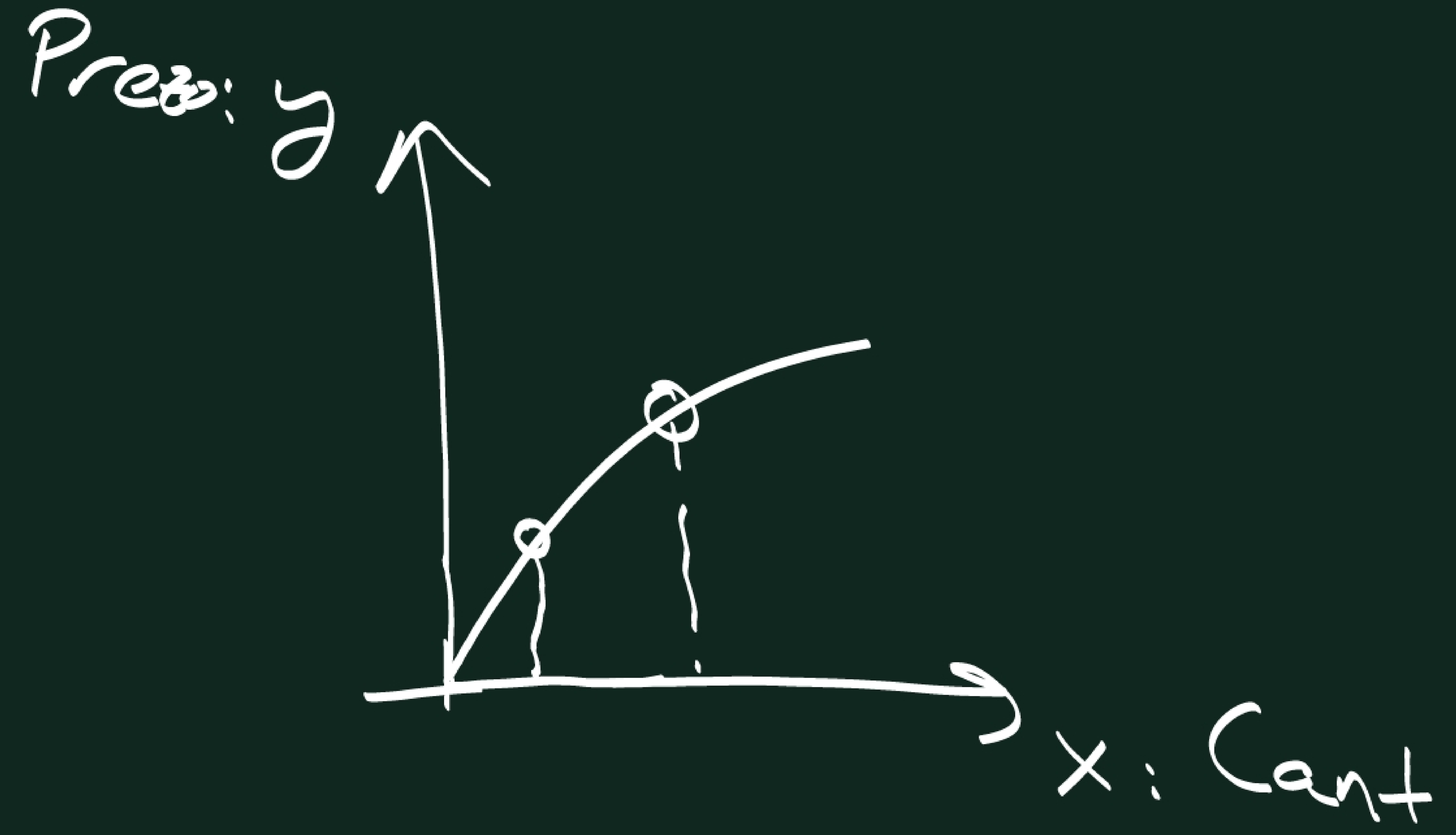
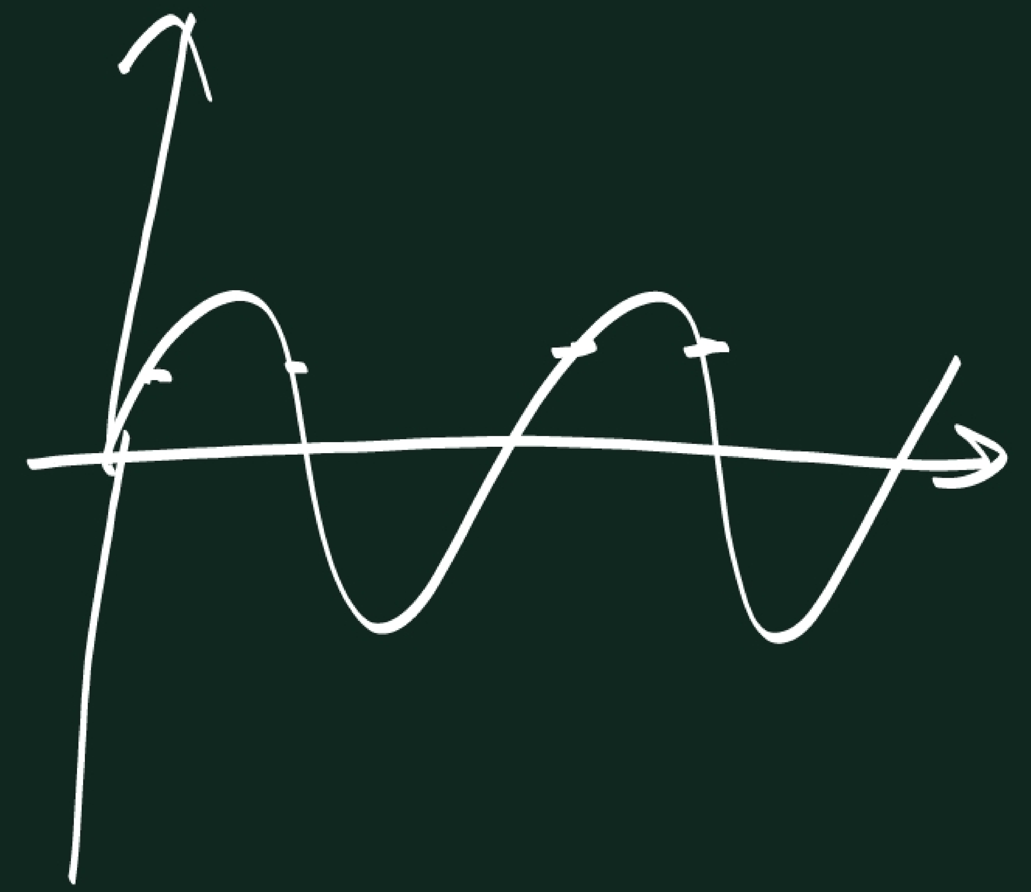
Tema 4 – Funcións

Relación entre dúas magnitudes A e B, tal que, a cada $x \in A$ correspóndelle un único $y \in B$.

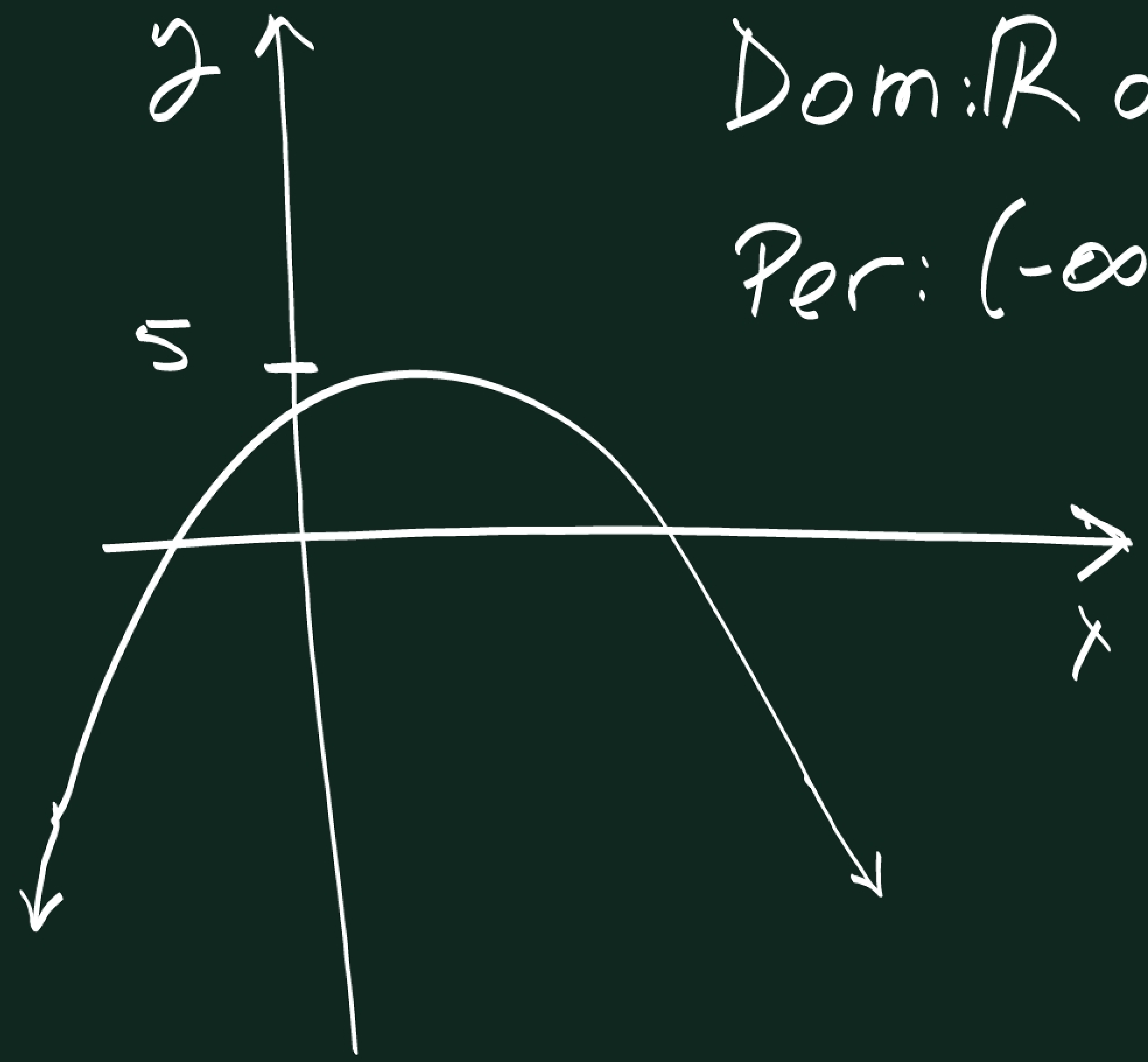
A función exprésase: $y = f(x)$





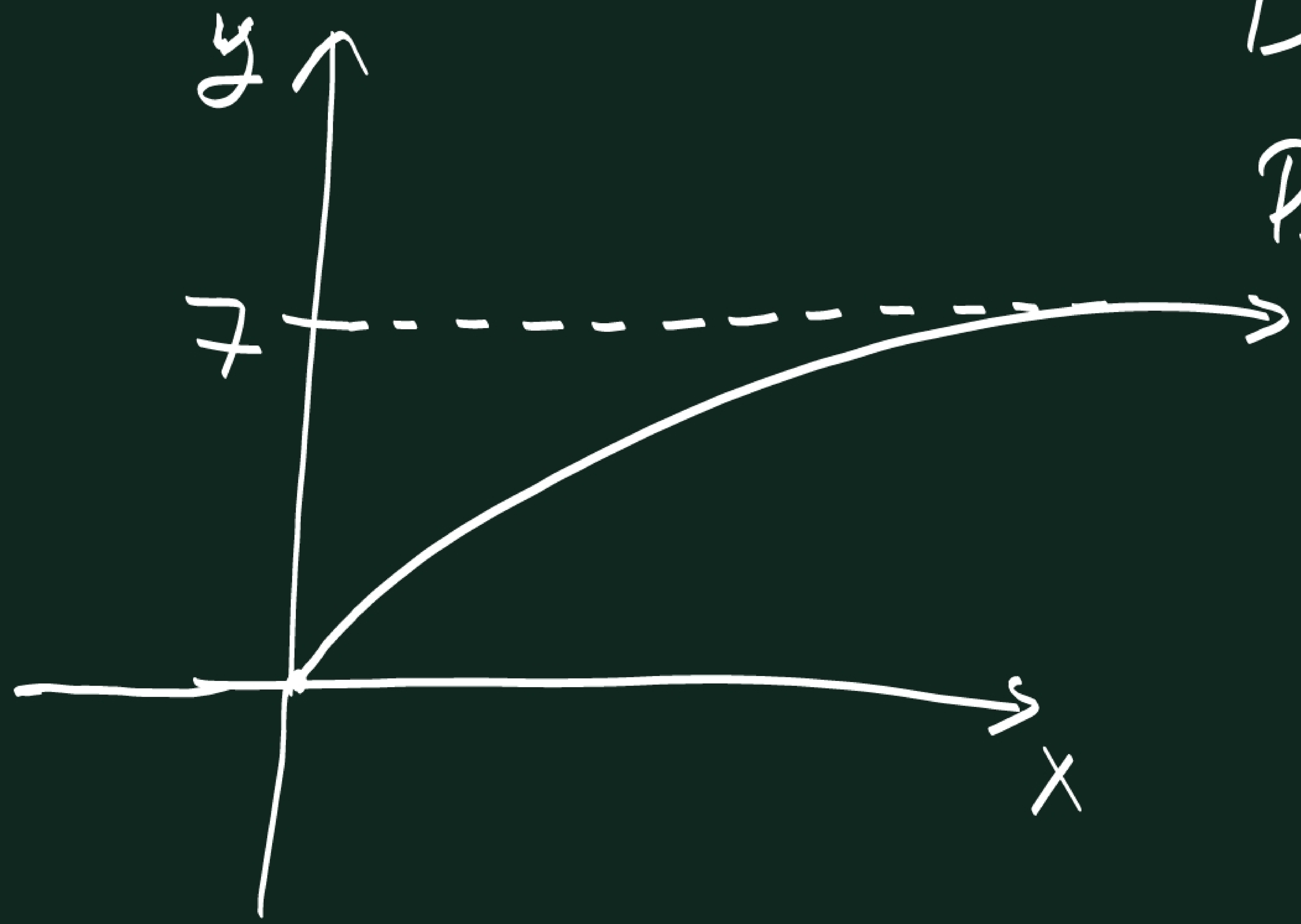
- O dominio dunha función son todos os valores de "x" para os que existe a función.

- O percorrido dunha función son todos os valores de "y" para os que existe a función.



Dom: $\mathbb{R}$ ou $(-\infty, +\infty)$

Per: $(-\infty, 5]$



Dom: $[0, +\infty)$

Per: $[0, 7)$

$f(x) = 7x^2 + 5$ $\quad$ Dom $f(x)$: $\mathbb{R}$

$g(x) = \dfrac{1}{x}$ $\quad$ Dom $g(x)$: $\mathbb{R} - \{0\}$

$h(x) = \sqrt{x-2}$ $\quad$ Dom $h(x)$: $[2, +\infty)$

$x - 2 = 0 \rightarrow x = 2$

$i(x) = 7 \log(3x-5) \longrightarrow$ Dom $i(x) = \left(5/3, +\infty\right)$

$3x - 5 = 0 \rightarrow x = 5/3$

a) $f(x) = \dfrac{2x^3-16}{x^2-6x+8}$ → Dom: $\mathbb{R}$

$$x^2-6x+8=0$$

$$x = \frac{6 \pm \sqrt{36-32}}{2} = \frac{6 \pm 2}{2} \left\langle \begin{matrix} 4 \\ 2 \end{matrix} \right.$$

Dom $f(x)$: $\mathbb{R} - \{2;4\}$

$(-\infty, 2) \cup (2,4) \cup (4, +\infty)$

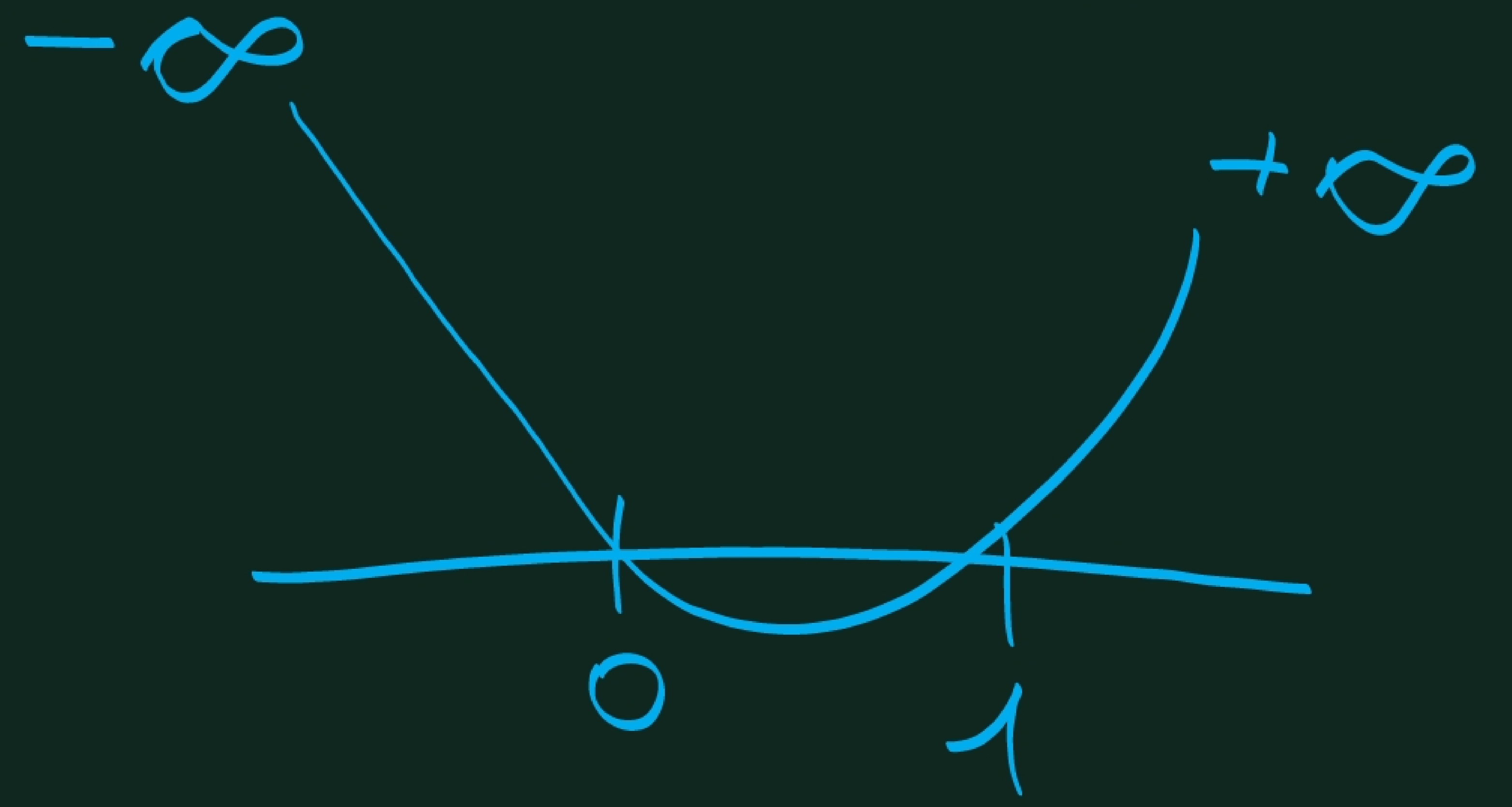
b) $f(x) = 2 \cdot \ln(x^2-x)$

$$x^2-x=0$$

$$x(x-1)=0$$

$$x=1$$

$$x=0$$



Dom $f(x)$: $(-\infty, 0) \cup (1, +\infty)$

c) $f(x) = \dfrac{2x+\sqrt{x+1}}{3-\sqrt{x}} \longrightarrow$ Dom $f(x)$: $[0,9)\cup(9,+\infty)$

Num:

$\cancel{2x+}\sqrt{x+1}$

$\sqrt{x+1}$

$x+1=0 \rightarrow x=-1$

Dom: $[-1,+\infty)$

Den:

