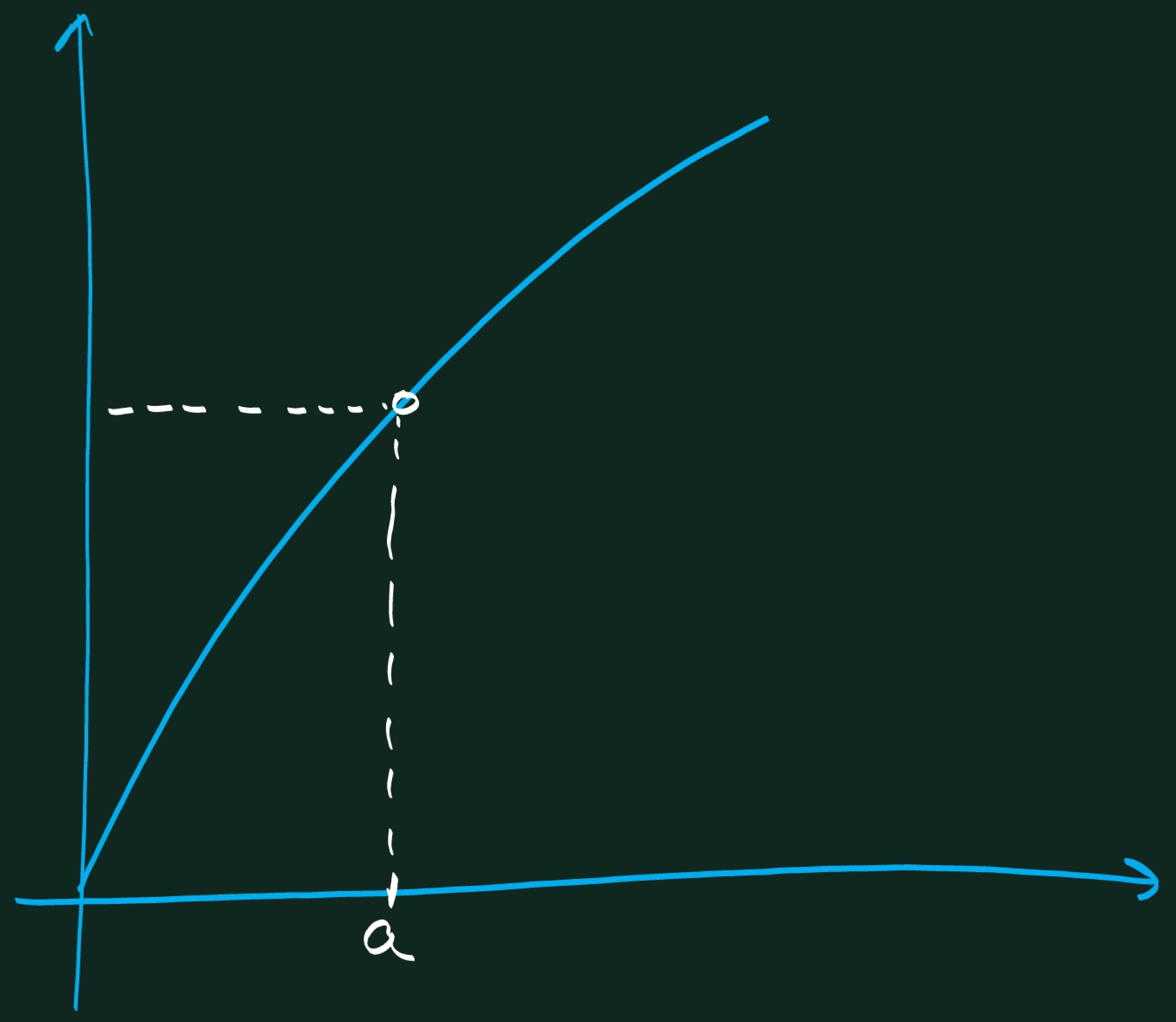
"Unha función ten límite cando x tende a a se:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$"

$$f(x)=\frac{2x^3-210x^2+10000}{x^2-50x-5000}+20$$

x	99	99,9	99,99	100.01	100.1	101
$f(x)$	138,2	139,82	139,98	140,02	140,18	141,8

$$\lim_{x \to 100^-} f(x) = 140 \qquad \lim_{x \to 100^+} f(x) = 140$$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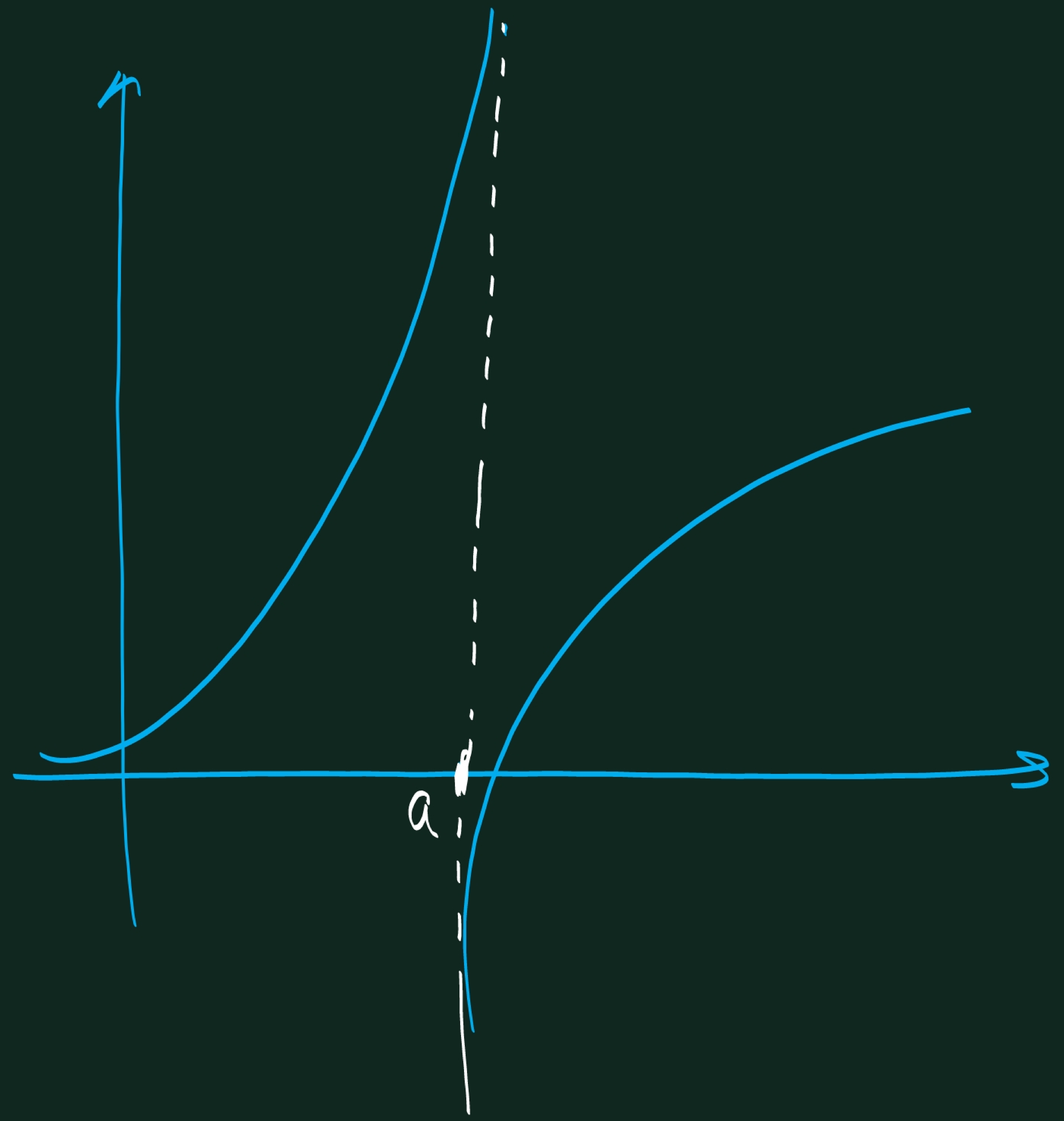
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$$\lim_{x\to 2} \frac{1}{x^2-4} = \frac{1}{4-4} = \frac{1}{0} = \infty \ ? \longrightarrow \lim_{x\to 2} \frac{1}{x^2-4} = \nexists$$

$$\left.\begin{array}{l} \displaystyle\lim_{x\to 2^-} \frac{1}{x^2-4} = \frac{1}{0^-} = -\infty \\[2em] \displaystyle\lim_{x\to 2^+} \frac{1}{x^2-4} = \frac{1}{0^+} = +\infty \end{array}\right\}$$