$3-\sqrt{x}$

$\sqrt{x} \longrightarrow$ Dom: $[0,+\infty)$

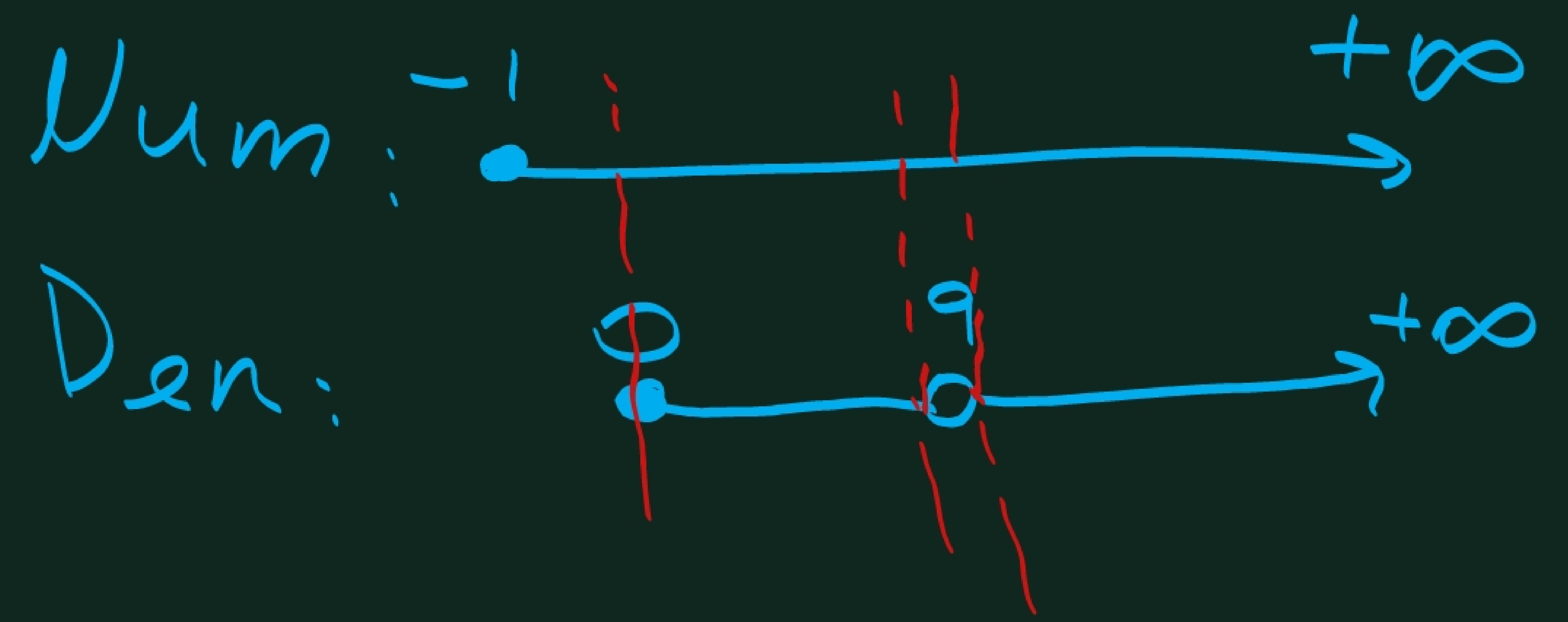
$3-\sqrt{x}=0$

$3=\sqrt{x}$

$9=x$

$\downarrow$

Anula ao denominador



$a(x) = x^2 - 3x + 1$ Dom: $\mathbb{R}$

$e(x) = e^{x^2}$ Dom: $\mathbb{R}$

$b(x) = \frac{-3}{2x+1}$ $\to$ $2x+1=0$, $x = -1/2$

Dom: $\mathbb{R} - \{-1/2\}$

$c(x) = \frac{x^2-1}{x-1}$ Dom: $\mathbb{R} - \{1\}$

$d(x) = \frac{3x^2+4}{x^3-6x^2+8x}$ Dom: $\mathbb{R} - \{0, 2, 4\}$

$x^3 - 6x^2 + 8x = 0$

$x \cdot (x^2 - 6x + 8) = 0 \to x = 0$

$x = \frac{6 \pm \sqrt{36-32}}{2} = \begin{cases} 4 \\ 2 \end{cases}$

$f(x) = \sqrt{\frac{x^4 + 5x^2 - 36}{7}}$

$a^2 + 5a - 36 = 0$

$a = \frac{-5 \pm \sqrt{25+144}}{2} = \begin{cases} -9 \\ 4 \end{cases}$

$x = \pm\sqrt{-9} = -\cancel{3}$

$x = \pm\sqrt{4} = \pm 2$

$-\infty$ $x=0$ $+\infty$

$(+)$ -2 $(-)$ $+2$ $(+)$

Dom $f(x) = (-\infty, -2] \cup [+2, +\infty)$

$g(x) = \sqrt[5]{x^2 + 5x - 6} \to$ Dom $g(x) = \mathbb{R}$

$h(x) = \sqrt{\log(9 - x^2)}$

$9 - x^2 \geq 1$

$9 - x^2 = 1$

$8 = x^2$

$x = \pm 2\sqrt{2}$

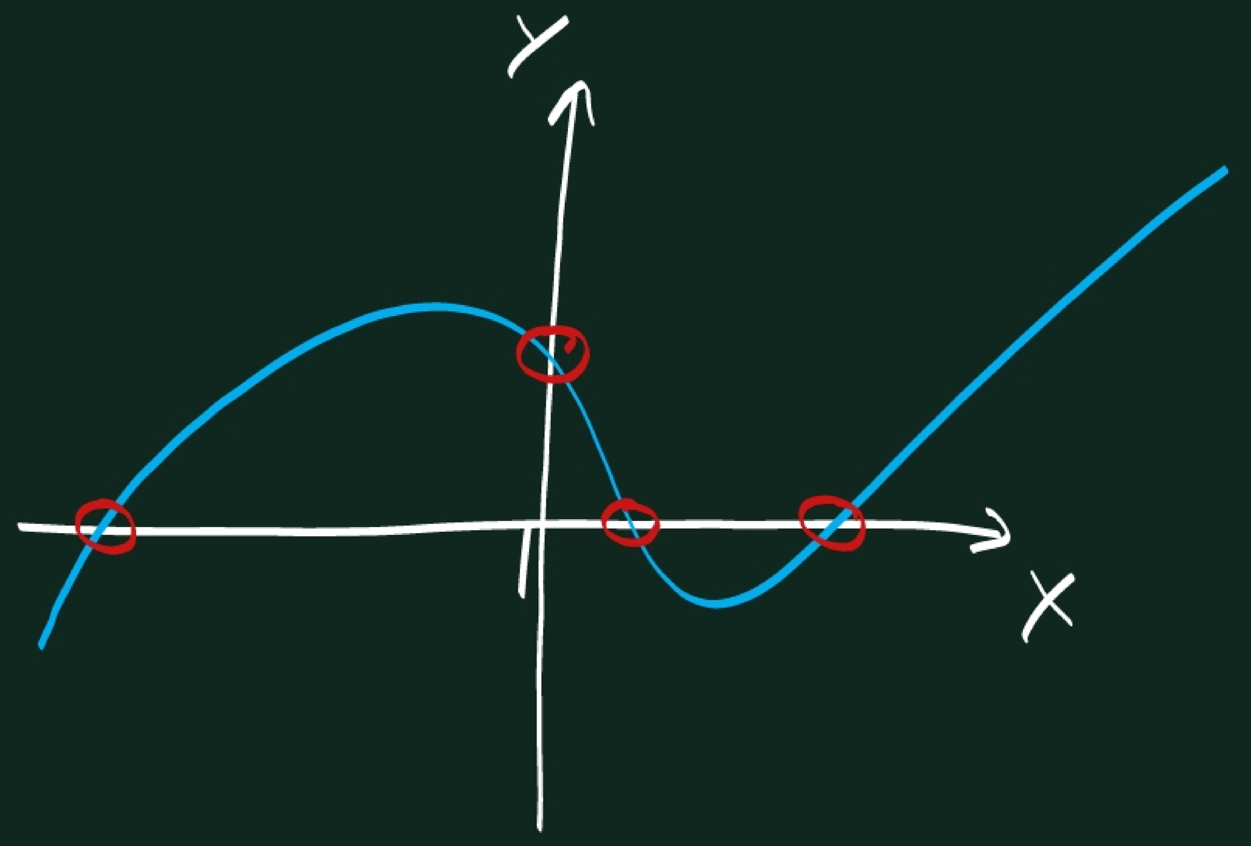
$(-)$ $-2\sqrt{2}$ $(+)$ $2\sqrt{2}$ $(-)$

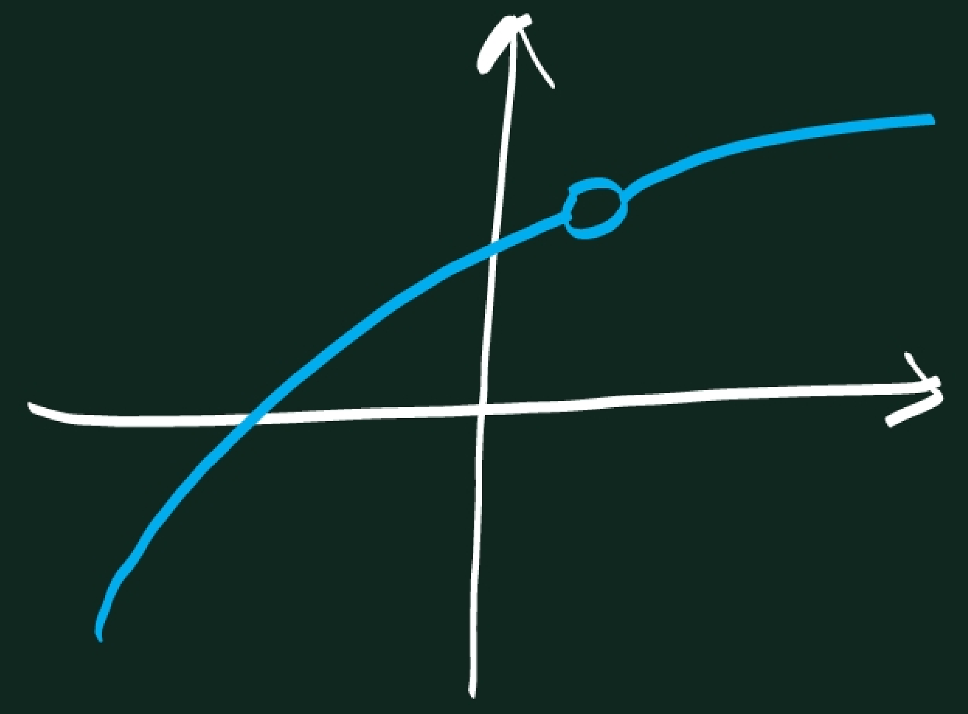
Dom $h(x) = [-2\sqrt{2}, +2\sqrt{2}]$

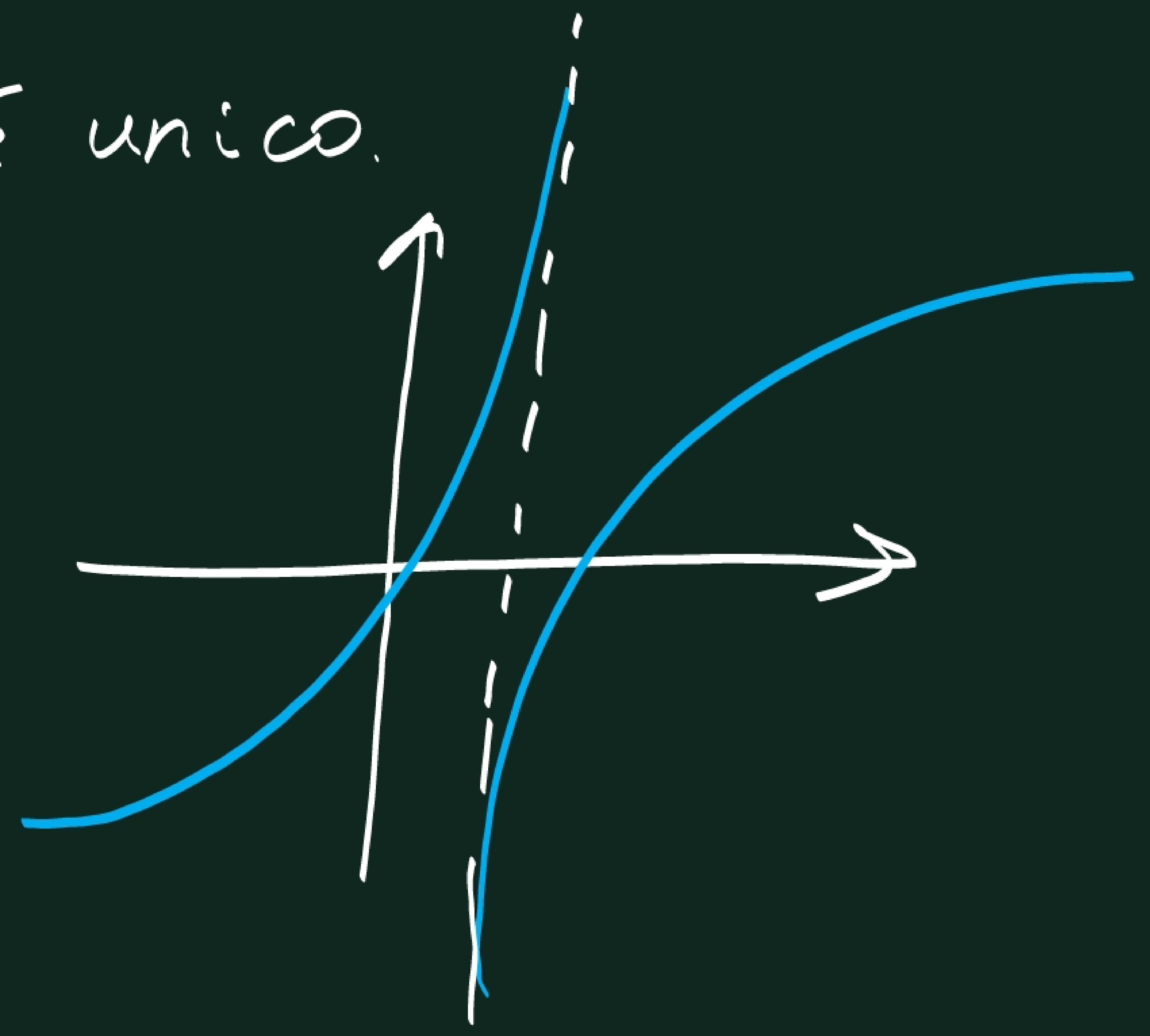
Características das funcións

• Puntos de Corte

- Co eixo X $\longrightarrow$ $f(x)=0$
- Co eixo Y $\longrightarrow$ $f(0)$, se existe, é unico.



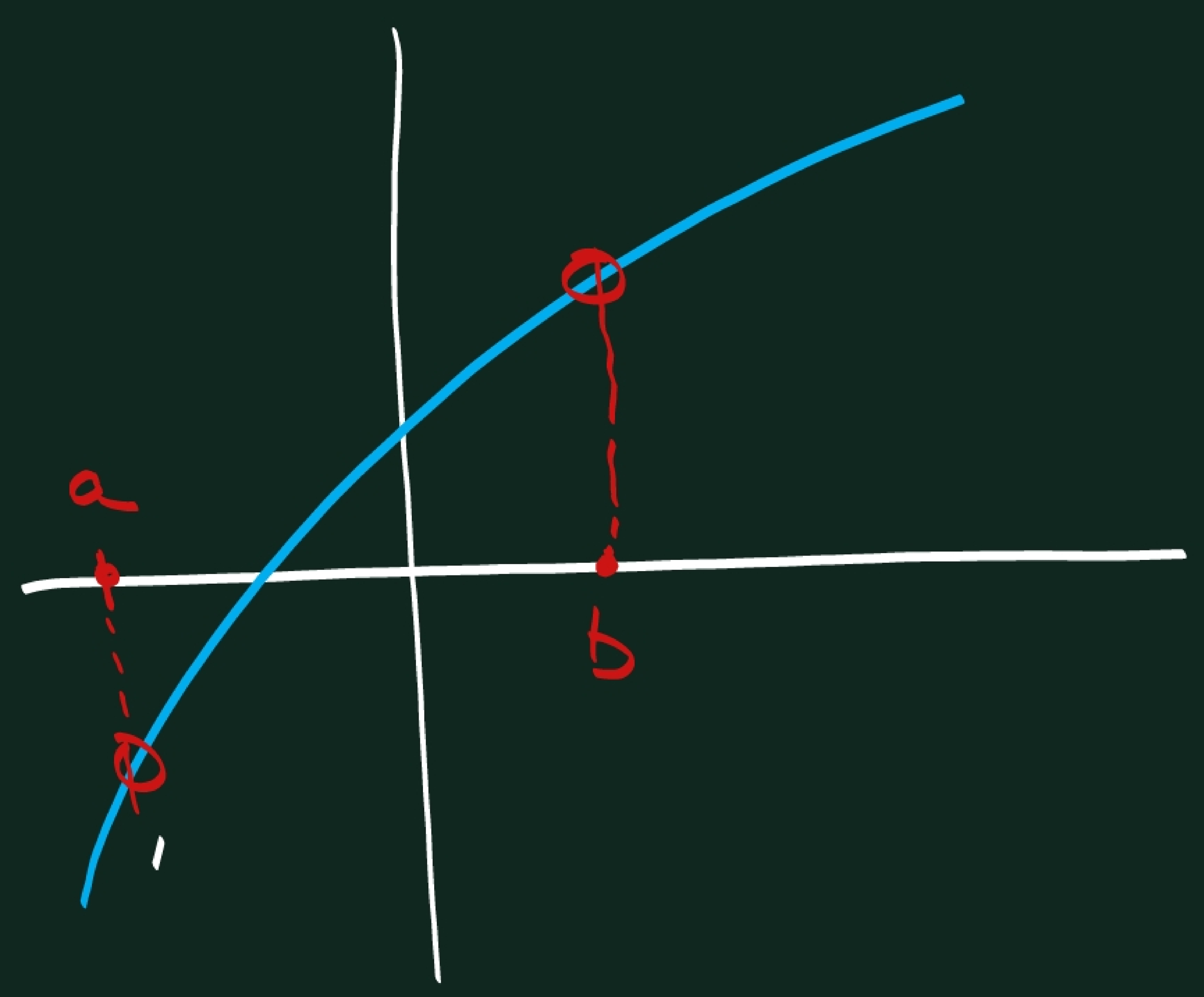


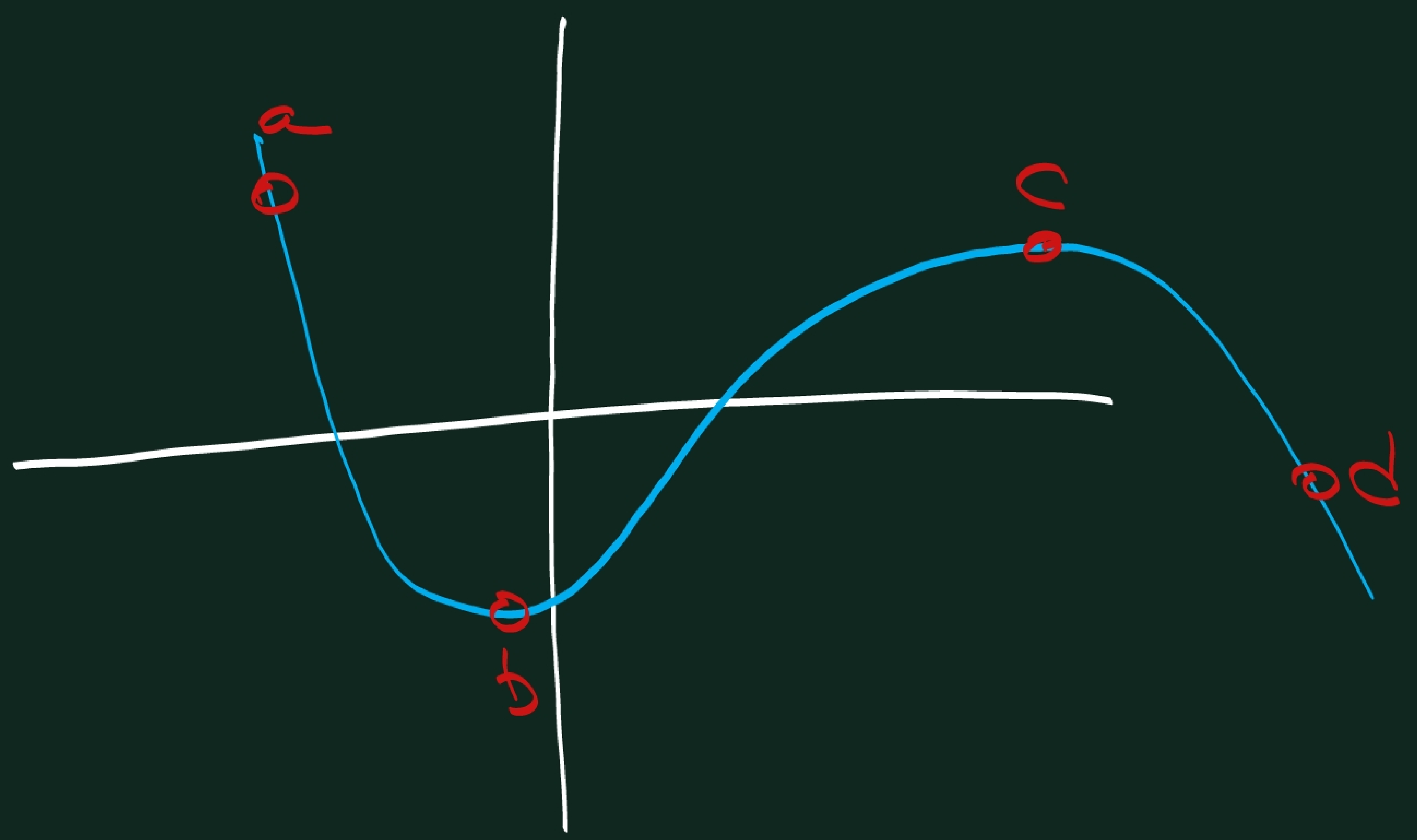


• Continuidade: Continua ou discontinua.

É continua cando a podo debuxar sen levantar o boli.

- Crecemento/Decrecemento:
  - Crece nun intervalo (a, b) se: $b > a$ e $f(b) > f(a)$
  - Decrece " " " se: $b > a$ e $f(b) < f(a)$

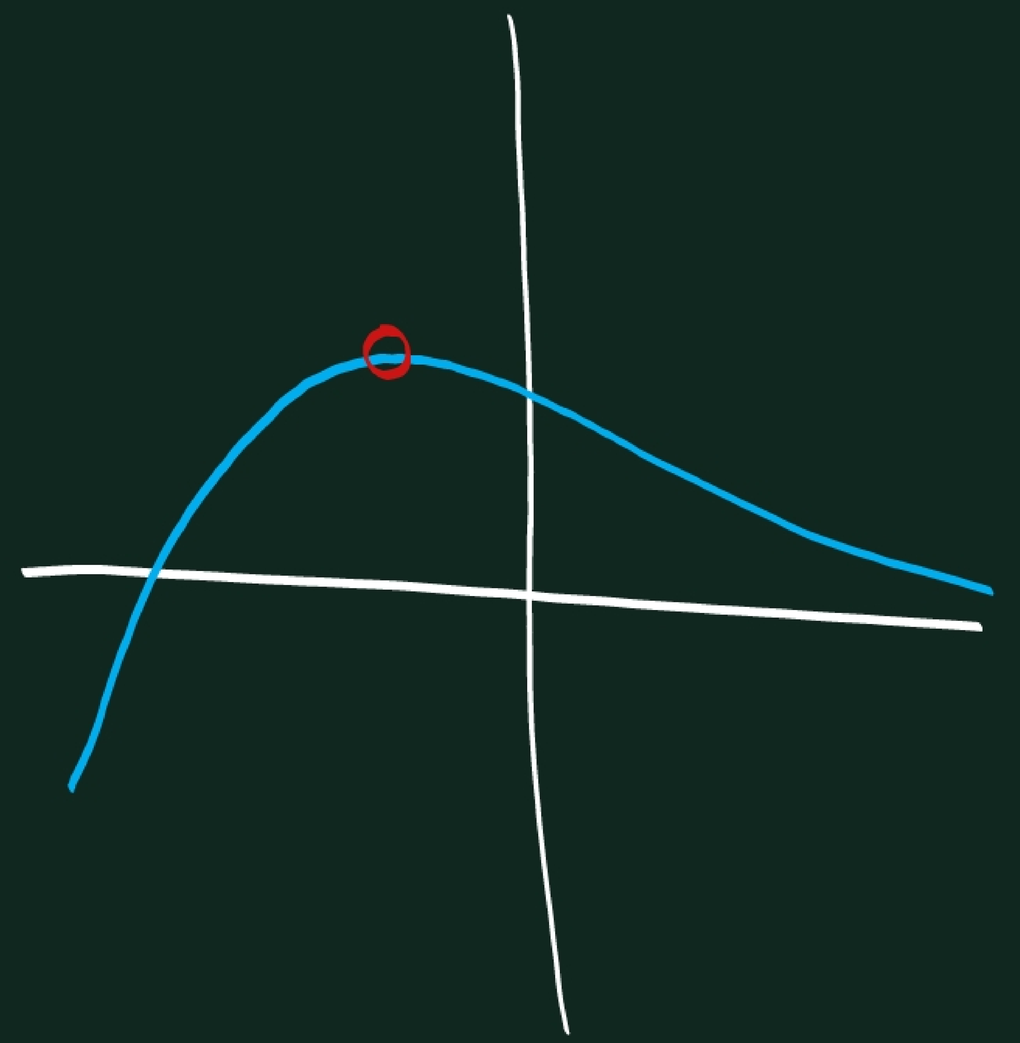


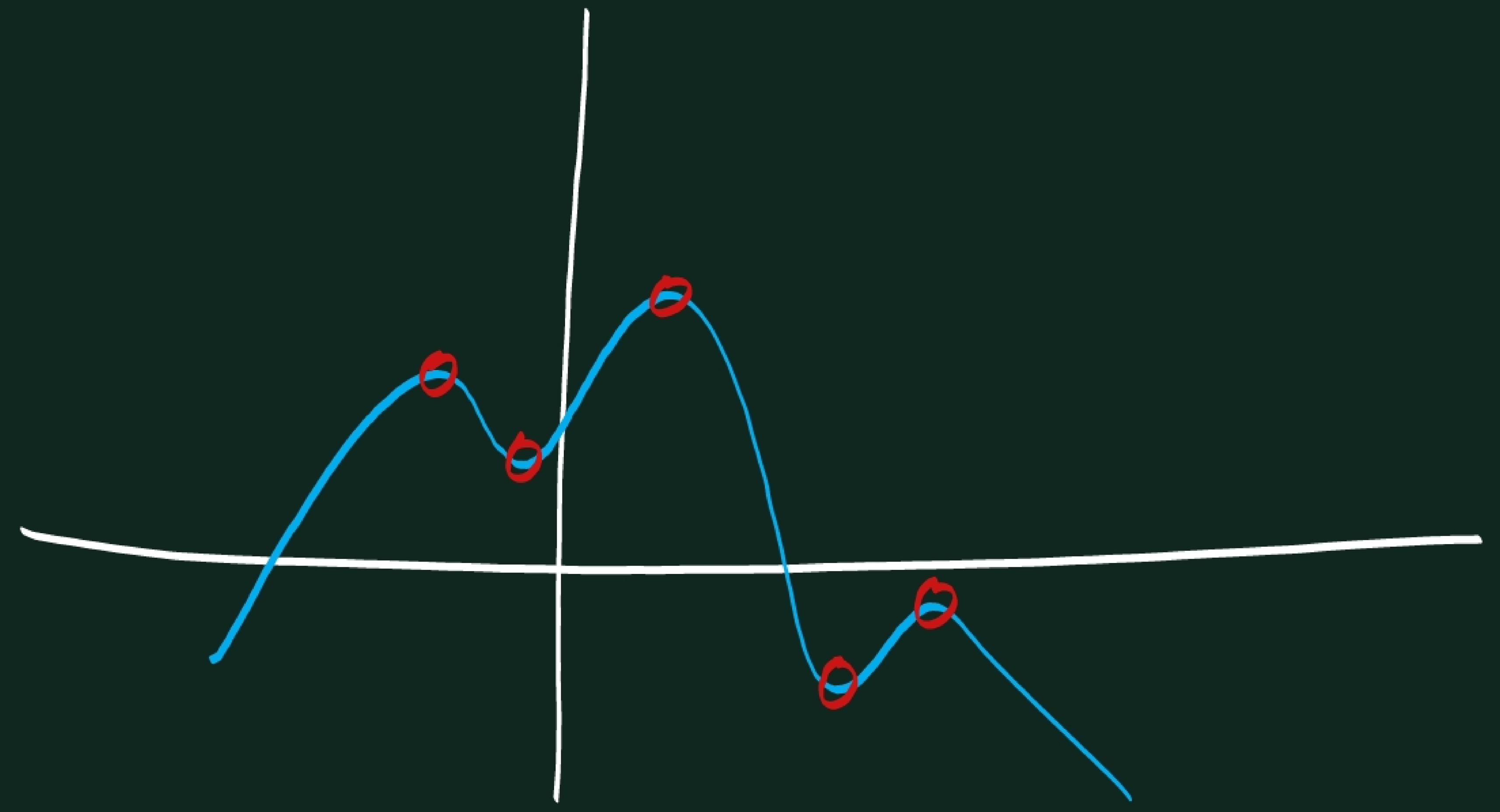


- Puntos Extremos ou críticos:

  - Máximo Relativo: punto onde a función pasa de crecer a decrecer, ou, punto máis alto da cotorna.

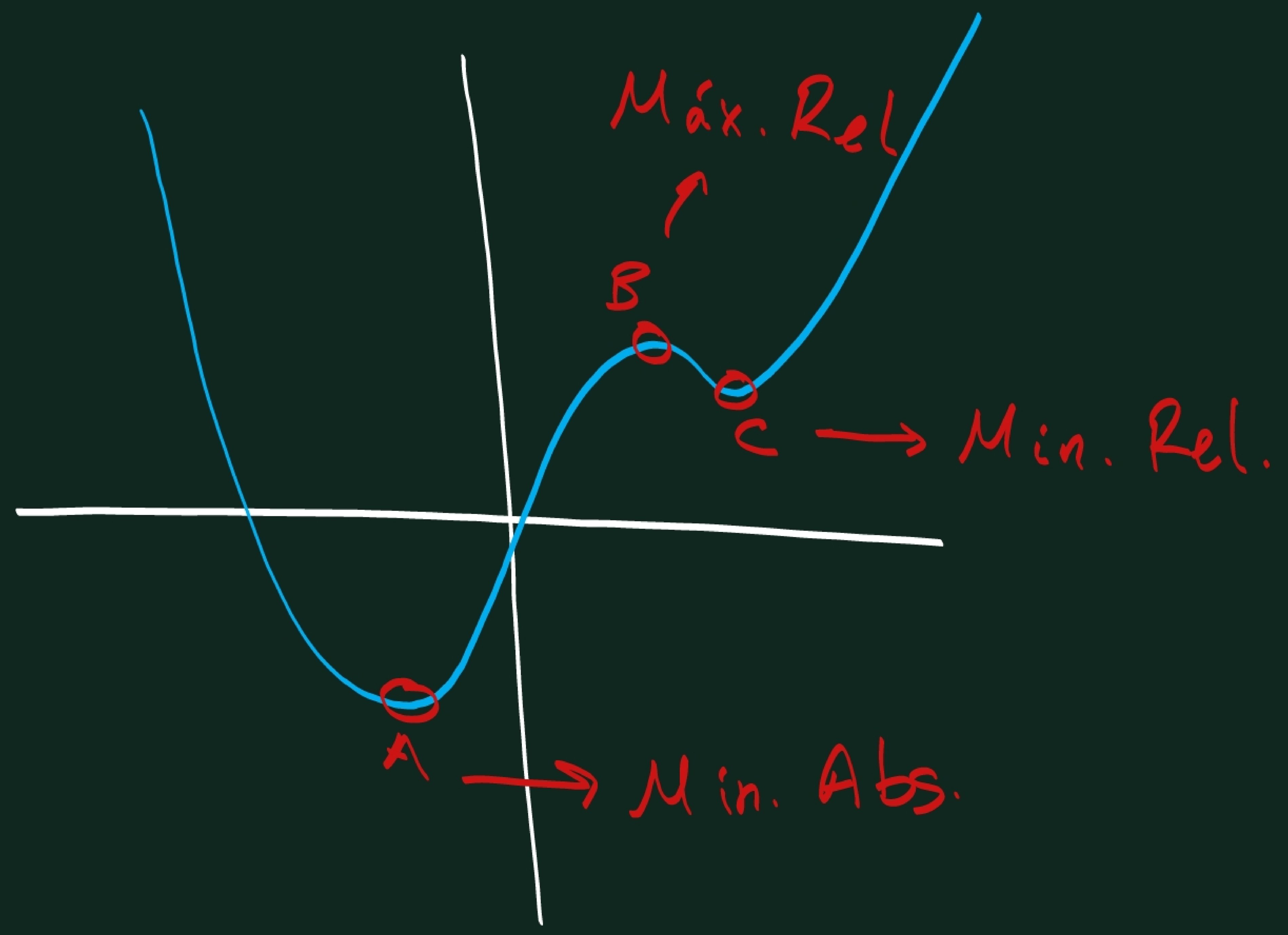
  - Mínimo Relativo: punto onde a función pasa de decrecer a crecer, ou, punto máis baixo da contorna.

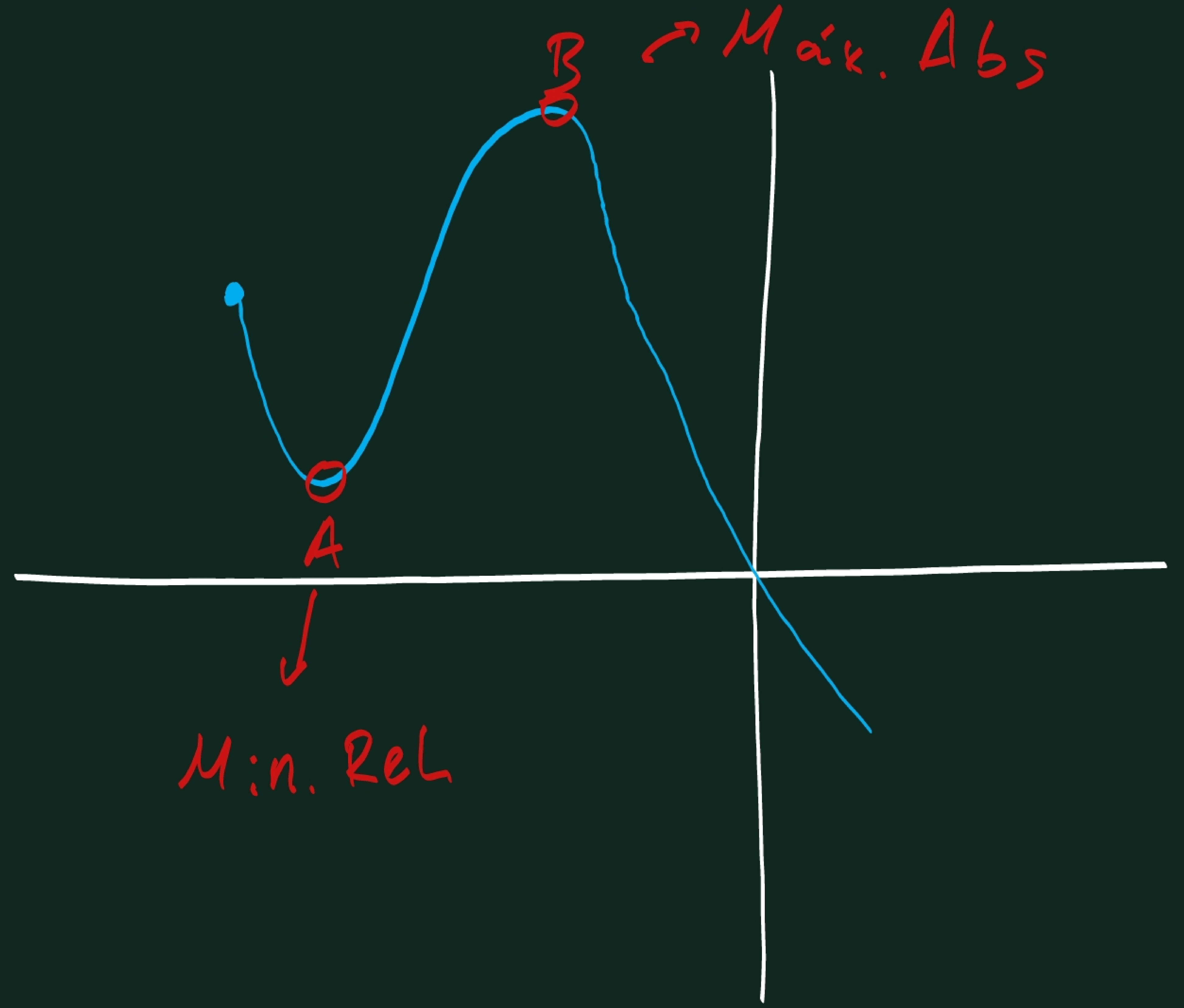


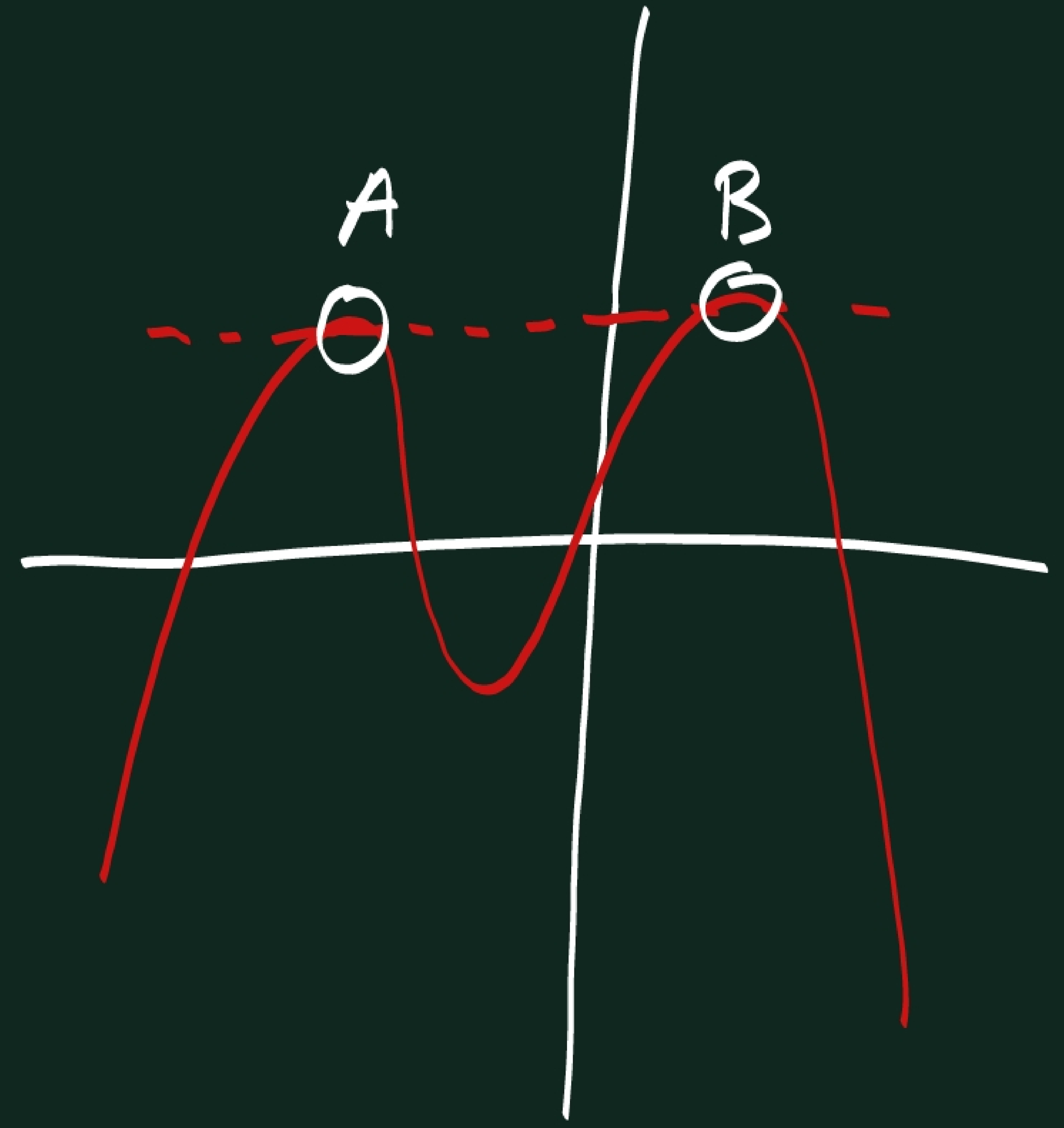


• Máximo Absoluto: é o valor máis alto da función.

• Mínimo Absoluto: é o valor máis baixo da función.

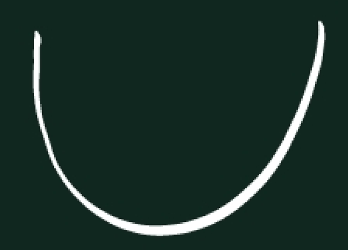




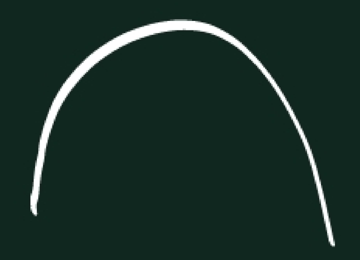


Curvatura:

- Convexa: se nun intervalo (a, b) a función queda por enriba das tanxentes.