Como os límites laterais non coinciden, a función non ten límite cando x tende a 2

Indeterminación do tipo $\frac{0}{0}$

Veñen de funcións tipo: $f(x) = \dfrac{A(x)}{B(x)}$

$$\lim_{x \to -2} \frac{2x^3+2x^2+8}{3x^2+5x-2} = \frac{-16+8+8}{12-10-2} = \frac{0}{0}$$

1º Factorizar arriba e abaixo usando o valor ao que tende **x**.

$$\begin{array}{r|rrrr} & 2 & 2 & 0 & 8 \\ -2 & & -4 & +4 & -8 \\ \hline & 2 & -2 & 4 & 0 \end{array} \qquad \begin{array}{r|rrr} & 3 & 5 & -2 \\ -2 & & -6 & +2 \\ \hline & 3 & -1 & 0 \end{array}$$

2º Reescribimos os polinomios factorizados, simplificamos e calculamos o límite.

$$\lim_{x \to -2} \frac{\cancel{(x+2)} \cdot (2x^2-2x+4)}{\cancel{(x+2)} \cdot (3x-1)} = \frac{8+4+4}{-6-1} = -\frac{16}{7}$$

a) $$\lim_{x \to -2} \frac{2x^3+2x^2+8}{3x^2+5x-2} = \frac{-16+8+8}{12-10-2} = \frac{0}{0}$$

	2	2	0	8
-2		-4	4	-8
	2	-2	4	$\boxed{0}$

	3	5	-2
-2		-6	+2
	3	-1	$\boxed{0}$

$$\lim_{x \to -2} \frac{\cancel{(x+2)} \cdot (2x^2-2x+4)}{\cancel{(x+2)} \cdot (3x-1)} = \frac{8+4+4}{-6-1} = -\frac{16}{7}$$

b) $\lim\limits_{x \to 1} \frac{x^2-x}{x^3-1} = \frac{1-1}{1-1} = \frac{0}{0}$

$$\begin{array}{c|ccc} & 1 & -1 & 0 \\ 1 & & 1 & 0 \\ \hline & 1 & 0 & \boxed{0} \end{array} \qquad \begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$

$$\lim_{x \to 1} \frac{x^2-x}{x^3-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}\,(x)}{\cancel{(x-1)}\,(x^2+x+1)} = \frac{1}{1+1+1} = \frac{1}{3}$$

c) $\lim\limits_{x \to 2} \dfrac{3x-5}{x+2} = \dfrac{6-5}{2+2} = \dfrac{1}{4}$

d) $\lim\limits_{x \to \sqrt{3}} \dfrac{e^{2x}}{2x^2-6} = \dfrac{e^{2\sqrt{3}}}{2 \cdot 3 - 6} = \dfrac{e^{2\sqrt{3}}}{0} = \infty$?

$\lim\limits_{x \to \sqrt{3}^-} \dfrac{e^{2x}}{2x^2-6} = \dfrac{e^{2\sqrt{3}}}{0^-} = -\infty$

$\lim\limits_{x \to \sqrt{3}^+} \dfrac{e^{2x}}{2x^2-6} = \dfrac{e^{2\sqrt{3}}}{0^+} = +\infty$

Non hai limite

cc) $\lim\limits_{x \to -\infty} \dfrac{5x^3 - 4x^2}{3x^4 - x} =$

$\lim\limits_{x \to +\infty} \dfrac{5(-x)^3 - 4(-x)^2}{3(-x)^4 - (-x)} = \lim\limits_{x \to +\infty} \dfrac{-5x^3 - 4x^2}{+3x^4 + x} = \dfrac{-\infty}{\infty}$

$\lim\limits_{x \to +\infty} \dfrac{-5x^3 \ldots}{3x^4 \ldots} = 0$

dd) $\lim_{x \to +\infty} \frac{\sqrt{x^3+3x^2}}{\sqrt[3]{5x^4-x}} = \frac{\sqrt{\infty^3+3\infty^2}}{\sqrt[3]{5\infty^4-\infty}} = \frac{\infty}{\infty} = \text{IND}$

$$\lim_{x \to \infty} \frac{x^{3/2} + \ldots}{5^{1/3} \cdot x^{4/3} + \ldots} = \infty$$

ee) $\lim_{x \to +\infty} \dfrac{\sqrt[3]{4x^2+3}}{\sqrt{6x+4}} = \dfrac{\infty}{\infty} = \text{IND}$

$$\lim_{x \to +\infty} \frac{4^{1/3}\, x^{2/3} \ldots}{6^{1/2}\, x^{1/2} \ldots} = \infty$$

ff) $$\lim_{x \to -\infty} \frac{3x^6+2x^2+3x}{2x^6+x^5-1} = \frac{\infty}{\infty} = \text{Indet.}$$

$$\lim_{x \to -\infty} \frac{3x^6+\dots}{2x^6+\dots} = \frac{3}{2}$$

$$\lim_{x \to -\infty} \frac{3\frac{x^6}{x^6}+2\frac{x^2}{x^6}+\frac{3x}{x^6}}{2\frac{x^6}{x^6}+\frac{x^5}{x^6}-\frac{1}{x^6}} = \lim_{x \to -\infty} \frac{3+\frac{2}{x^4}+\frac{3}{x^5}}{2+\frac{1}{x}-\frac{1}{x^6}} = \frac{3+0+0}{2+0-0} = \frac{3}{2}$$

Asíntotas

"É unha recta á que a función se aproxima infinitamente, pero nunca alcanza".