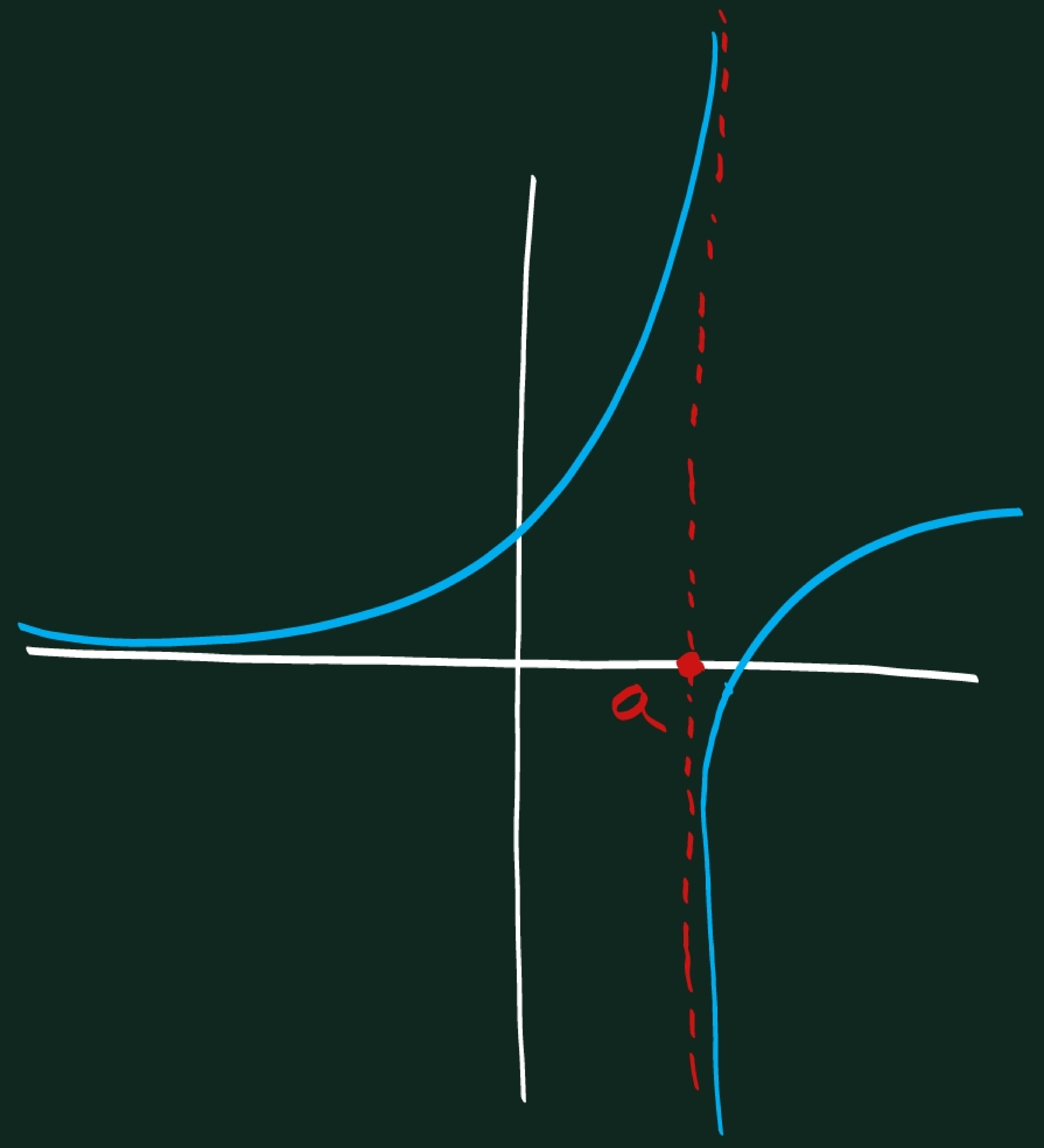


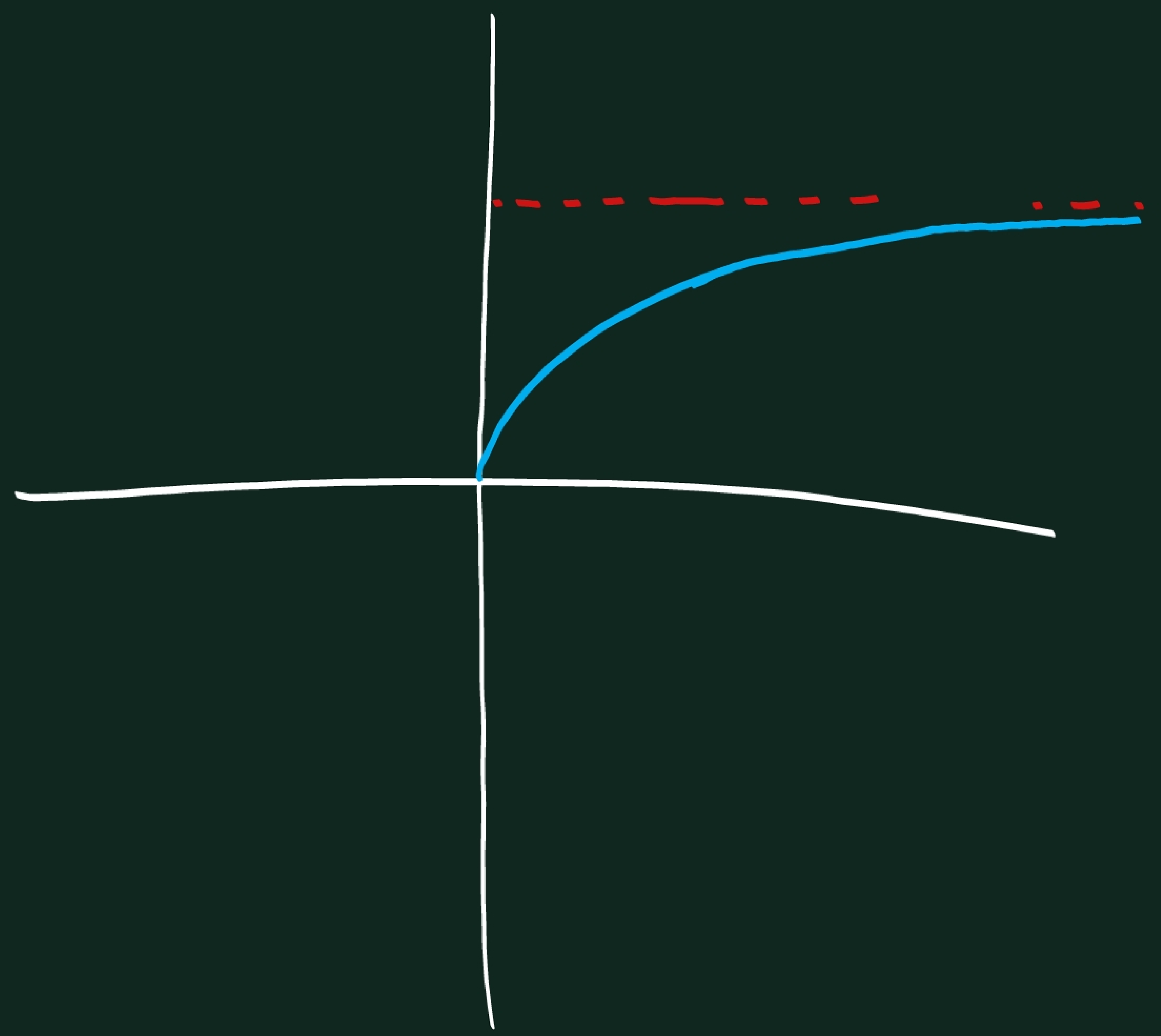
- Cóncava: se nun intervalo (a, b) a función queda por debaixo das tanxentes.

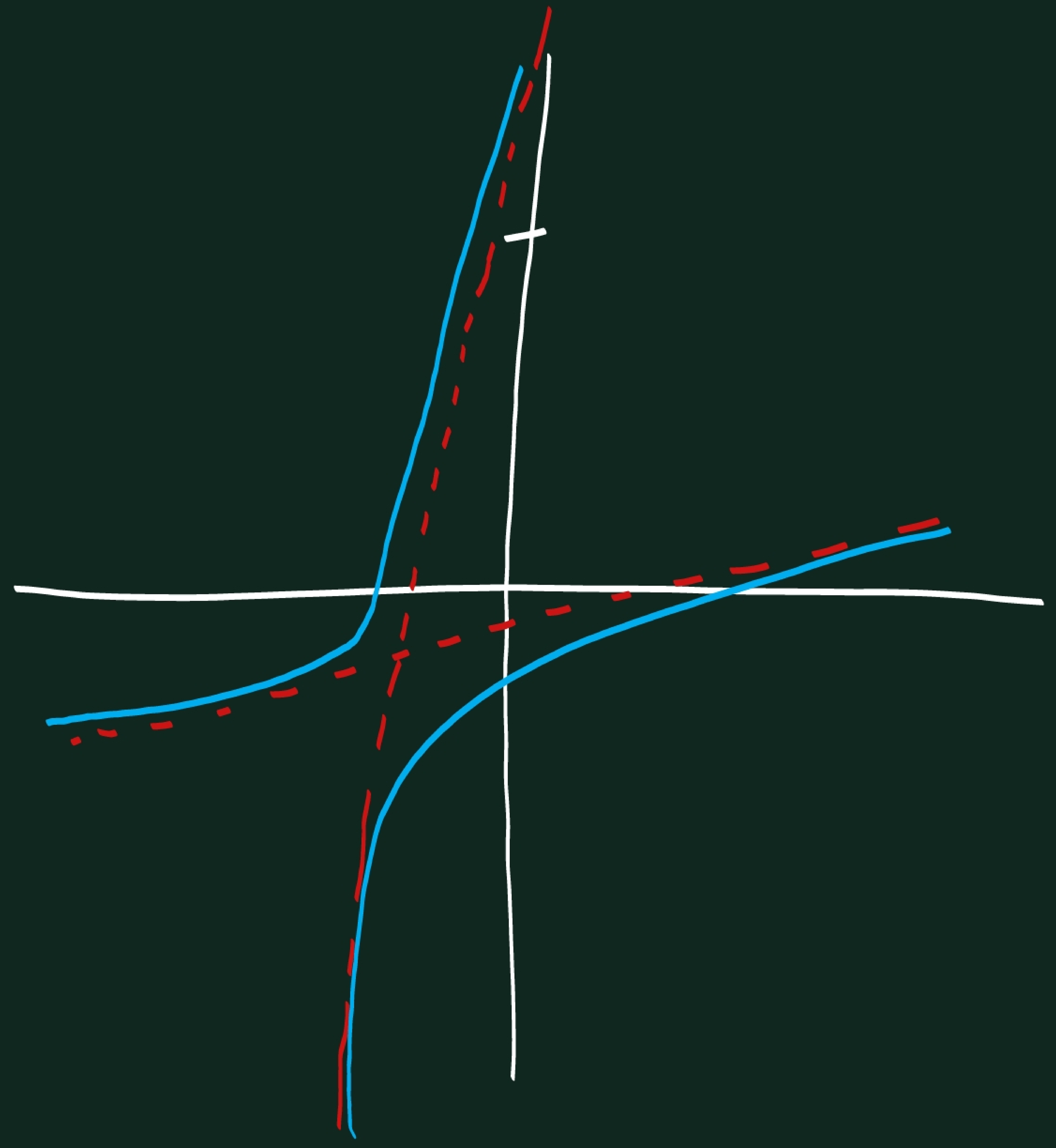


- Punto de inflexión: punto onde a función pasa de convexa a cóncava ou viceversa.

• Asíntota: recta á que se aproxima unha función pero que nunca chega a tocar.







② a) $\dfrac{3x^2-6}{x+1}$

Ptos. Corte Eixo X ($y=0$):

$$\frac{3x^2-6}{x+1}=0$$

$$3x^2-6=0$$

$$x^2=\frac{6}{3}=2$$

$$x=\pm\sqrt{2}$$

$(+\sqrt{2},0)$

$(-\sqrt{2},0)$

Cando $x=+\sqrt{2}$

$$y=\frac{3\cdot 2-6}{\sqrt{2}+1}=\frac{0}{\ldots}=0$$

Ptos. Corte Eixo Y ($x=0$):

$$y=\frac{-6}{1}=-6 \longrightarrow (0,-6)$$

b) $2^{1-2x} \cdot (x^2+x)$

Corte Eixo X:

$$0 = 2^{1-2x} \cdot (x^2+x)$$

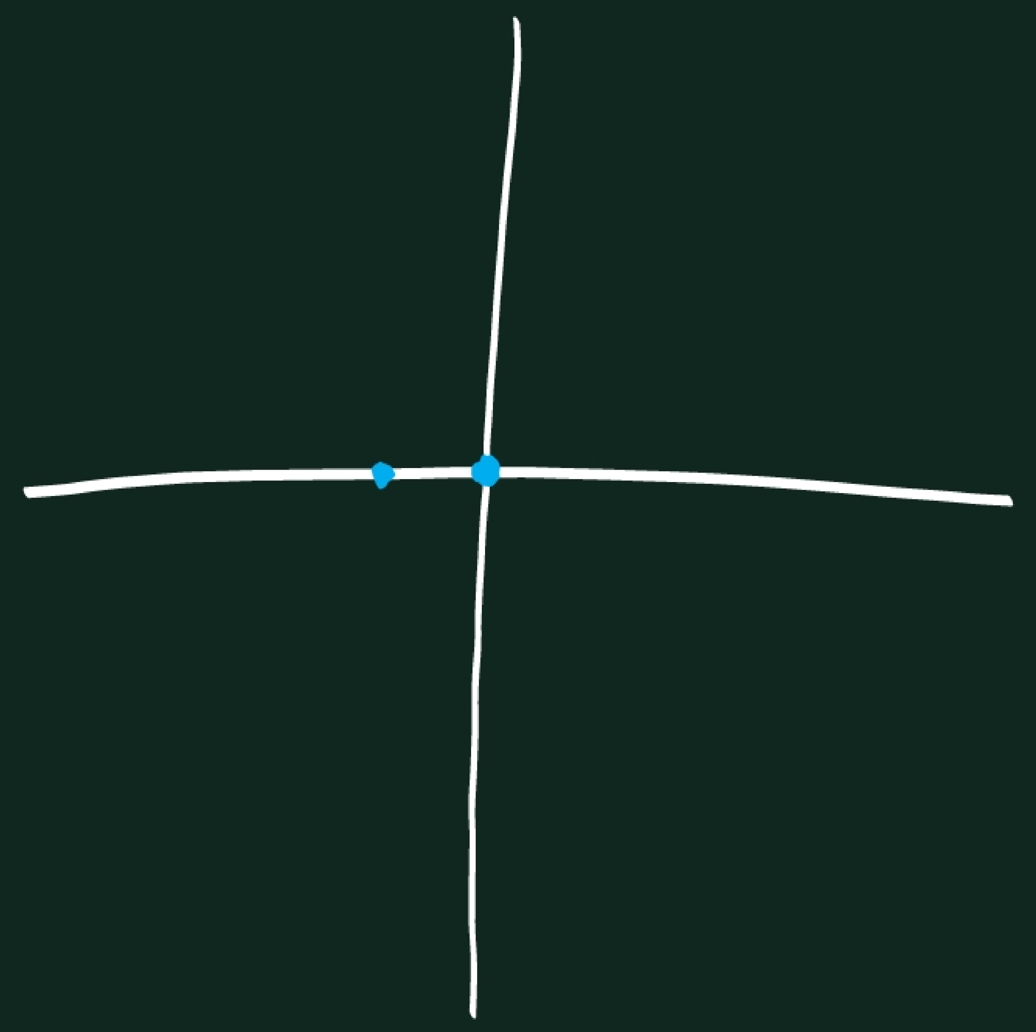
$$\cancel{2^{1-2x} = 0}$$

$$(x^2+x) = 0$$

$$x(x+1) = 0 \begin{cases} x = 0 \to (0,0) \\ x = -1 \to (-1,0) \end{cases}$$

Corte Eixo Y:

$$y = 2^{1-2\cdot 0} \cdot (0^2+0) = 0 \longrightarrow (0,0)$$



c) $\ln(x+2)\cdot(x^3-5x+4)$

Eixo X $\rightarrow y=0$

$\boxed{\begin{matrix}(1,0)\\(1{,}56;0)\\(-2{,}56;0)\\(-1,0)\end{matrix}}$

$\ln(x+2)\cdot(x^3-5x+4)=0$

- $\ln(x+2)=0 \rightarrow e^0=x+2$

$1=x+2$

$\boxed{x=-1}$

- $x^3-5x+4=0$

$\boxed{x=1}$

$$\begin{array}{c|cccc} & 1 & 0 & -5 & 4 \\ 1 & & 1 & 1 & -4 \\ \hline & 1 & 1 & -4 & \boxed{0} \end{array}$$

$x^2+x-4=0 \rightarrow x=\dfrac{-1\pm\sqrt{17}}{2}$ $\boxed{1{,}56}$ $\boxed{-2{,}56}$

Eixo Y $\rightarrow x=0$ $\boxed{(0;2{,}73)}$

$y=\ln(2)\cdot(0-0+4)=$

$=\ln(2)\cdot4=2{,}73$

d) $3x^3-5x^2+x-6$

Eixo X $\longrightarrow y=0$ $\quad(2,0)$

$3x^3-5x^2+x-6=0$

$\boxed{x=2}$

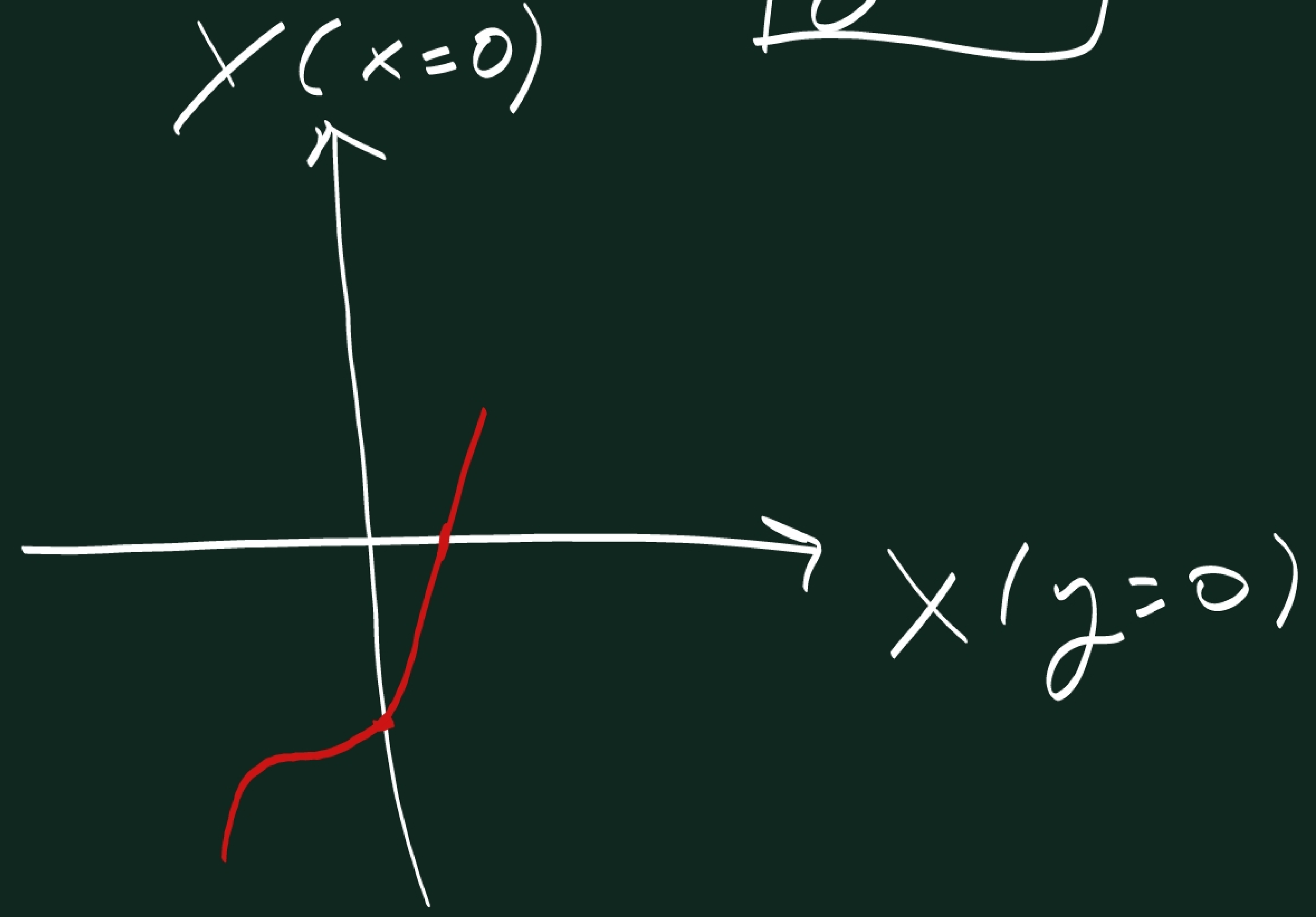
	3	-5	1	-6
2		6	2	6
	3	1	3	0

$3x^2+x+3=0 \longrightarrow x=\nexists$

Eixo Y $\longrightarrow x=0$

$y=0-0+0-6$

$\boxed{y=-6} \longrightarrow (0,-6)$

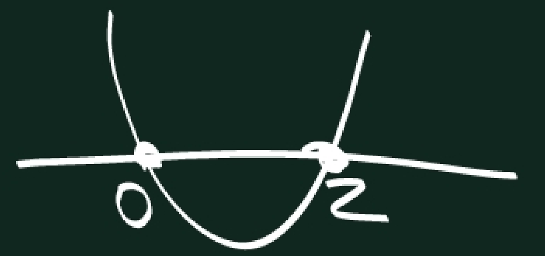


1. a) $f(x) = \log(x^2 - 2x)$

$$x^2 - 2x > 0$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0 \begin{cases} x = 0 \\ x = 2 \end{cases}$$



$\text{Dom } f(x) = (-\infty, 0) \cup (2, +\infty)$

b) $f(x) = \dfrac{x}{2 + \log_3(1-3x)}$

• $\log_3(1-3x)$

$$1 - 3x > 0$$

$$1 > 3x$$

$$\frac{1}{3} > x \rightarrow (-\infty, 1/3) \longrightarrow \text{Dom} = \left(-\infty, \frac{8}{27}\right) \cup \left(\frac{8}{27}, \frac{1}{3}\right)$$

• $2 + \log_3(1-3x) = 0$

$$\log_3(1-3x) = -2 \rightarrow 3^{-2} = 1 - 3x$$

$$\frac{1}{9} = 1 - 3x$$

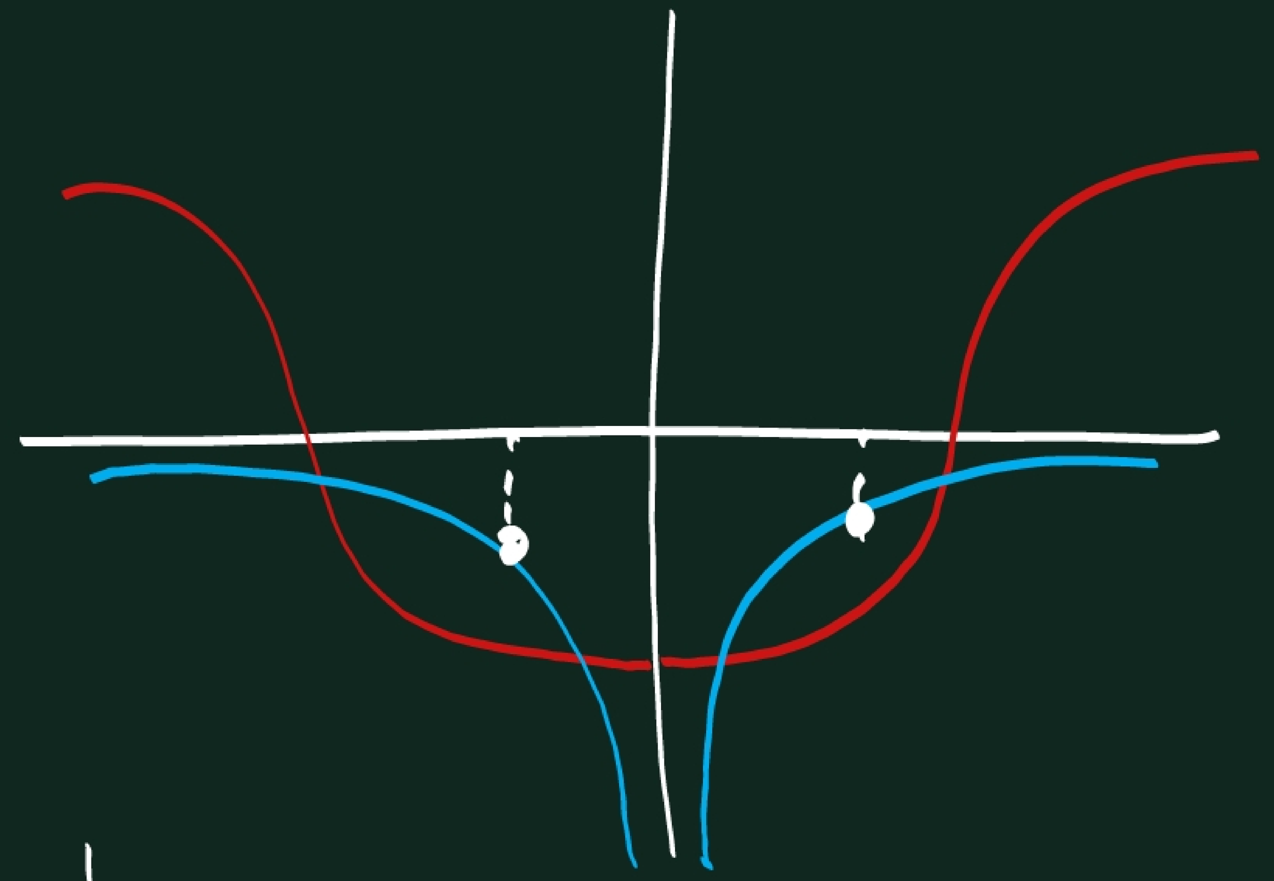
$$\frac{1}{9} - \frac{9}{9} = \underbrace{\frac{-8}{9} = -3x}$$

$$\frac{8}{27} = x$$

- Simetría:

- Función par: simétricas respecto ao eixo Y.

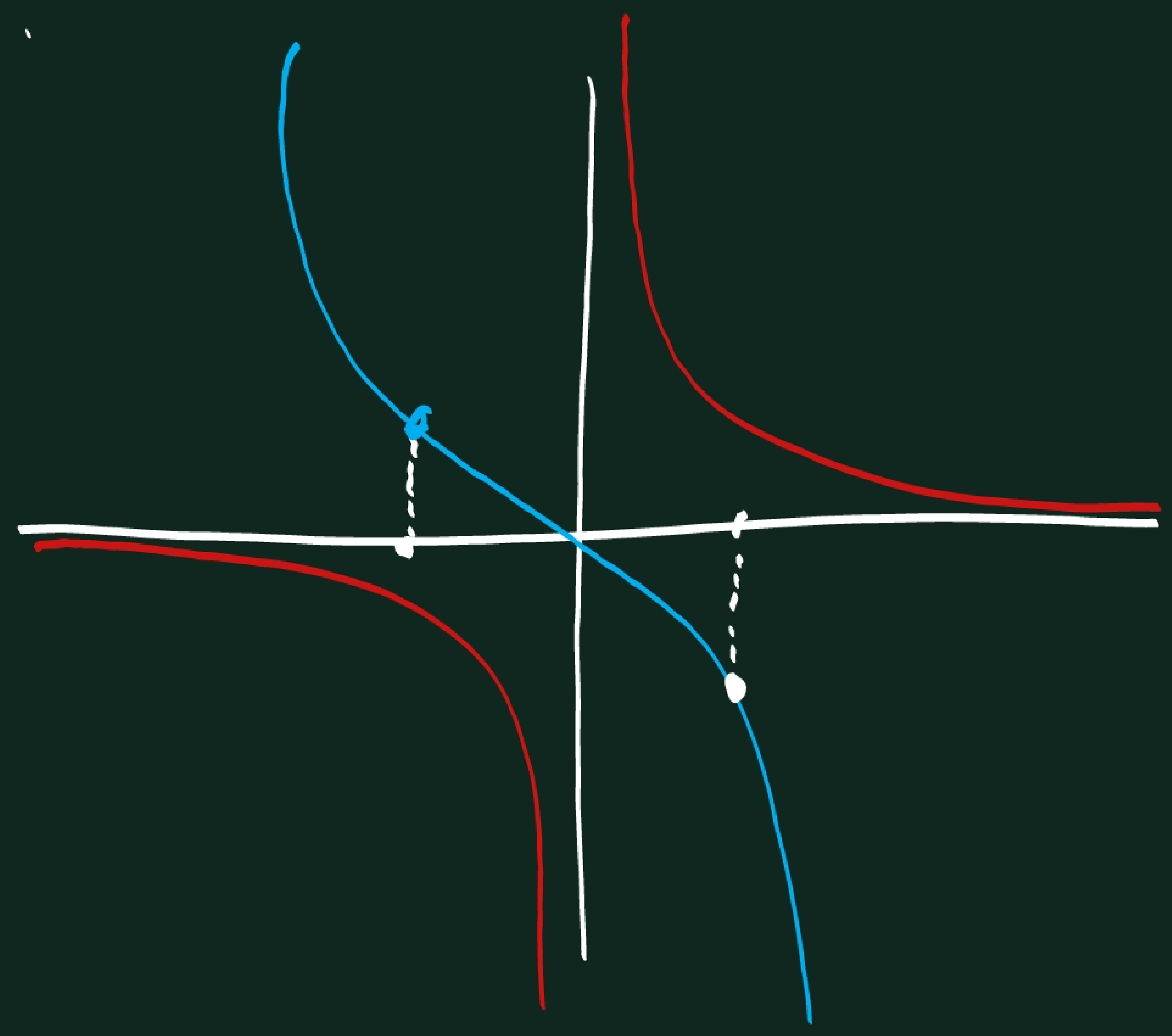
$$f(x) = f(-x)$$
$$f(3) = f(-3)$$



- Función impar: simétricas respecto á orixe de coordenadas.

$$-f(x) = f(-x)$$

$$-f(3) = f(-3)$$



1 c) $f(x) = \dfrac{x^2-5}{x^3-3x}$

$x^3 - 3x = 0$

$x(x^2-3) = 0$

$x = 0$

$x = \pm\sqrt{3}$

$\text{Dom } f(x) = \mathbb{R} - \{-\sqrt{3}, 0, +\sqrt{3}\}$

d) $f(x) = \dfrac{2^{1-2x} \cdot (1-2x)}{x^2-4x}$

$x^2 - 4x = 0$

$x(x-4) = 0 < \begin{matrix} x = 0 \\ x = 4 \end{matrix}$

$\text{Dom } f(x) = \mathbb{R} - \{0, 4\}$

③ a) $f(x) = x^3 - 3x + 5$

$f(-x) = (-x)^3 - 3\cdot(-x) + 5 = -x^3 + 3x + 5 \longrightarrow$ Non hai Sim. Par

$-f(x) = -x^3 + 3x - 5 \longrightarrow$ Non hai sim. impar

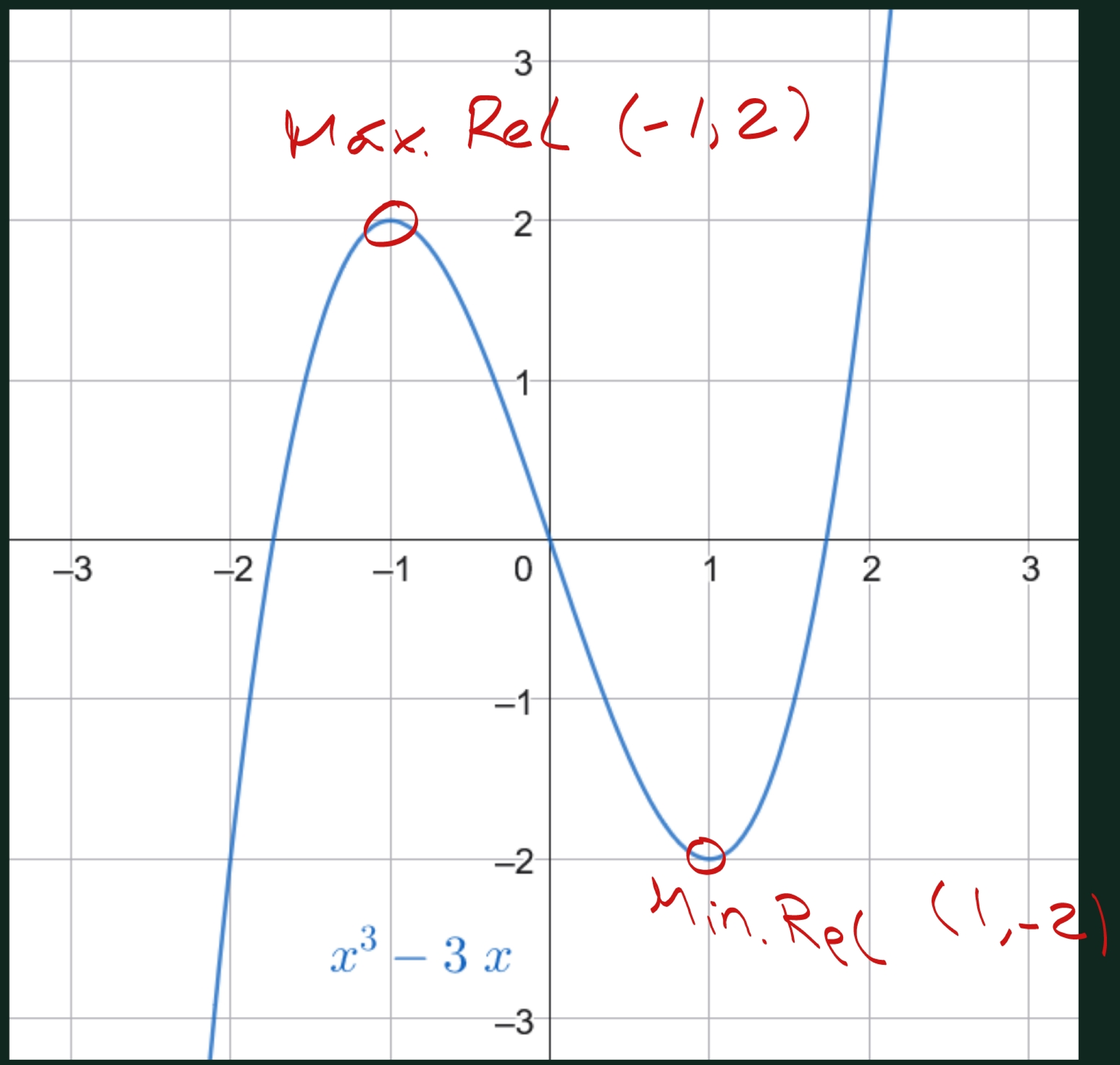
b) $f(x) = \sqrt{x^2+1}$

$f(-x) = \sqrt{(-x)^2+1} = \sqrt{x^2+1} \longrightarrow$ Hai sim. par

c) $f(x) = 3x^3 - 2x$

$f(-x) = -3x^3 + 2x$

$-f(x) = -3x^2 + 2x \longrightarrow$ Sim. impar



Ptos. Corte X: $(-1,75;0)$

$(0,0) \rightarrow$ Corte Y

$(1,75;0)$

Crece: $(-\infty,-1) \cup (1,+\infty)$

Decrece: $(-1, 1)$

Sim. Impar ou respecto á orixe de coord.

Operacións con funcións

Suma e diferenza: $(f \pm g)(x) = f(x) \pm g(x)$

$$\mathrm{Dom}\,(f \pm g)(x) = \mathrm{Dom}\, f(x) \cap \mathrm{Dom}\, g(x)$$

Multiplicación: $(f \cdot g)(x) = f(x) \cdot g(x)$

$$\mathrm{Dom}\,(f \cdot g)(x) = \mathrm{Dom}\, f(x) \cap \mathrm{Dom}\, g(x)$$

División: $(f/g)(x) = f(x)/g(x)$

$$\mathrm{Dom}\,(f/g)(x) = \mathrm{Dom}\, f(x) \cap \mathrm{Dom}\, g(x) \overbrace{\cap \{x \in \mathbb{R} / g(x) \neq 0\}}^{\text{Que o denominador non sexa 0}}$$

- Composición de funcións:

$$\left.\begin{array}{l} f \circ g\,(x) = f(g(x)) \\ g \circ f(x) = g(f(x)) \end{array}\right\} f(g(x)) \neq g(f(x))$$

$$f(x) = \sqrt{x^2 + 1} \qquad f \circ g(x) = \sqrt{(2x+3)^2 + 1} = \sqrt{4x^2 + 12x + 10}$$

$$g(x) = 2x + 3 \qquad g \circ f(x) = 2\sqrt{x^2 + 1} + 3$$

• Elemento neutro: $\boxed{Id(x) = x}$

$f(x)$ a que sexa.

$$f \circ Id(x) = f(x)$$

$$Id \circ f(x) = f(x)$$

$$f(x) = \sqrt{x} \cdot \log(x^2-2)$$

$$Id(x) = x$$

$$\left.\begin{array}{l} \\ \end{array}\right\} \begin{array}{l} f \circ Id(x) = \sqrt{x} \cdot \log(x^2-2) \\ Id \circ f(x) = \sqrt{x} \cdot \log(x^2-2) \end{array}$$

• Función Inversa: $f^{-1}(x)$ é a función inversa de $f(x)$ se,

$$f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$$

$$f(x) = \sqrt{x^2 - 5}$$

Cálculo de $f^{-1}(x)$:

1º - Substitúo $f(x)$ por "x" e "x" por "y".

2º - Despexo "y":

$$x = \sqrt{y^2 - 5}$$

$$x^2 = y^2 - 5$$

$$x^2 + 5 = y$$

$$\sqrt{x^2+5} = y \longrightarrow f^{-1}(x) = \sqrt{x^2+5}$$

⑥ $f(x) = e^{x-1}$ $\qquad g(x) = \dfrac{1}{x^2+x}$

a) $g \circ f(x) = \dfrac{1}{(e^{x-1})^2 + e^{x-1}}$

b) $f \circ g(x) = e^{\frac{1}{x^2+x} - 1} = e^{\frac{1}{x^2+x} - \frac{x^2+x}{x^2+x}} = e^{\frac{1-x^2-x}{x^2+x}}$

8. $z(c) = 10c + \sqrt{2c+4}$

a) z cando $x=4$

$c(x) = 5 + \frac{x}{2} - \frac{x^2}{16}$

b) $z(x) = ?$

a) $c(4) = 5 + \frac{4}{2} - \frac{4^2}{16} = 5 + 2 - 1 = 6$

$z \circ c(4) = 10 \cdot 6 + \sqrt{2 \cdot 6 + 4} = 60 + \sqrt{16} = 60 + 4 = \boxed{64}$

b) $z \circ c(x) = 10 \cdot \left(5 + \frac{x}{2} - \frac{x^2}{16}\right) + \sqrt{2 \cdot \left(5 + \frac{x}{2} - \frac{x^2}{16}\right) + 4} =$

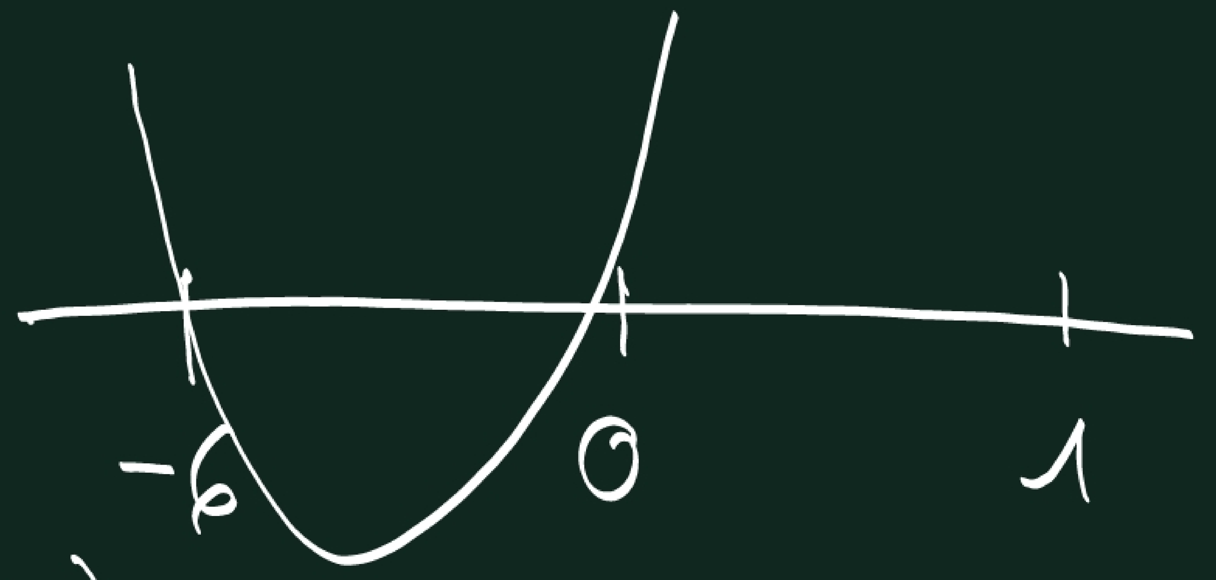
$= 50 + 5x - \frac{5x^2}{8} + \sqrt{14 + x - \frac{x^2}{8}}$

1 e) f)

e) $\dfrac{\log(x^2+6x)}{x-1} \rightarrow x^2+6x=0 \rightarrow x(x+6)=0 \begin{cases} x=0 \\ x+6=0 \\ x=-6 \end{cases}$