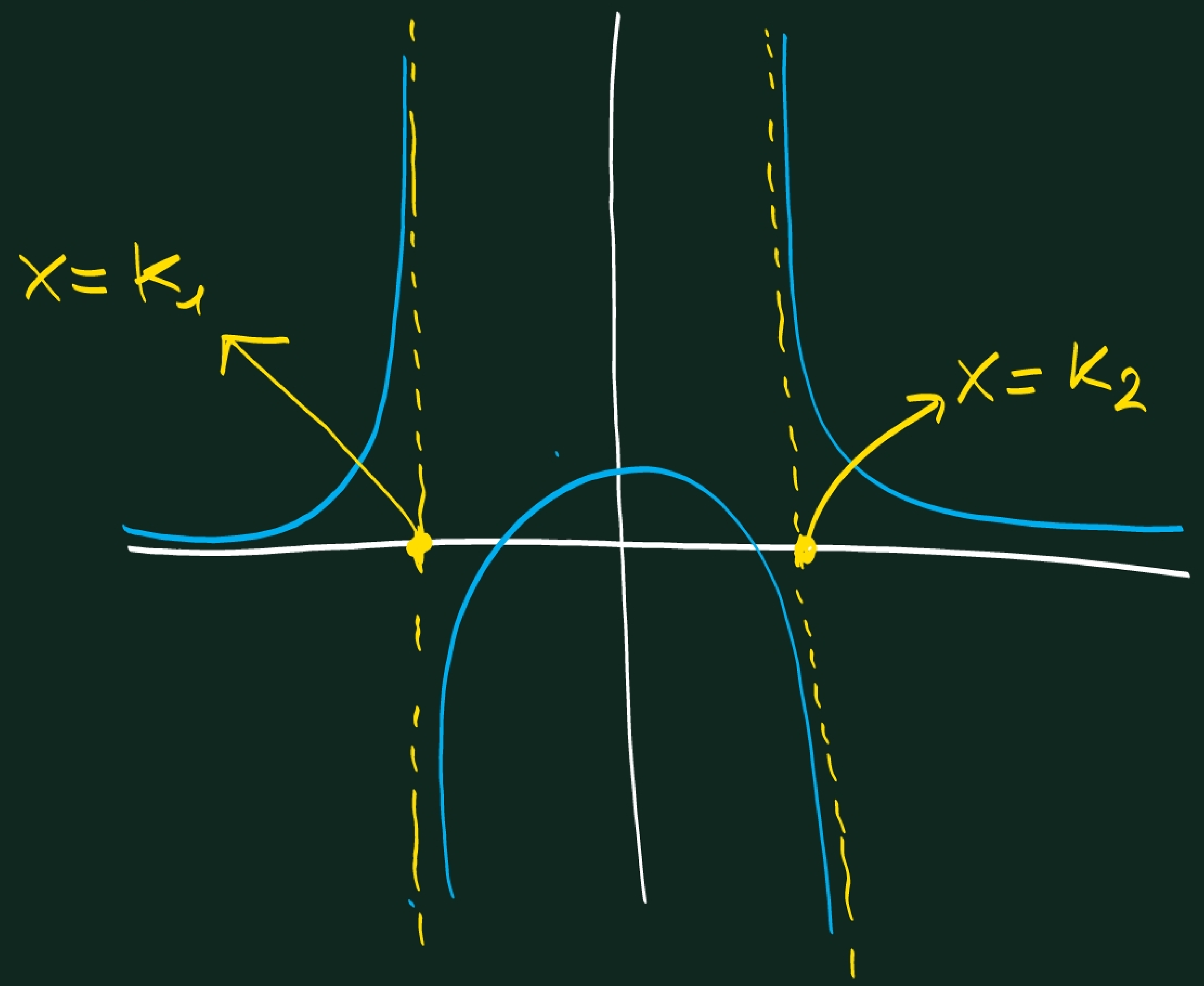
- Verticais
- Horizontais
- Oblicuas

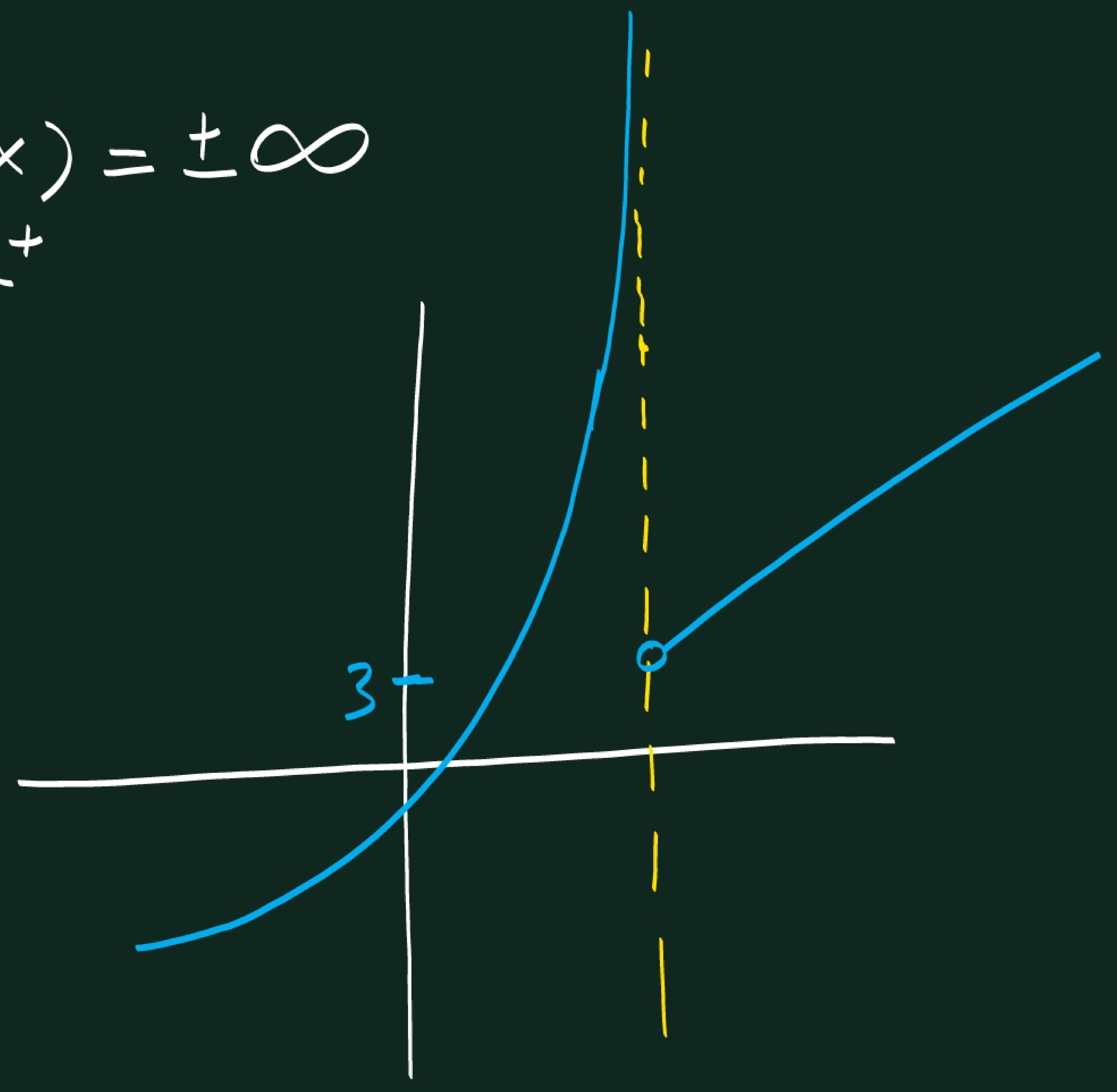
A. Verticais

Poden existir nos valores de "x" onde a función non está definida.

Son rectas do tipo: $x = k$, existen cando:

$$\lim_{x \to k^-} f(x) = \pm\infty \quad \text{e/ou} \quad \lim_{x \to k^+} f(x) = \pm\infty$$



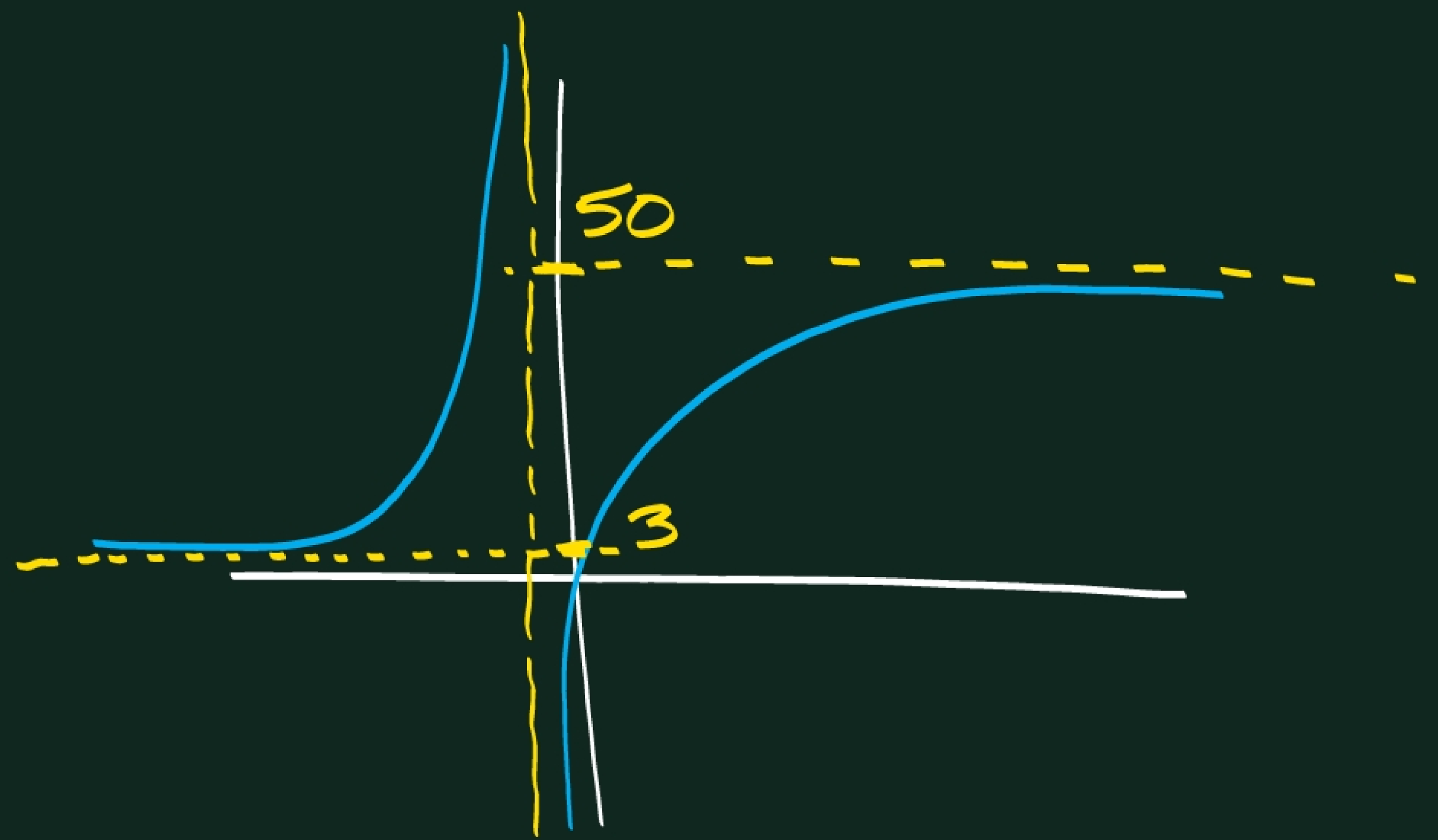


A. Horizontais

Poden existir no infinito.

Son rectas do tipo: $y = k$, existen cando:

$\lim\limits_{x \to -\infty} f(x) = K$ ou $\lim\limits_{x \to +\infty} f(x) = K$; sendo "k" un número real.



$\lim\limits_{x \to -\infty} f(x) = 3 \longrightarrow$ AH: $y = 3$

$\lim\limits_{x \to +\infty} f(x) = 50 \longrightarrow$ AH: $y = 50$

- Posición da función

$$\lim_{x \to \pm\infty} \left(f(x) - k \right) = \begin{cases} 0^+ : & f(x) \text{ está por enriba da A.H.} \\ 0^- : & f(x) \text{ está por abaixo da A.H.} \end{cases}$$

Asíntotas Oblicuas

A función tende a unha recta do tipo: $y = mx + n$

$$m = \lim_{x \to \pm\infty} \frac{f(x)}{x} \quad ; \quad n = \lim_{x \to \pm\infty} \left(f(x) - mx \right)$$

$$\lim_{x \to \pm\infty} \left(f(x) - (mx + n) \right) \begin{cases} 0^- : f(x) \text{ está por debaixo} \\ 0^+ : f(x) \text{ está por enriba} \end{cases}$$

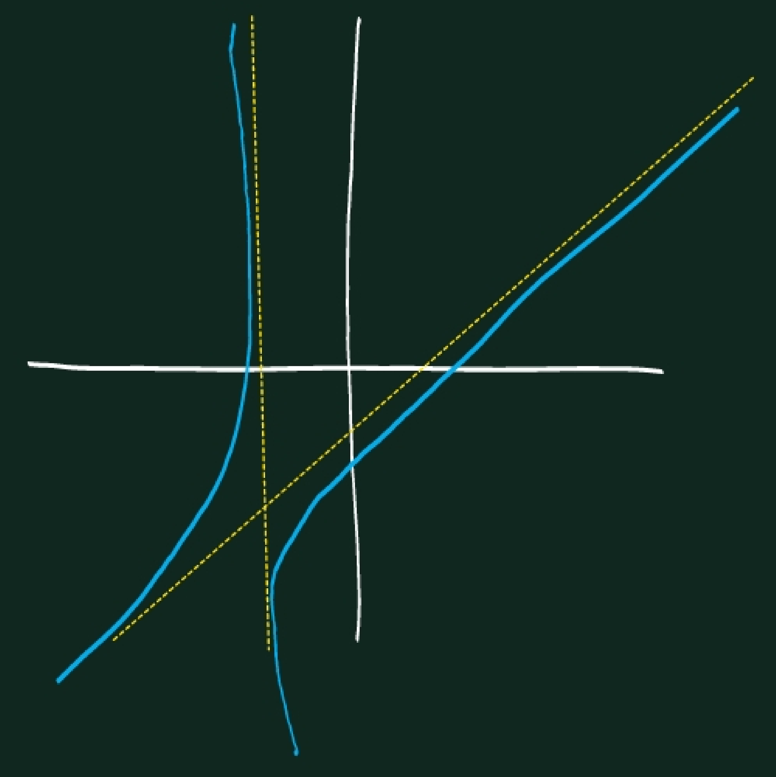
$$f(x)=\frac{x^2-5}{x+2} \longrightarrow Dom = \mathbb{R}-\{-2\}$$

A.V. $\longrightarrow x=-2$

$$\lim_{x\to -2}\frac{x^2-5}{x+2}=\frac{4-5}{-2+2}=\frac{-1}{0}=\infty \longrightarrow x=-2 \text{ é unha A.V.}$$

$$\lim_{x\to -2^-}\frac{x^2-5}{x+2}=\frac{-1}{0^-}=+\infty$$

$$\lim_{x\to -2^+}\frac{x^2-5}{x+2}=\frac{-1}{0^+}=-\infty$$



A.H. $\longrightarrow$ Non hai.

$$\lim_{x\to -\infty}\frac{x^2-5}{x+2}=\lim_{x\to +\infty}\frac{x^2-5}{-x+2}=\frac{\infty}{-\infty}=Ind.$$

$$\lim_{x\to -\infty}\frac{\frac{x^2}{x^2}-\frac{5}{x^2}}{\frac{x}{x^2}+\frac{2}{x^2}}=\lim_{x\to -\infty}\frac{1-0}{0+0}=+\infty$$

$$\lim_{x\to +\infty}\frac{x^2-5}{x+2}=+\infty$$

A.O.

$\frac{x^2-5}{x+2}:x$

$x\to +\infty$

$$m=\lim_{x\to +\infty}\frac{\frac{x^2-5}{x+2}}{x}=\lim_{x\to +\infty}\frac{x^2-5}{x^2+2x}=\lim_{x\to +\infty}\frac{x^2+\dots}{x^2+\dots}=1$$

$$n=\lim_{x\to +\infty}\left(\frac{x^2-5}{x+2}-x\right)=\lim_{x\to +\infty}\left(\frac{x^2-5-x^2-2x}{x+2}\right)=\lim_{x\to +\infty}\frac{-2x-5}{x+2}=-2$$

$y=mx+n$

$y=x-2$

1

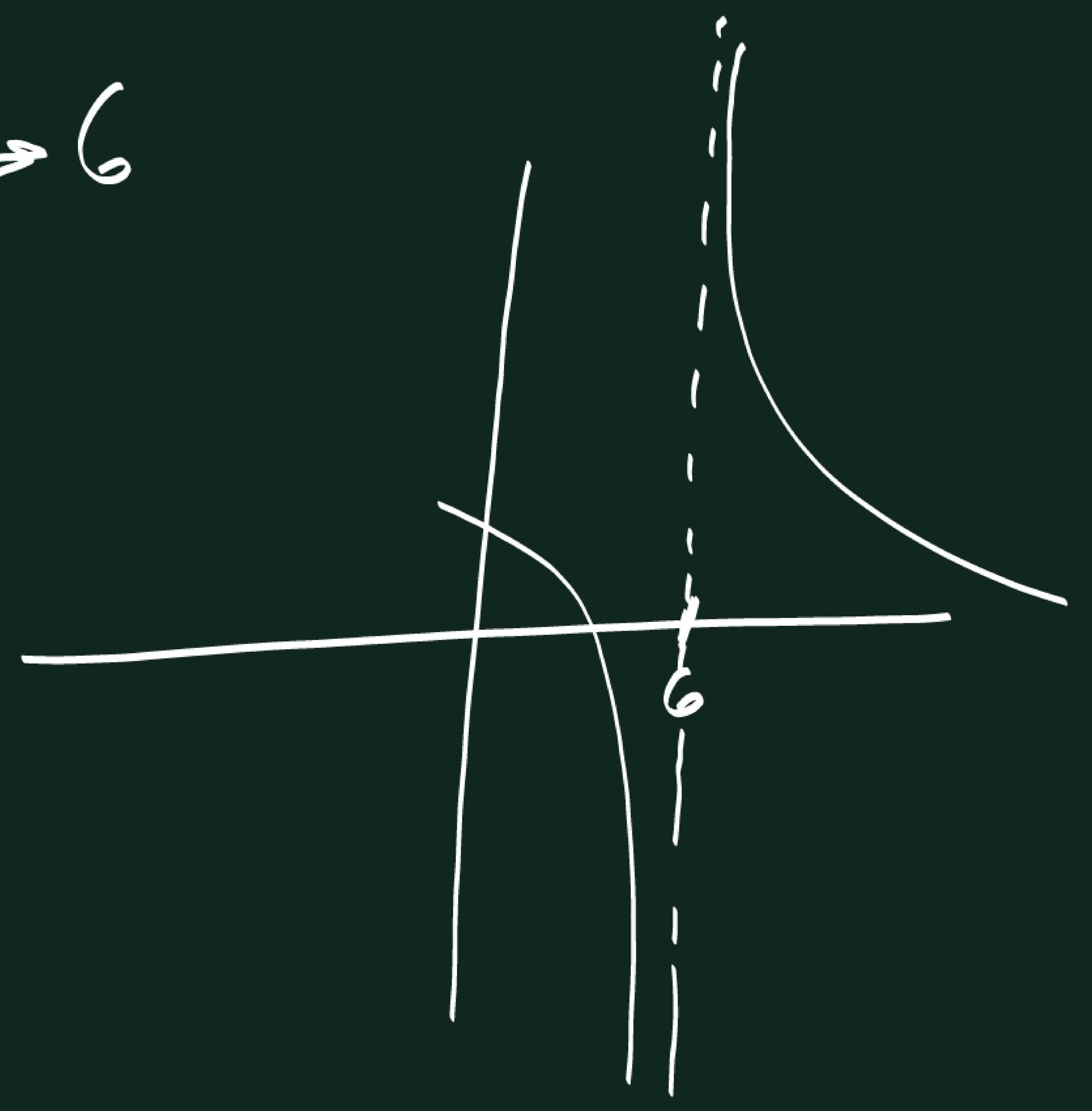
a) $$\lim_{x \to -1} \frac{x^2+3x+2}{x^2+5x^2-2} = \frac{x^2+3x+2}{6x^2-2} = \frac{-1^2+3(-1)+2}{6(1^2)-2} = \frac{1-3+2}{4-2} = \frac{0}{4} = 0$$