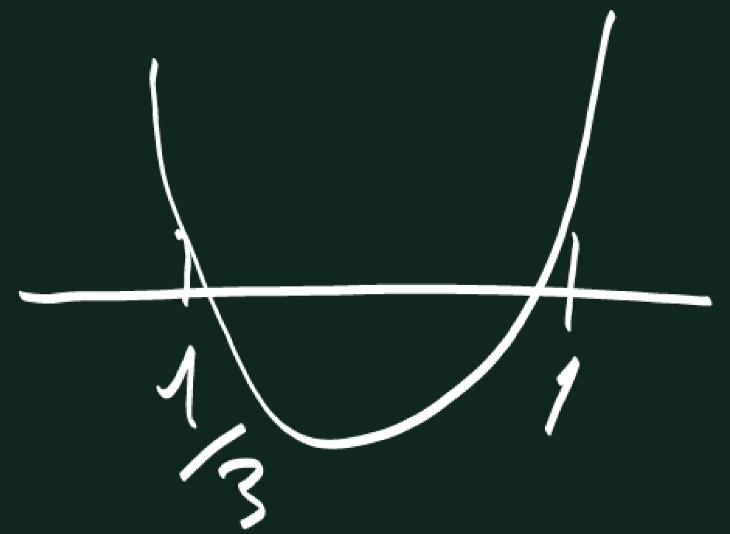
$\hookrightarrow x-1=0$

$x=1$



Dom: $(-\infty,-6)\cup(0,1)\cup(1,+\infty)$

f) $\ln(1-4x+3x^2) \rightarrow 3x^2-4x+1=0 \begin{cases} 1 \\ 1/3 \end{cases}$



Dom: $(-\infty,\frac{1}{3})\cup(1,+\infty)$

② e) f)

e) $f(x) = \dfrac{-6x}{x^2-16}$ Cortes eje x $\Rightarrow y=0$

$$\frac{-6x}{x^2+16} = 0$$

$$-6x = 0$$

$$x = 0$$

$(0,0)$

f) $\sqrt{3x(x-\sqrt{2})}$ Cortes eje x $\Rightarrow y=0$

$$\sqrt{3x(x-\sqrt{2})} = 0$$

$$3x \cdot (x-\sqrt{2}) = 0$$

$3x = 0 \rightarrow \boxed{x=0}$

$x-\sqrt{2}=0$

$\boxed{x=\sqrt{2}}$

$(0,0)$

$(\sqrt{2},0)$

③ c)

$$3x^3 - 2x$$

$$f(-x) = 3(-x)^3 - 2(-x) = -3x^3 + 2x$$

$$-f(x) = -3x^3 + 2x \rightarrow$$ Hay simetría impar

d) $\sqrt{x^2 - 2}$

$$f(-x) = \sqrt{(-x)^2 - 2} = \sqrt{x^2 - 2} \rightarrow$$ Hay simetría par

6) $g(x) = \dfrac{1}{x^2+x}$; $h(x) = x+3$

c) $g \circ h(x) = \dfrac{1}{(x+3)^2 + x+3} = \dfrac{1}{x^2+6x+9+x+3} = \dfrac{1}{x^2+7x+12}$

d) $h \circ g(x) = \dfrac{1}{x^2+x} + 3 = \dfrac{1}{x^2+x} + \dfrac{3x^2+3x}{x^2+x} = \dfrac{3x^2+3x+1}{x^2+x}$

a) Crece: $(-\infty, 0) \cup (0, +\infty)$

Ptos. Corte X: $(1,0), (-1,0)$

" " Y: Non hai

Ten sim. Impar

b) Crece: $(-\infty,-1), (-1,0)$

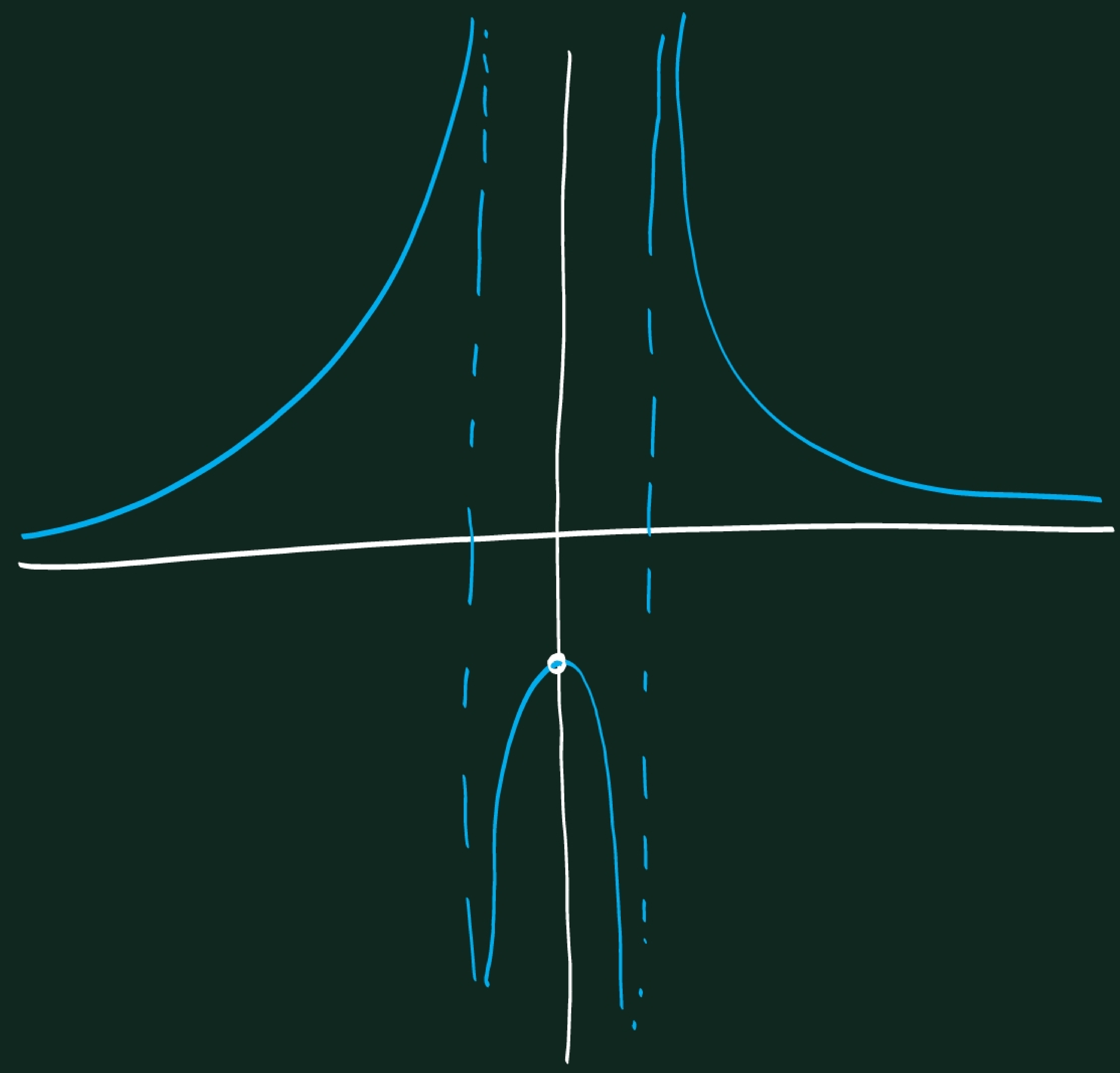
Decrece: $(1,+\infty), (0,1)$

Máx. Relativo: $(0,-1)$

Puntos Corte X: Non hai

Puntos Corte Y: $(0,-1)$

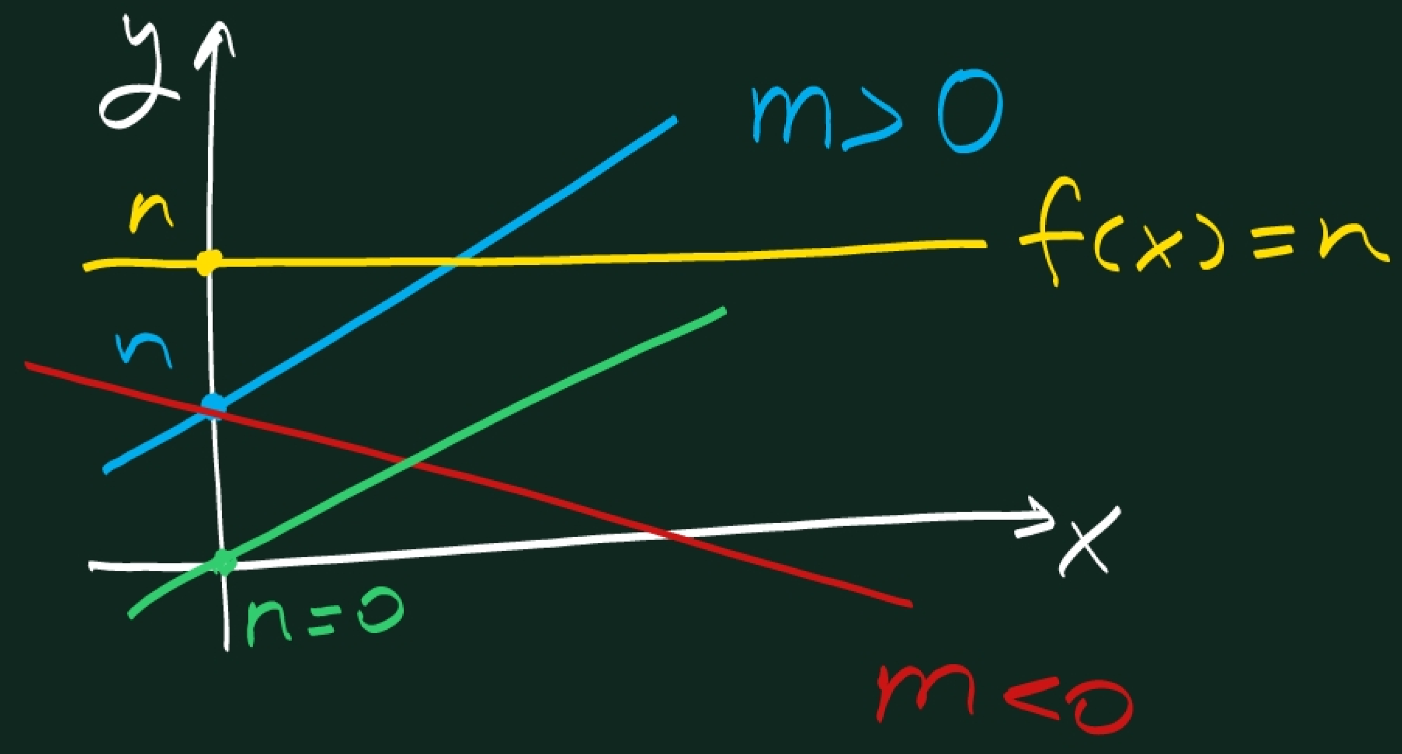
Simetría Par



Funcións Elementais

$f(x) = 7x^4 - 2x^3 + 5$

- Función polinómica : $f(x) = P(x)$



- Función Lineal: $f(x) = mx + n$

  Función constante: $f(x) = n$
  $m = 0$

- Función cuadrática: $f(x) = ax^2 + bx + c$

  $V = (V_1, V_2) = (V_x, V_y)$ → $V_x = \frac{-b}{2a}$; $V_y = f(V_x)$

Se $a > 0$ → Parábola positiva: $\cup$

Se $a < 0$ → Parábola negativa: $\cap$

$$f(x) = x^2 - 6x + 3$$

- Vértice
- Ptos. de corte
- Outros puntos

$$V = (V_x, V_y) = (3, -6)$$

$$V_x = \frac{+6}{2} = 3 \longrightarrow V_y = (3)^2 - 6 \cdot (3) + 3 = 9 - 18 + 3 = -6$$

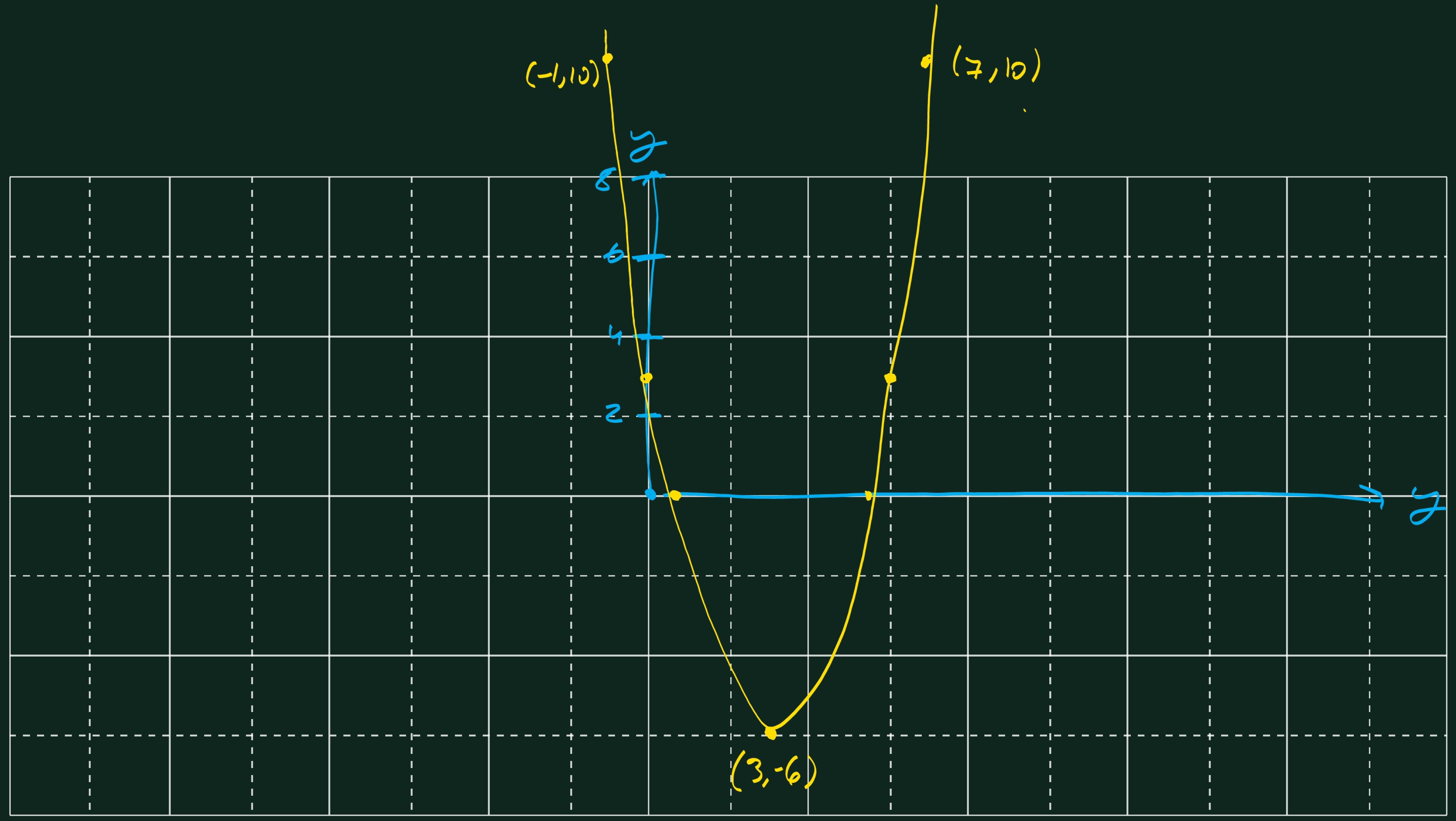
Corte X: $\rightarrow y = 0 \begin{cases} (5{,}45 ; 0) \\ (0{,}55 ; 0) \end{cases}$

$$x^2 - 6x + 3 = 0$$

$$x = \frac{+6 \pm \sqrt{36-12}}{2} = \frac{+6 \pm 4{,}9}{2} \begin{cases} 5{,}45 \\ 0{,}55 \end{cases}$$

Corte Y: $\rightarrow x = 0 \rightarrow 0^2 - 6 \cdot 0 + 3 = 3 \rightarrow (0, 3)$

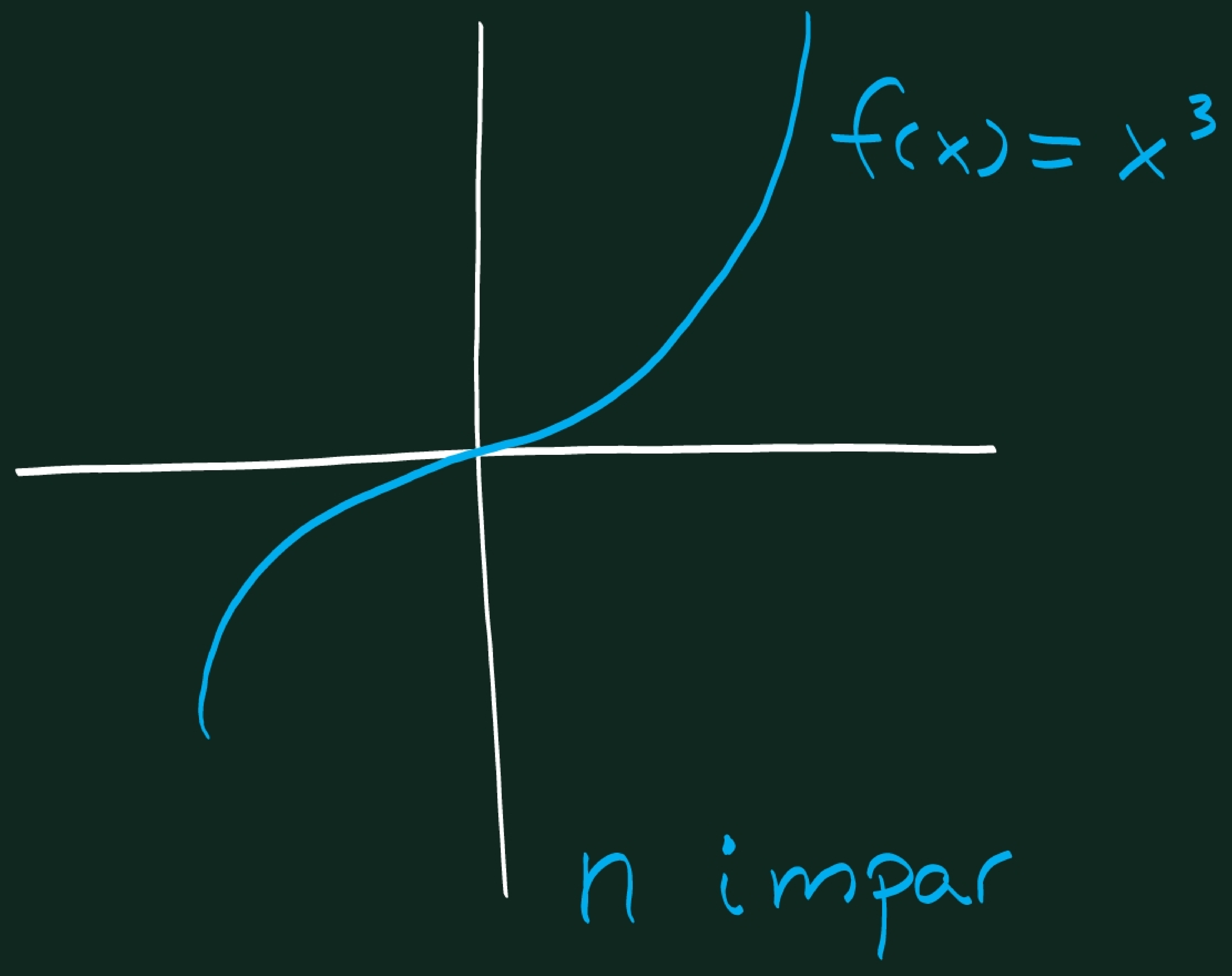
x	y	
-1	10	
0	3	Corte Y
0,55	0	Corte X
3	-6	Vertice
5,45	0	Corte X
6	3	Simetrico
7	10	

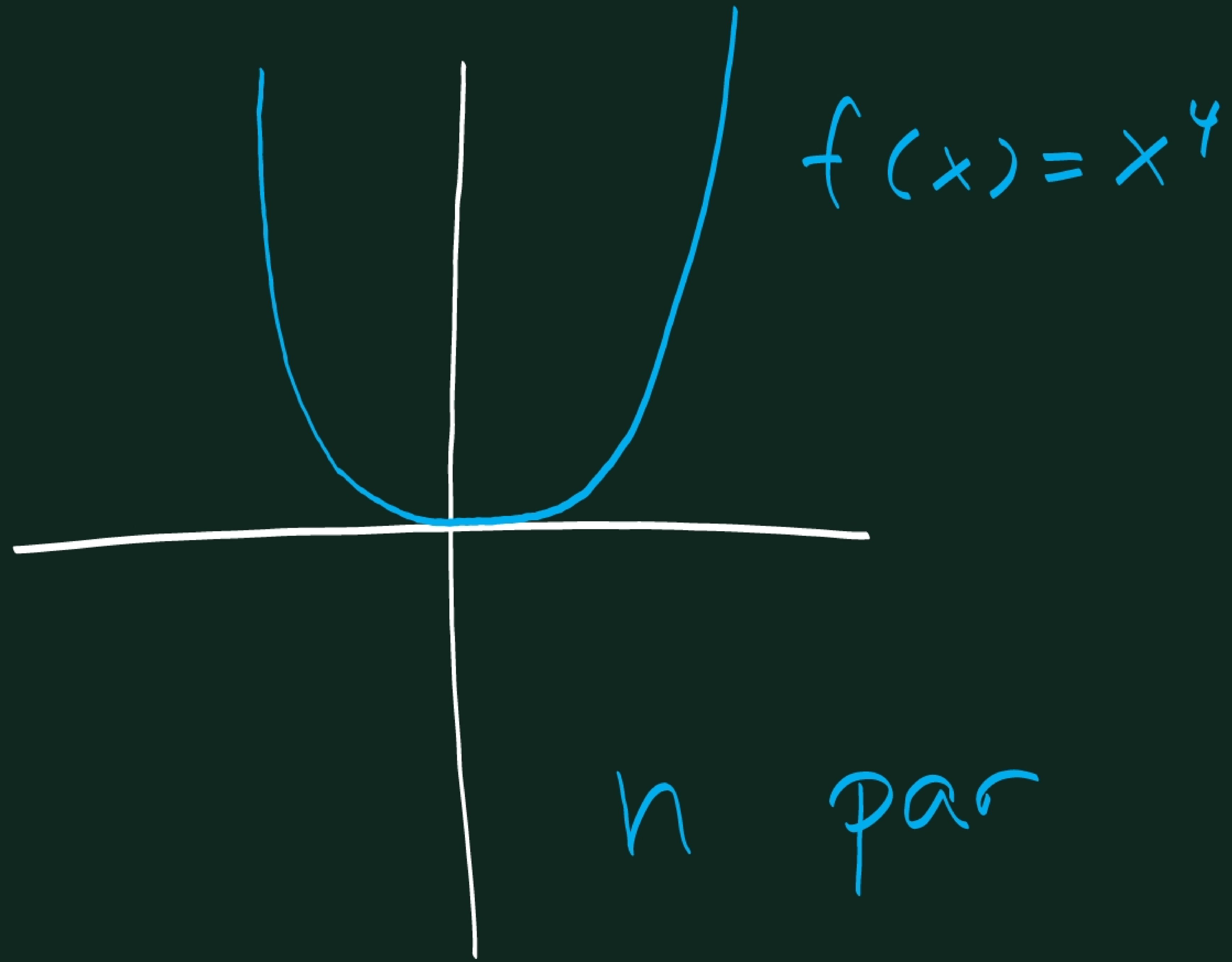


(-1,10)
(7,10)
y
8
6
4
2
y
(3,-6)