b) $$\lim_{x \to 6} \frac{x^3-36x}{x^2-12x+36} = \frac{x(x^2-36)}{(x-6)^2} = \frac{x\cancel{(x-6)}(x+6)}{(x-6)^{\cancel{2}}} = \frac{x(x+6)}{x-6} = \frac{6(6+6)}{6-6} = \frac{72}{0} = \infty$$

$$\lim_{x \to 6^+} f(x) = \frac{\sim}{0^+} = +\infty$$

$$\lim_{x \to 6^-} f(x) = \frac{\cdots}{0^-} = -\infty$$

No existe limite cuando $x \to 6$



② a) $f(x) = \dfrac{x^2+9}{x^3+x}$

$$\lim_{x \to \pm\infty} f(x) \begin{cases} \lim\limits_{x \to +\infty} \dfrac{x^2+9}{x^3+x} = \lim\limits_{x \to +\infty} \dfrac{x^2+\dots}{x^3+\dots} = \lim\limits_{x \to +\infty} \dfrac{1}{x} = 0 \\ \lim\limits_{x \to -\infty} \dfrac{x^2+9}{x^3+x} = \dots\dots\dots = 0 \end{cases}$$

A.H. $\rightarrow y = 0$

③ a) $f(x) = \dfrac{7x-5}{x^3-6x^2+12x-8}$

$Dom = \mathbb{R} - \{2\}$

$x^3 - 6x^2 + 12x - 8 = 0$

$$\begin{array}{c|cccc} & 1 & -6 & +12 & -8 \\ 2 & & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}$$

$x^2 - 4x + 4 = 0 \longrightarrow x = \dfrac{4 \pm \sqrt{16-16}}{2} = \dfrac{4 \pm 0}{2} = 2$

$$\lim_{x \to 2^-} f(x) = \frac{14-5}{8^- - 24^- + 24 - 8} = \frac{9}{0^-} = -\infty$$

$\downarrow$

A.V. $\longrightarrow x = 2$

4 a) $f(x) = \dfrac{4x^2-2x}{x+3}$

$$A.O. \rightarrow y = mx+n \longrightarrow \boxed{y = 4x - 14}$$

$$m = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{4x^2-2x}{x^2+3x} = IND \rightarrow \lim_{x\to\pm\infty} \frac{4x^2+\cdots}{x^2+\cdots} = \frac{4}{1} = 4$$

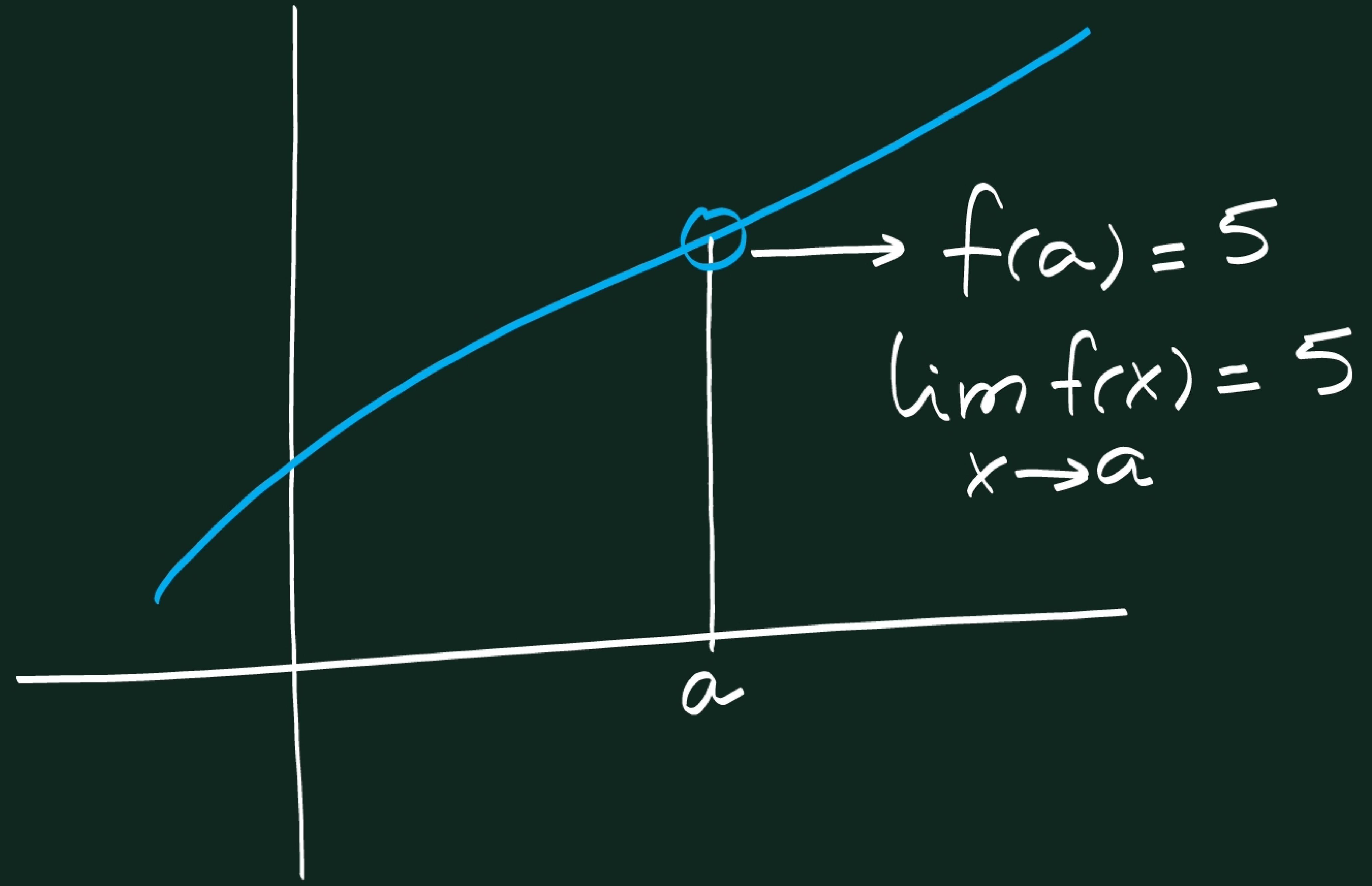
$$n = \lim_{x\to\pm\infty} \left(f(x) - mx \right) = \lim_{x\to\pm\infty} \frac{4x^2-2x}{x+3} - 4x = \lim_{x\to\pm\infty} \frac{4x^2-2x-4x^2-12x}{x+3} =$$

$$\lim_{x\to\pm\infty} \frac{-14x}{x+3} = IND \rightarrow \lim_{x\to\pm\infty} \frac{-14x}{x+\cdots} = -14$$

Continuidade

Unha función e continua nun punto se:

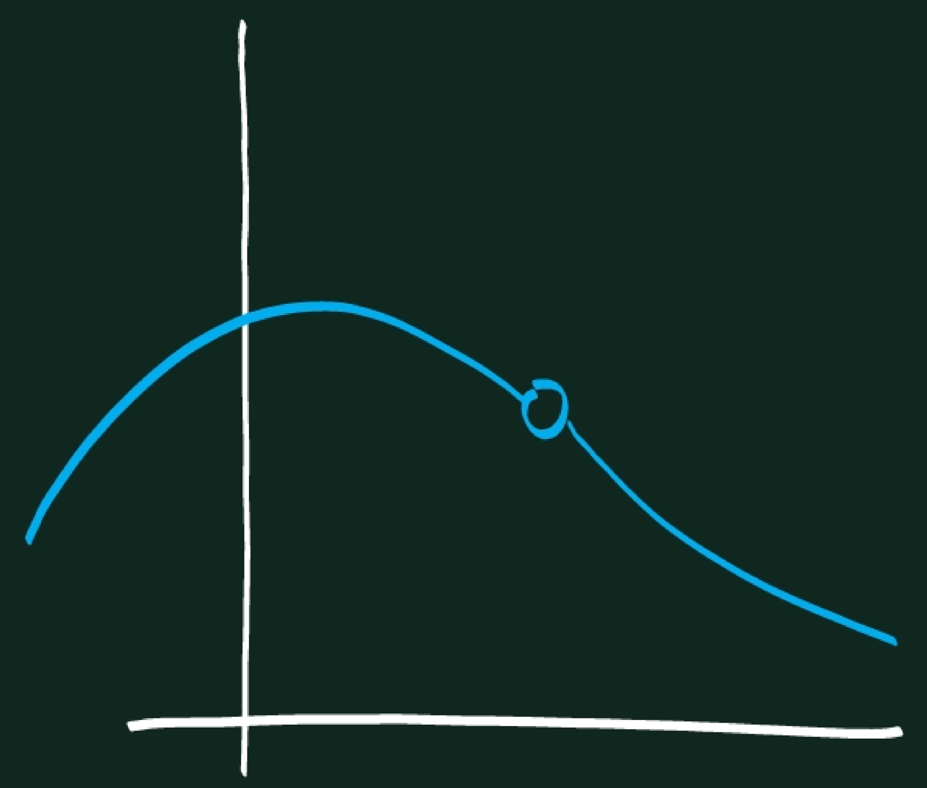
- Existe $f(a)$
- Existe $\lim_{x \to a} f(x)$
- $f(a) = \lim_{x \to a} f(x)$



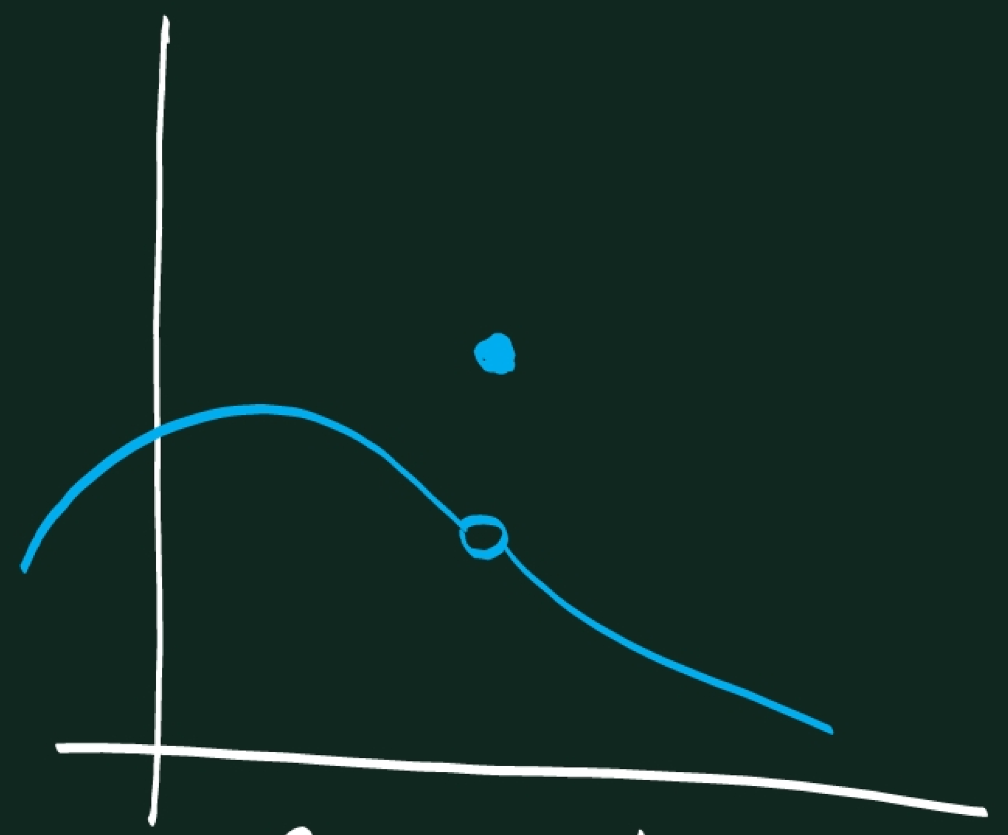
Discontinuidades

Evitables

Existe $\lim\limits_{x\to a} f(x)$



$f(a)$ non existe

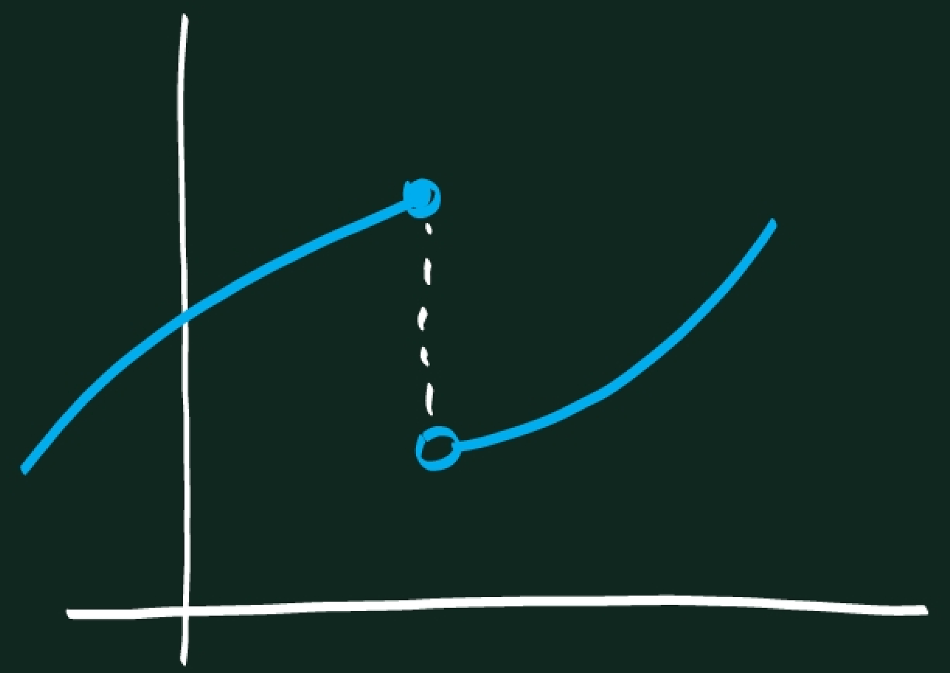


$f(a) \neq \lim\limits_{x\to a} f(x)$

Non Evitables

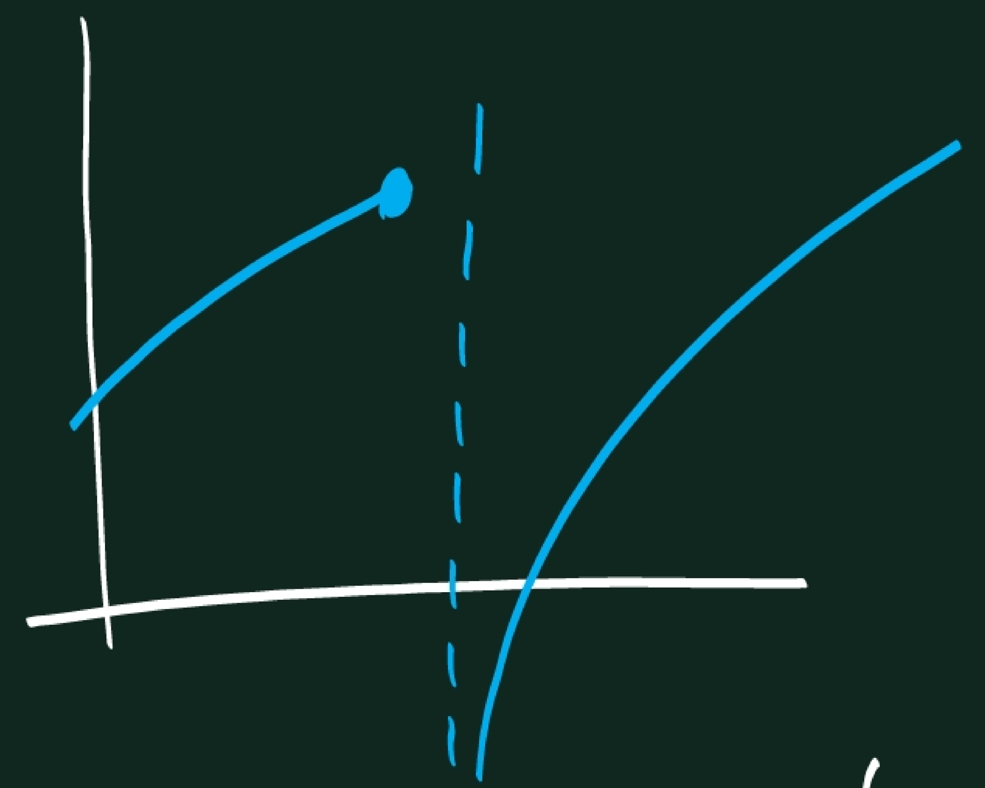
Non existe $\lim\limits_{x\to a} f(x)$

Salto Finito



$\lim\limits_{x\to a^-} f(x) \neq \lim\limits_{x\to a^+} f(x)$

Salto Infinito



$\lim\limits_{x\to a^-} f(x) = \pm\infty$ ou $\lim\limits_{x\to a^+} f(x) = \pm\infty$

5 a) $f(x)=\dfrac{3x^2+x+1}{x^2-5} \longrightarrow x^2-5=0 \to x=\pm\sqrt{5}$

$\text{Dom}=\mathbb{R}-\{-\sqrt{5},+\sqrt{5}\}$

A.V.

$$\lim_{x\to-\sqrt{5}} f(x)=\frac{3\cdot5-\sqrt{5}+1}{5-5}=\frac{16-\sqrt{5}}{0}=\infty \begin{cases} \lim_{x\to-\sqrt{5}^-} f(x)=\dfrac{16-\sqrt{5}}{0^+}=+\infty \\ \lim_{x\to-\sqrt{5}^+} f(x)=\dfrac{16-\sqrt{5}}{0^-}=-\infty \end{cases}$$

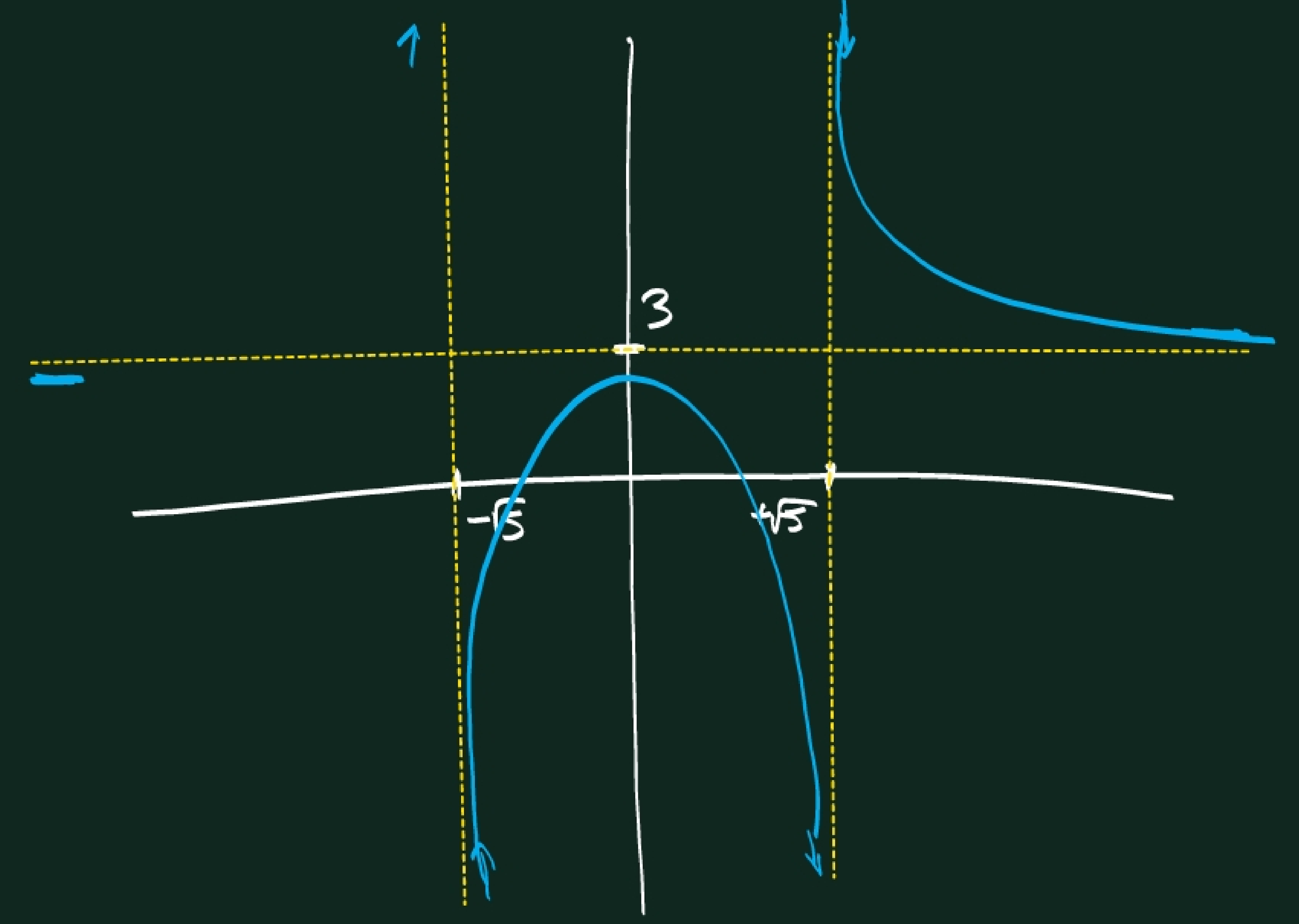
$$\lim_{x\to+\sqrt{5}} f(x)=\infty \begin{cases} \lim_{x\to\sqrt{5}^-} f(x)=\dfrac{16+\sqrt{5}}{0^-}=-\infty \\ \lim_{x\to\sqrt{5}^+} f(x)=\dfrac{\cdots}{0^+}=+\infty \end{cases}$$

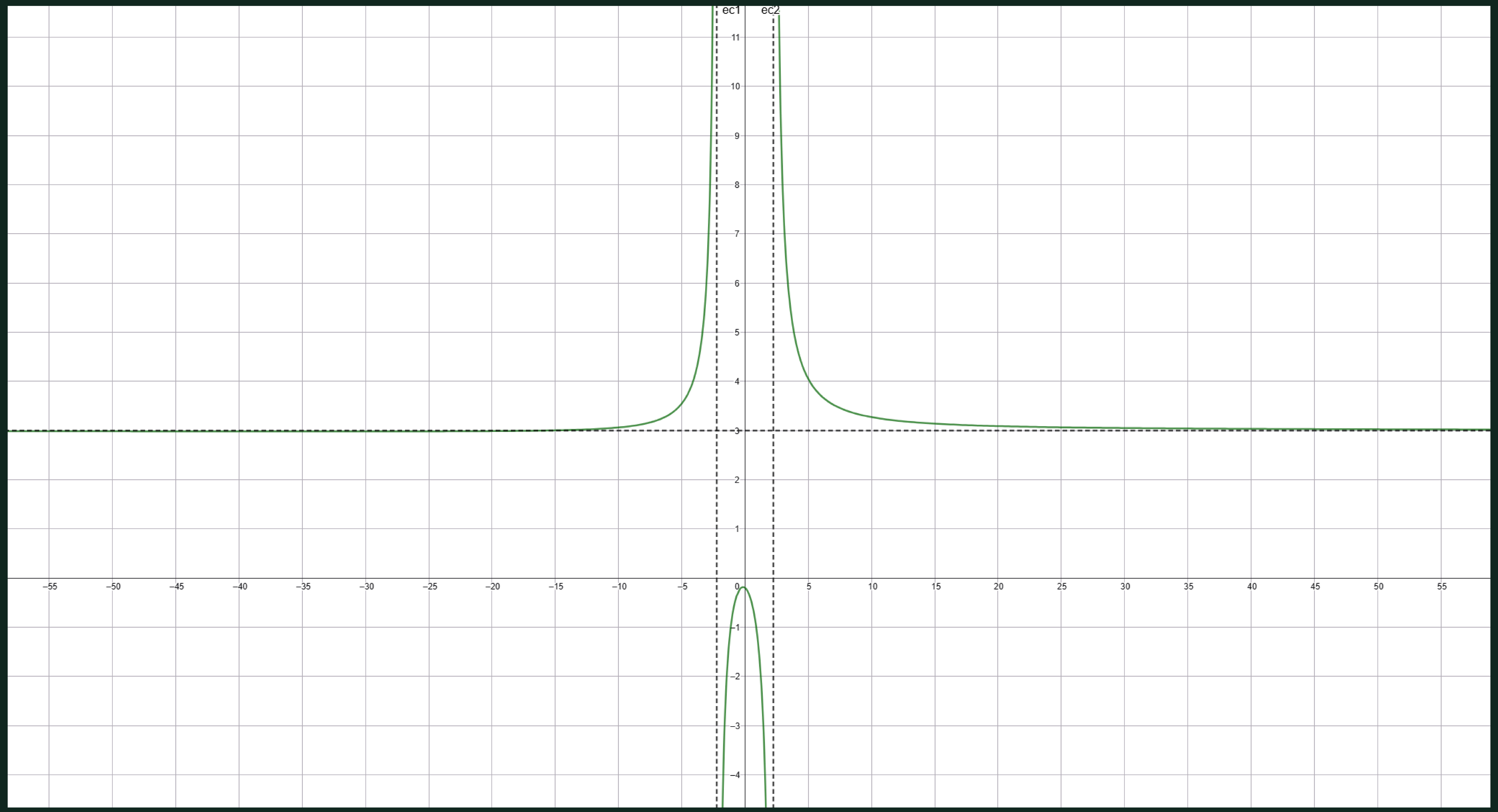
A.H. $\to y=3$

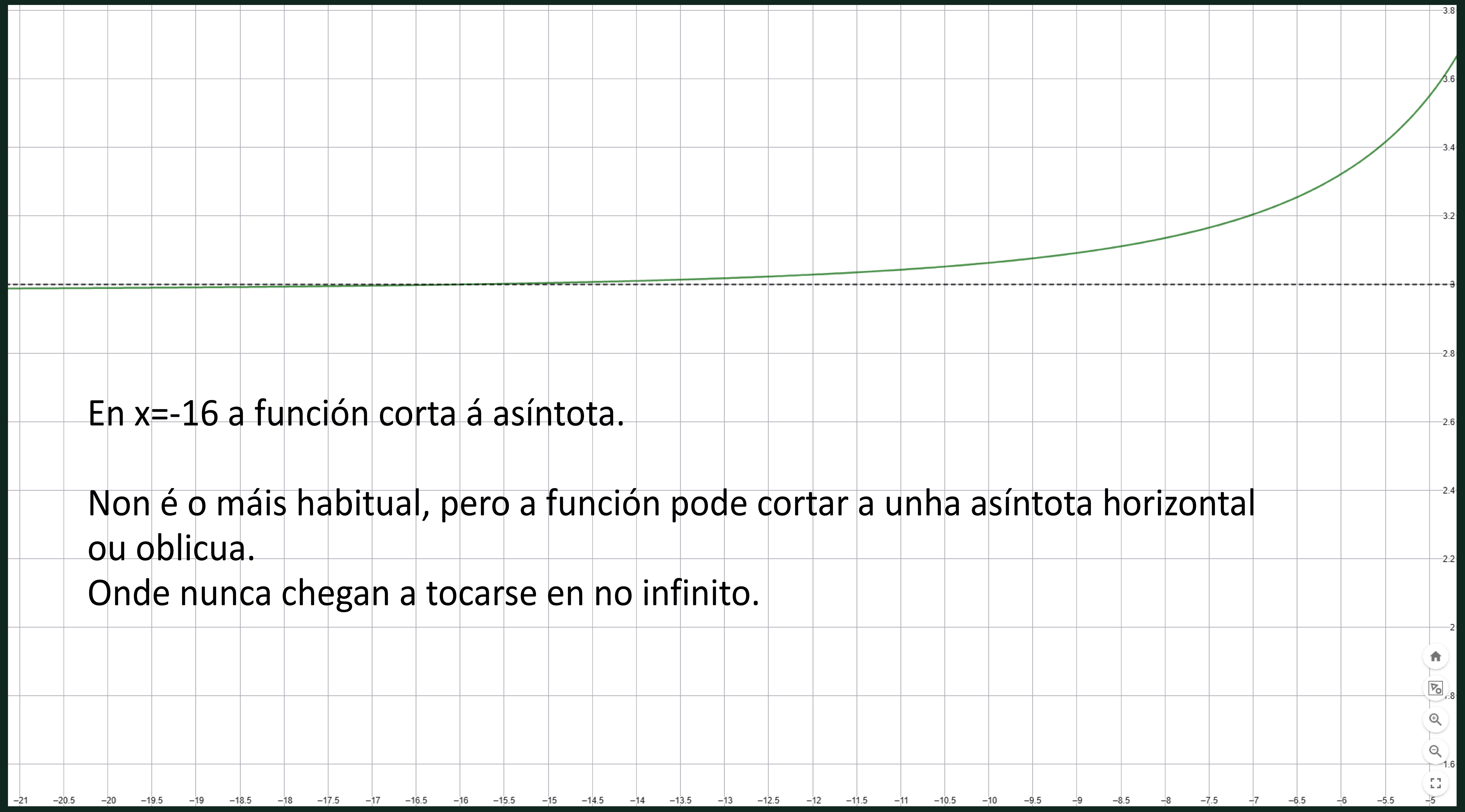
$$\lim_{x\to\pm\infty}\frac{3x^2+x+1}{x^2-5}=\frac{\infty}{\infty}=IND\to\lim_{x\to\pm\infty}\frac{3x^2+\cdots}{x^2+\cdots}=3$$

$$\lim_{x\to-\infty} f(x)-3=\lim_{x\to-\infty}\frac{3x^2+x+1}{x^2-5}-\frac{3x^2-15}{x^2-5}=\lim_{x\to-\infty}\frac{x+16}{x^2-5}=0^-$$

$$\lim_{x\to+\infty} f(x)-3=\cdots=\lim_{x\to+\infty}\frac{x+16}{x^2-5}=0^+$$







En x=-16 a función corta á asíntota.

Non é o máis habitual, pero a función pode cortar a unha asíntota horizontal ou oblicua.
Onde nunca chegan a tocarse en no infinito.

rr) $$\lim_{x\to+\infty}\left(\frac{x^2+2x}{x^2+1}\right)^{\frac{x^2+1}{x-3}} = 1^{\infty} = IND$$

$$\lim_{x\to+\infty}\left(1+\frac{x^2+2x}{x^2+1}-1\right)^{\frac{x^2+1}{x-3}} = \lim\left(1+\frac{x^2+2x-x^2-1}{x^2+1}\right)^{\frac{x^2+1}{x-3}} =$$

$$\lim\left(1+\frac{2x-1}{x^2+1}\right)^{\frac{x^2+1}{x-3}} = \lim\left(\left(1+\frac{1}{\frac{x^2+1}{2x-1}}\right)^{\frac{x^2+1}{2x-1}}\right)^{\frac{1}{x-3}\cdot(2x-1)} =$$

$$= e^{\lim\limits_{x\to+\infty}\frac{2x-1}{x-3}} = e^2$$