- Funcións de grado > 2

$$f(x) = a\,x^n$$

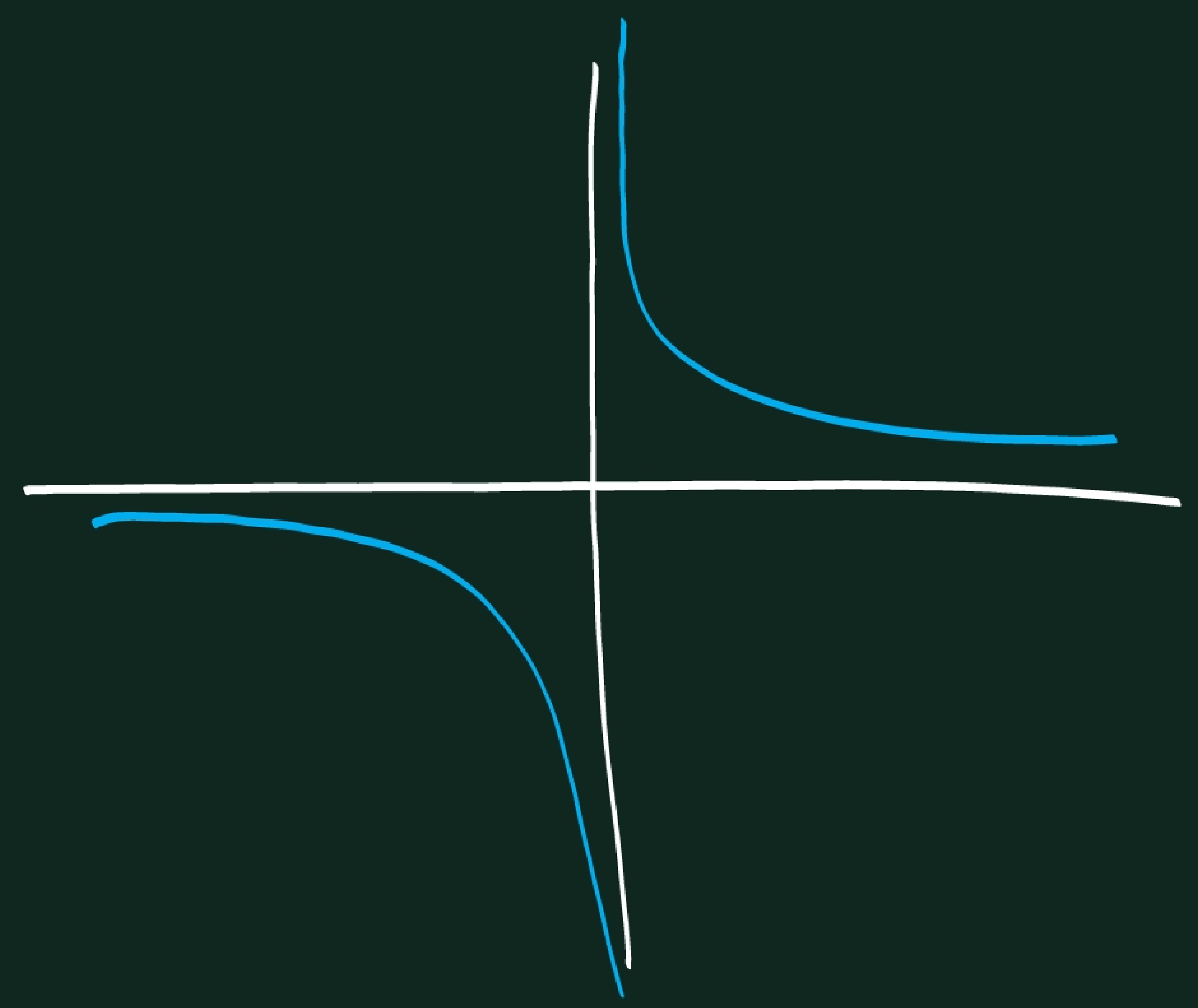




- Funcións Racionais:

$$f(x) = \frac{P(x)}{Q(x)} \quad ; \quad \text{Grado } Q(x) > 0$$

$$f(x) = \frac{a}{bx \pm c} \rightarrow \text{Función de prop. inversa}$$

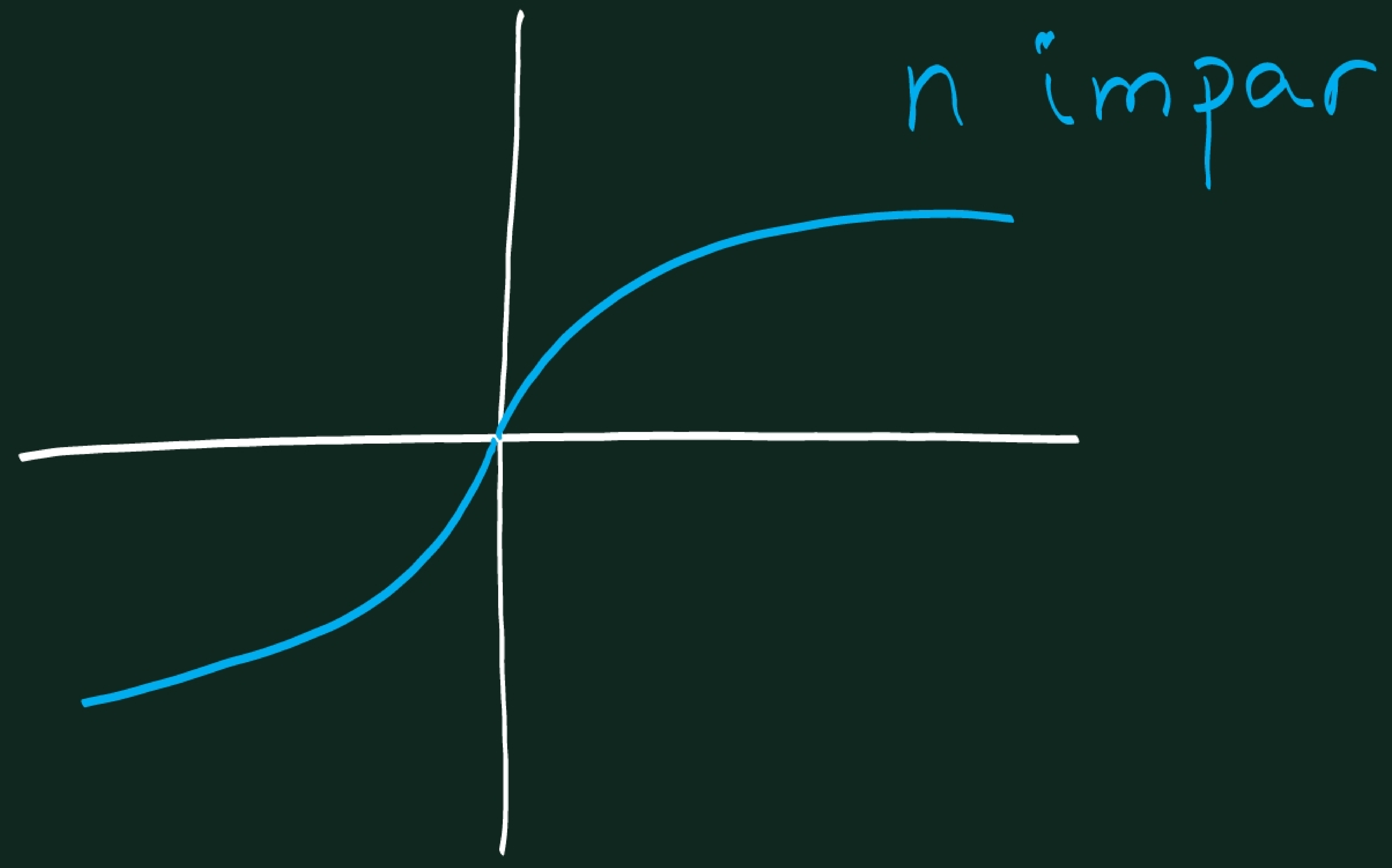


- Funcións Radicais:

$$f(x)=\sqrt{P(x)}$$

$$f(x)=\sqrt[n]{ax\pm b}$$

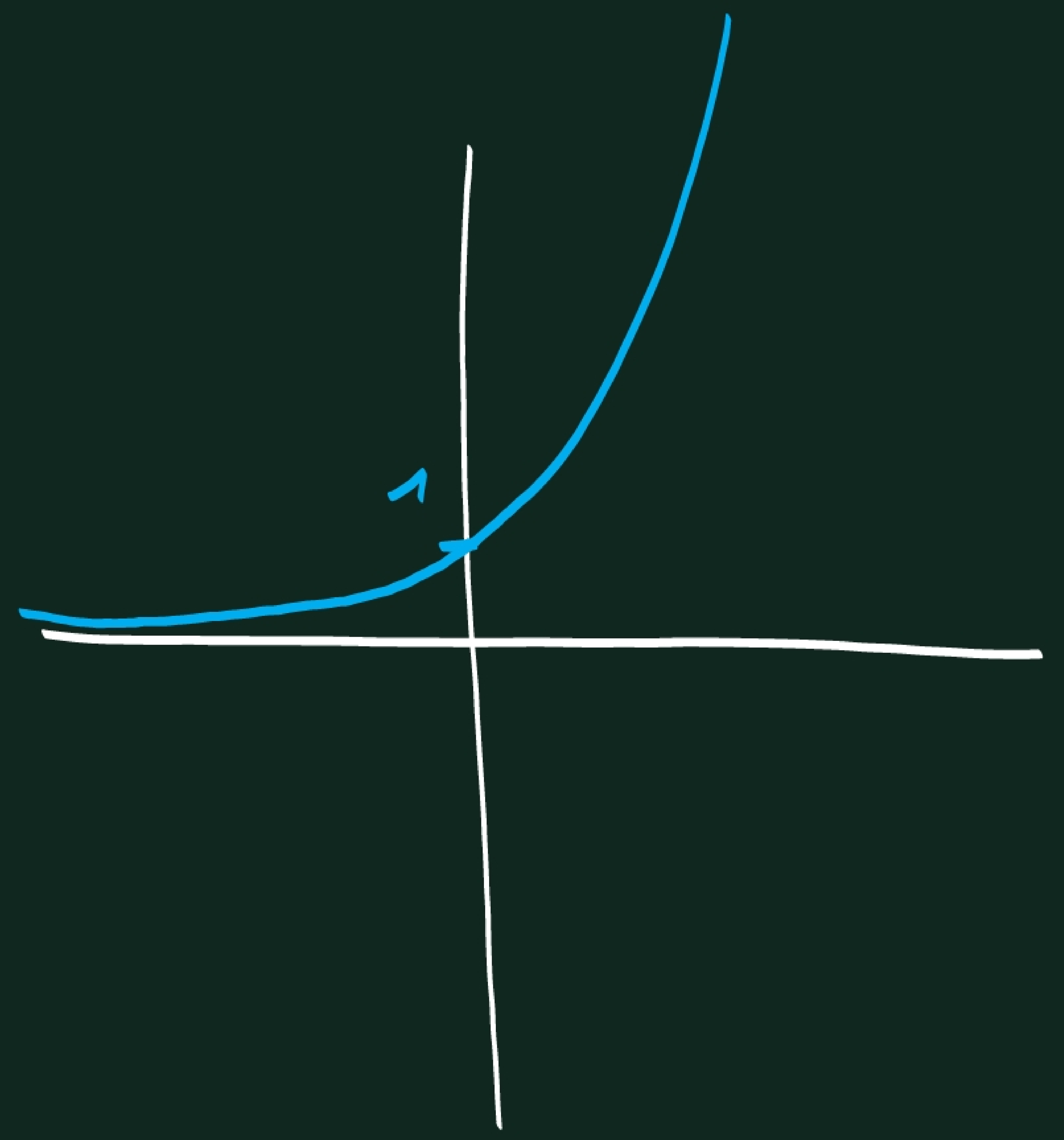
$f(x)=\sqrt[n]{x}$



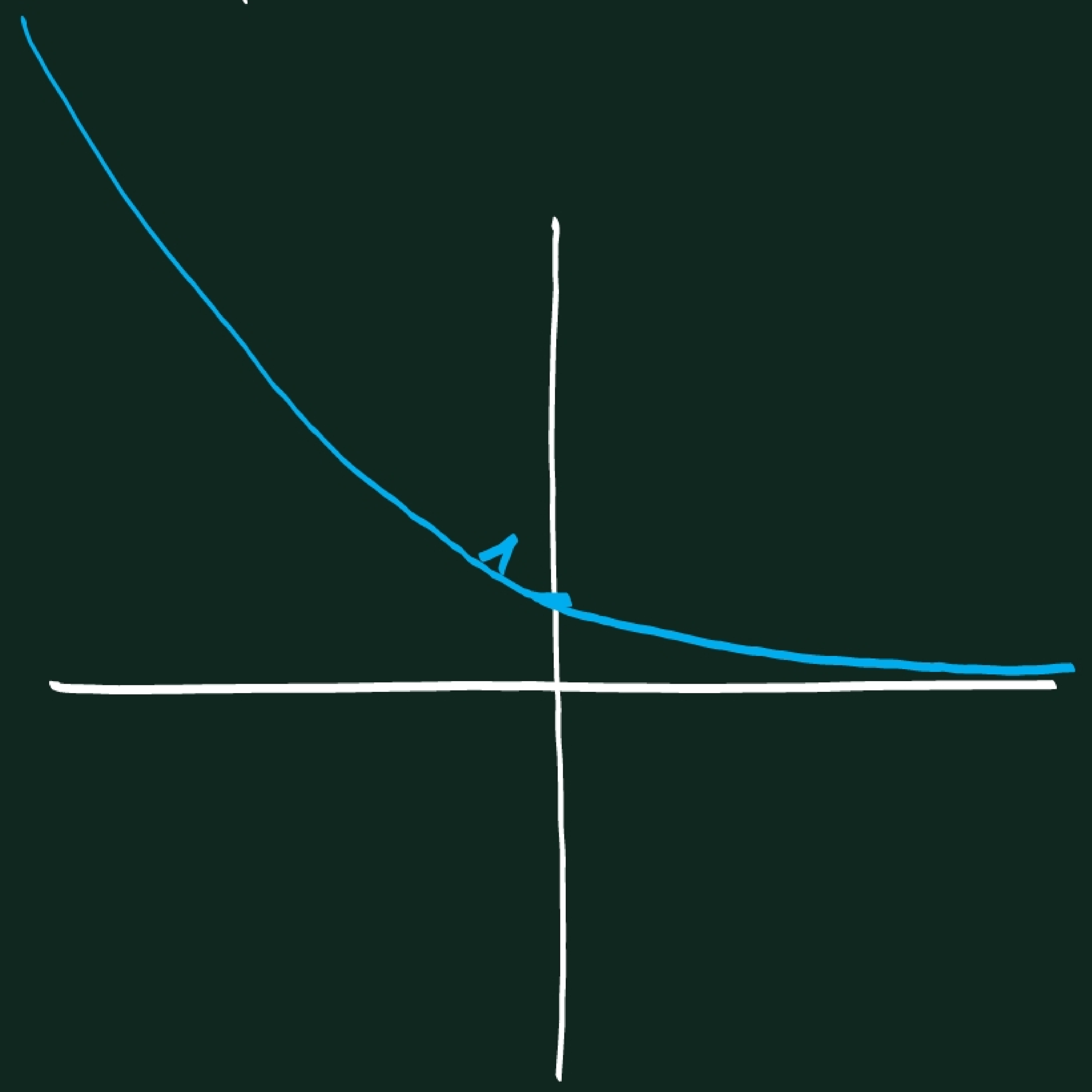
• Funcións Exponenciais

$$f(x) = a^x \quad ; \quad a > 0$$

$a > 1$



$1 > a > 0$

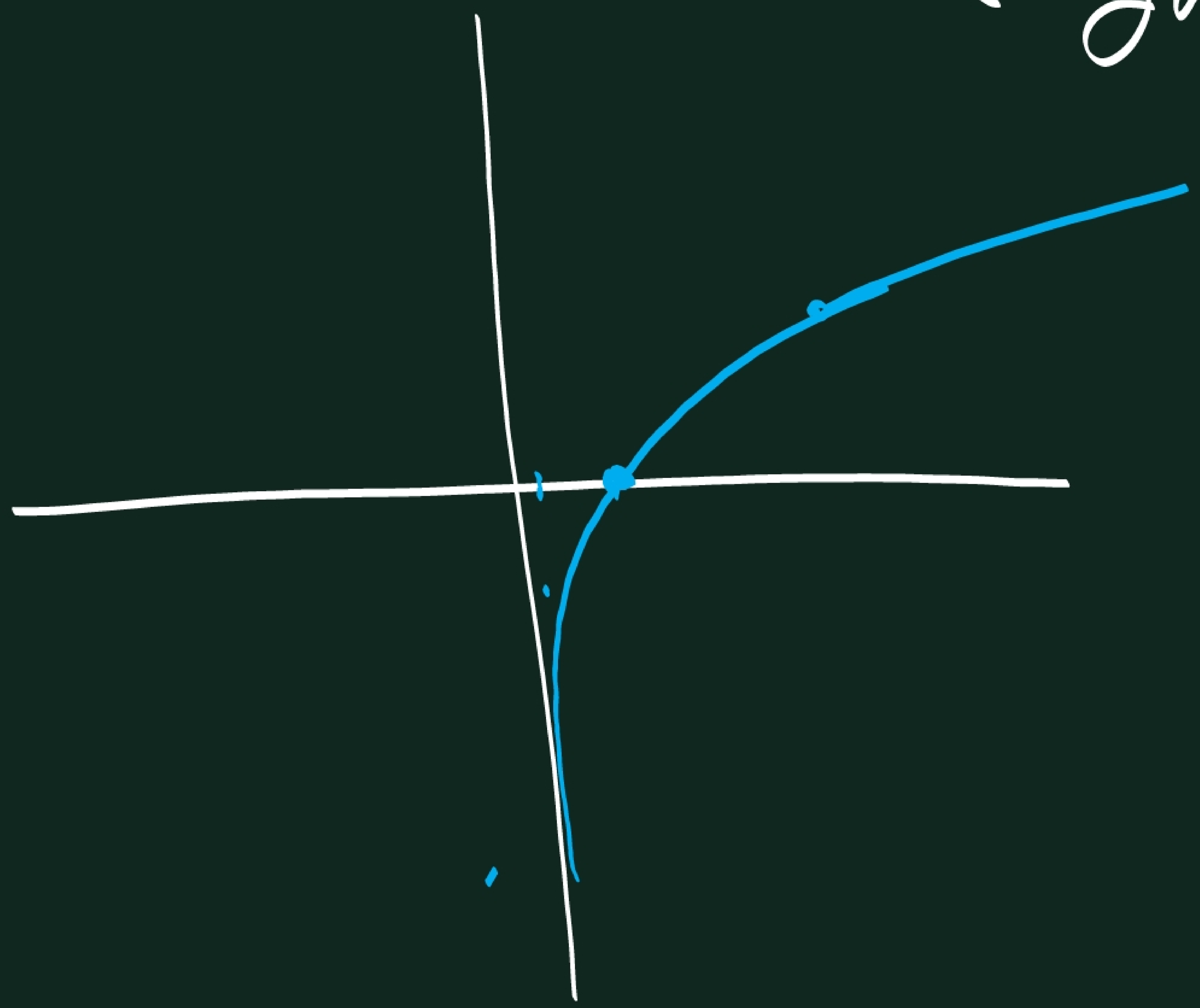


- **Funcións Logarítmicas**

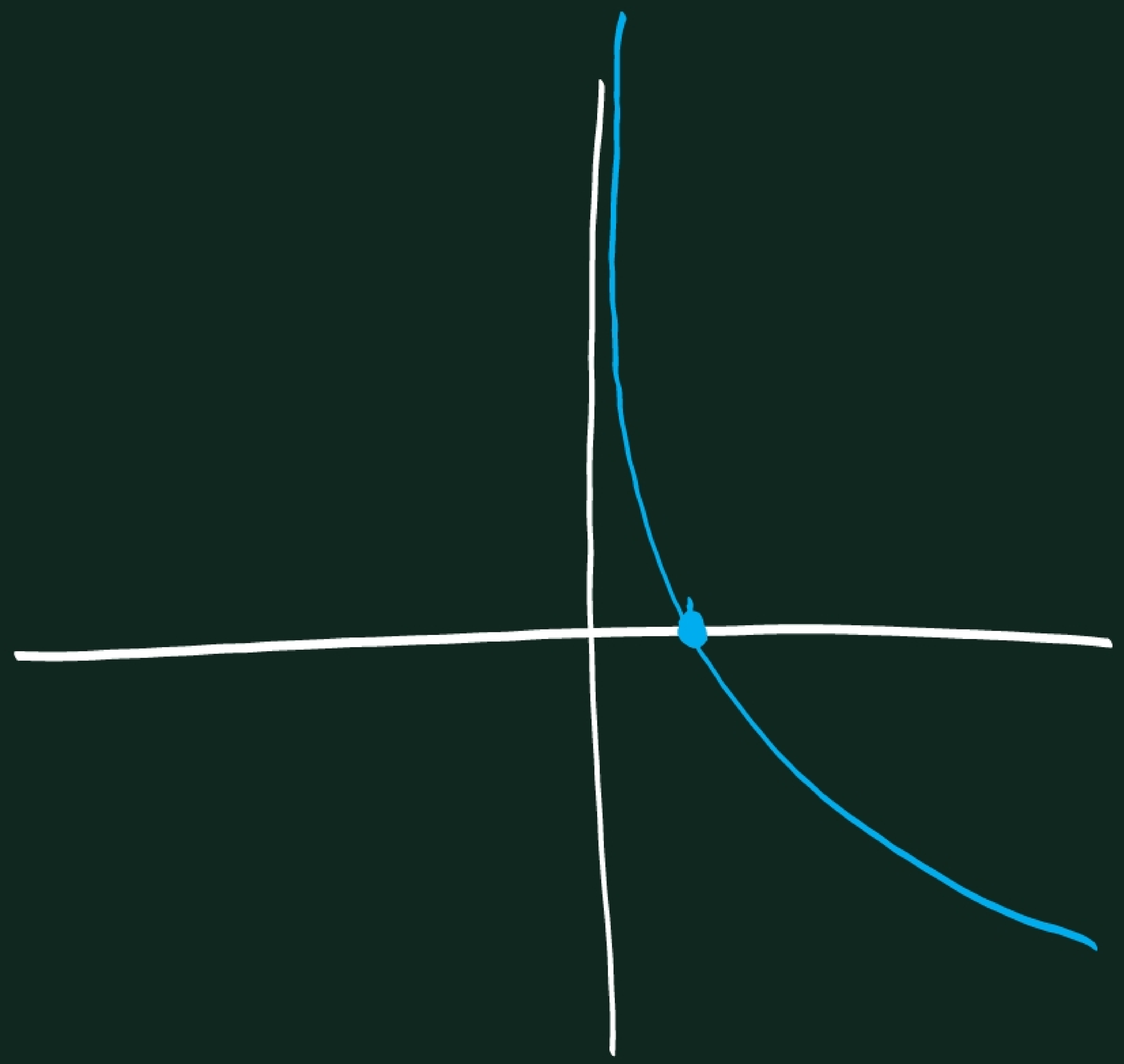
$$f(x) = \log_a x \iff a^{f(x)} = x$$

$a > 1$

$\log_a x$



$1 > a > 0$



③ a) $y = f(x) = \sqrt{x^2 - 4}$

Ptos. Corte Eixo X:

$y = 0$

$0 = \sqrt{x^2 - 4}$

$0 = x^2 - 4$

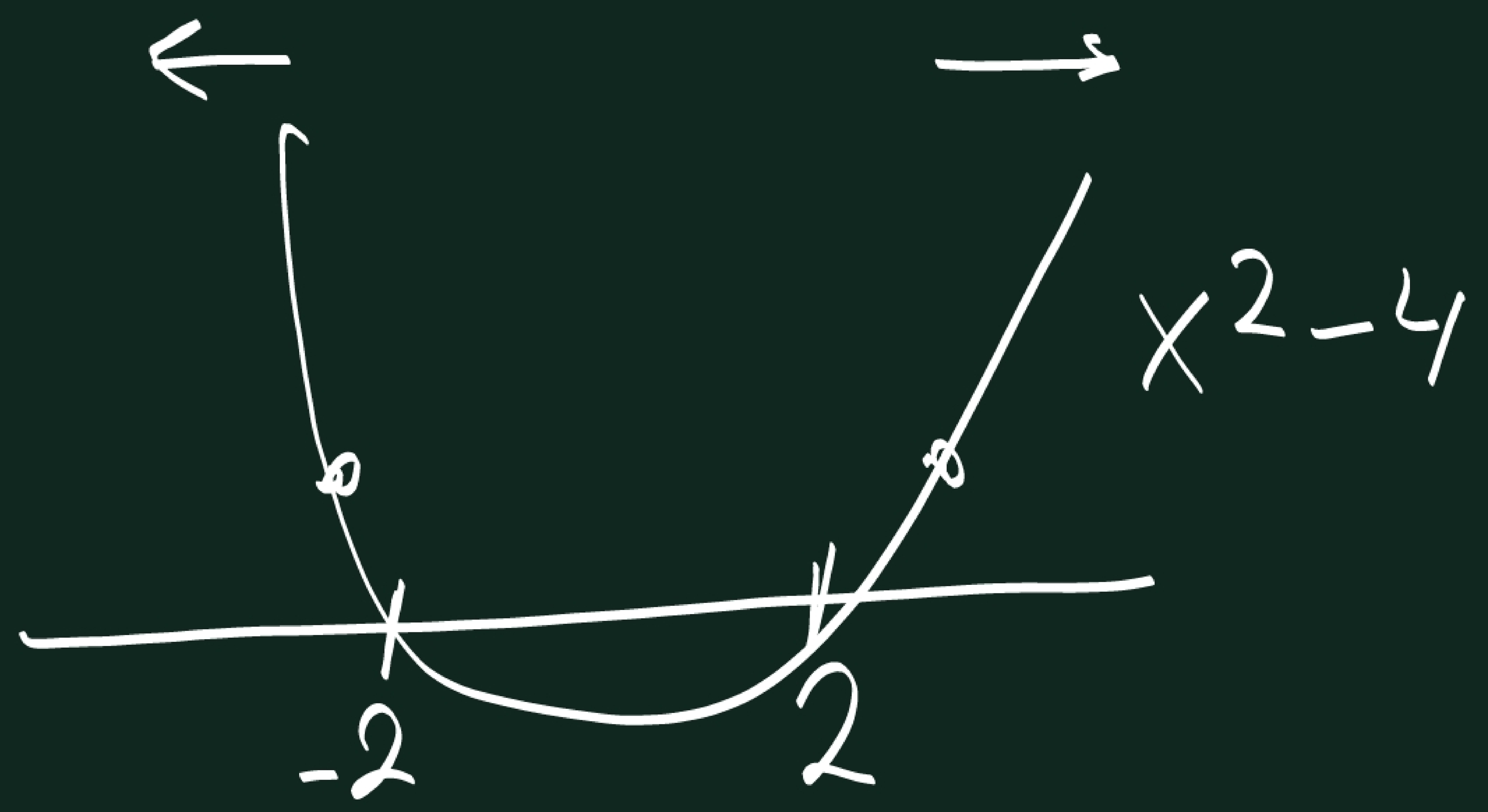
$4 = x^2 \longrightarrow x = \sqrt{4} = \pm 2$

$(-2,0)\ (2,0)$

Corte Eixo Y:

$x = 0$

$y = \sqrt{-4} = \not\exists$



x	y
-10	9,8
-6	5,66
-4	3,46
-2	0
2	0
4	3.46
6	5.66
10	9.80

(2) a) $x^2 - 2x + 3$

Vértice (V_x, V_y)

$V_x = \frac{-b}{2a} \to \frac{+2}{2} \to 1$

$V_y = 1^2 - 2 \cdot 1 + 3 \to 2$

$V = (1,2)$

Puntos Corte

Eixo X $(y=0)$

$0 = x^2 - 2x + 3$

$$\frac{+2 \pm \sqrt{-2^2 - 4 \cdot 1 \cdot 3}}{2} = \nexists$$

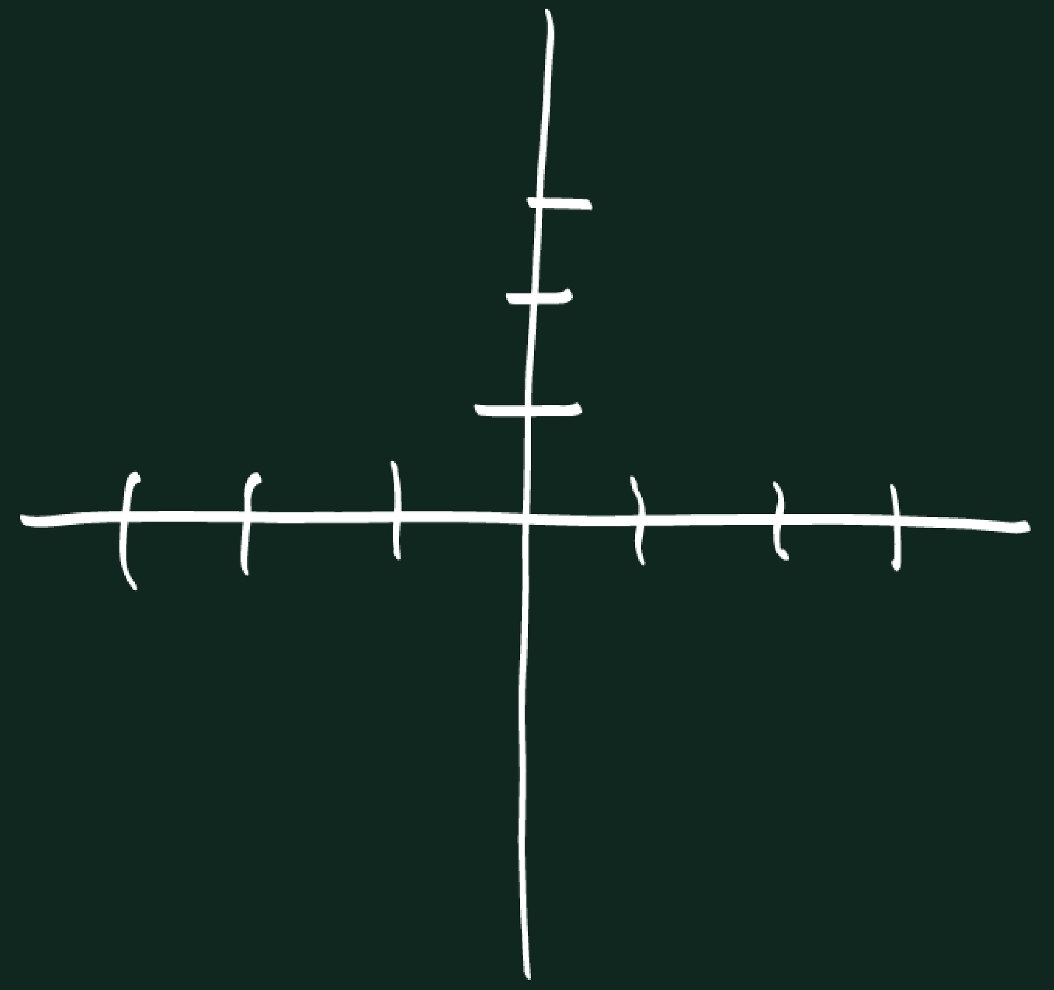
Eixo Y $(x=0)$

$0^2 - 2 \cdot 0 + 3 = 3$

$(0,3)$

Outros puntos

x	y
0	3
1	2
2	3



b) $-x^2+2x-3$

$V(V_x, V_y) = (1,-2)$

$V_x = \frac{-b}{2a} = \frac{-2}{-2} = +1$

$V_y = -1+2-3 = -2$

Corte Eixo Y:

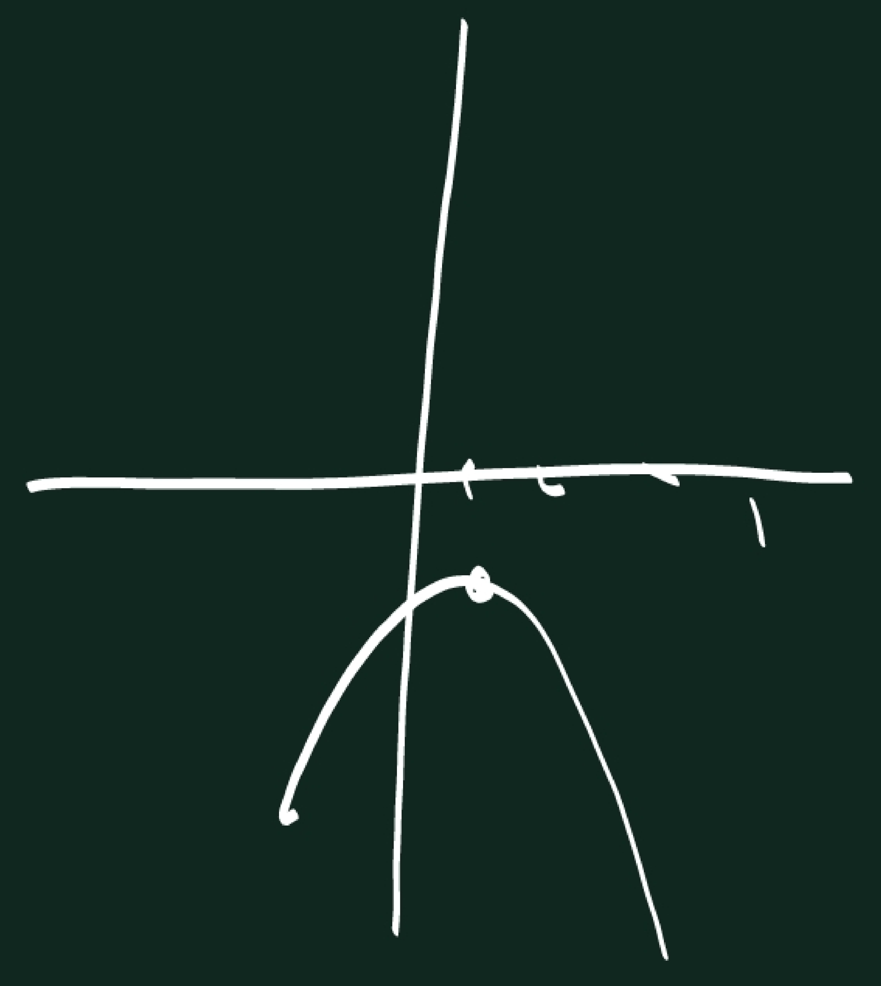
$x=0$

$y = 0+0-3 = -3$

$(0,-3)$

x	y
-2	-11
-1	-6
0	-3
1	-2
2	-3
3	$-3^2+2\cdot 3-3=-6$
4	$-4^2+2\cdot 4-3=-11$

Vértice



Funcións Definidas a Trozos

Teñen expresións alxebraicas diferentes en intervalos distintos do seu dominio.

$$f(x)=\begin{cases} 4x+1 & \text{si } x<-1 \\ 1 & \text{si } -1\leq x<2 \\ \log x & \text{si } x\geq 2 \end{cases}$$

$4x+1$

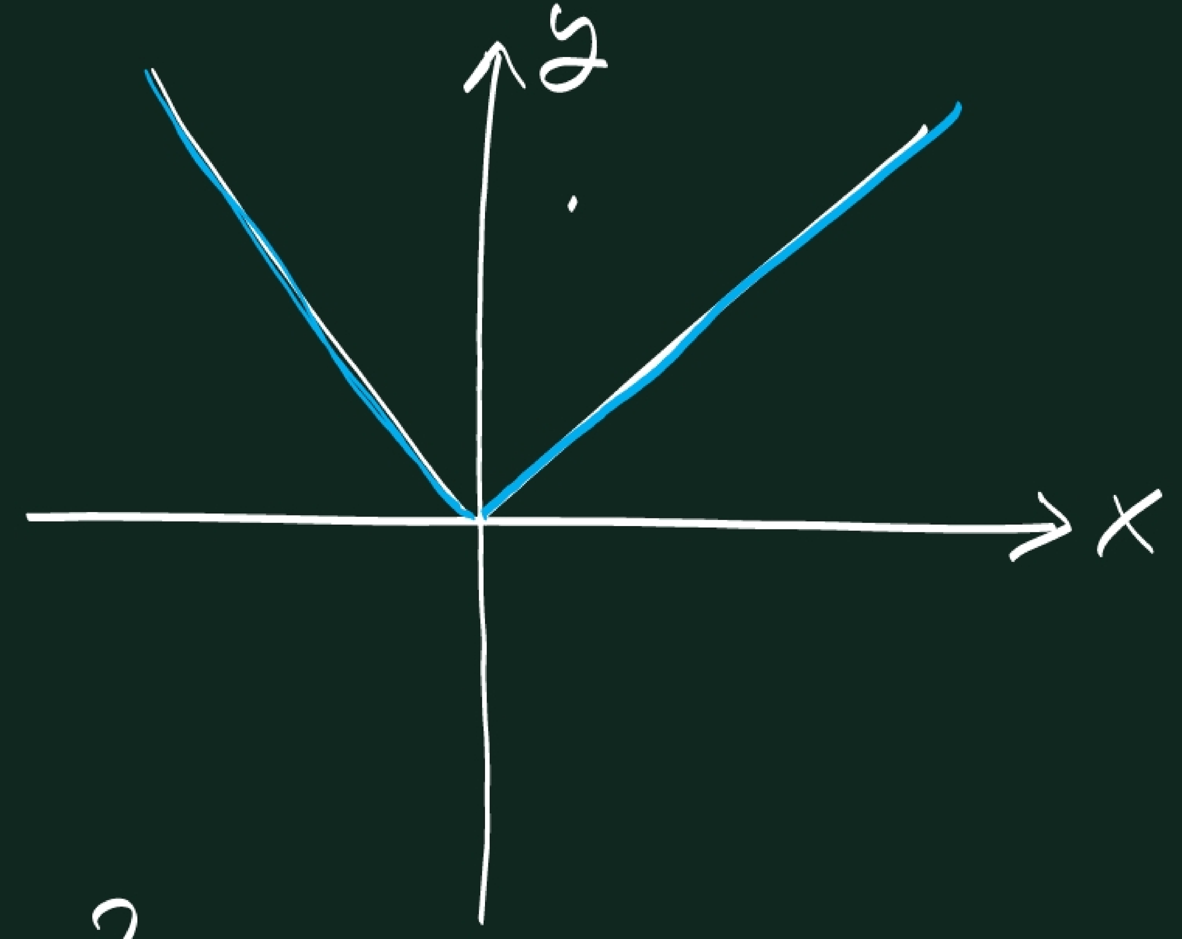
x	y
-3	-11
-1	-3

$\log x$

x	y
2	0,3
5	0,7
10	1

Función valor absoluto

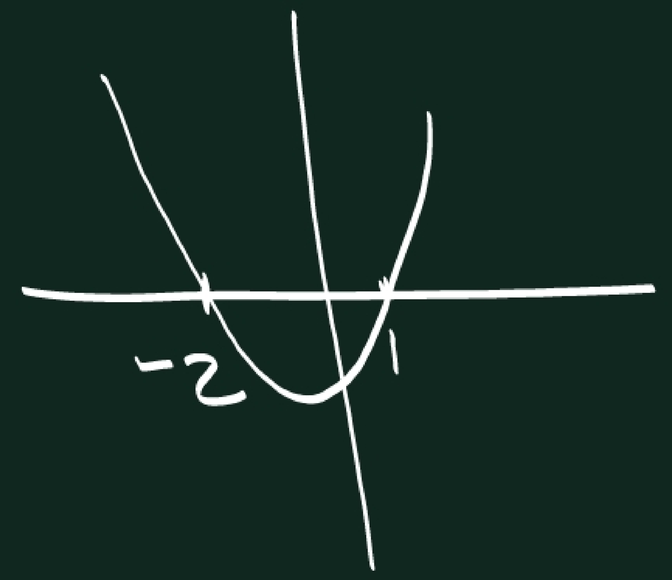
$$f(x) = |x| = \begin{cases} -x & \text{se} \quad x<0 \\ x & \text{se} \quad x \geq 0 \end{cases}$$



$$f(x) = |x^2+x-2| \quad \begin{cases} x^2+x-2 & \text{se} \quad x \leq -2 \\ -x^2-x+2 & \text{se} \quad -2 < x \leq 1 \\ x^2+x-2 & \text{se} \quad 1 < x \end{cases}$$

$$x^2+x-2=0$$

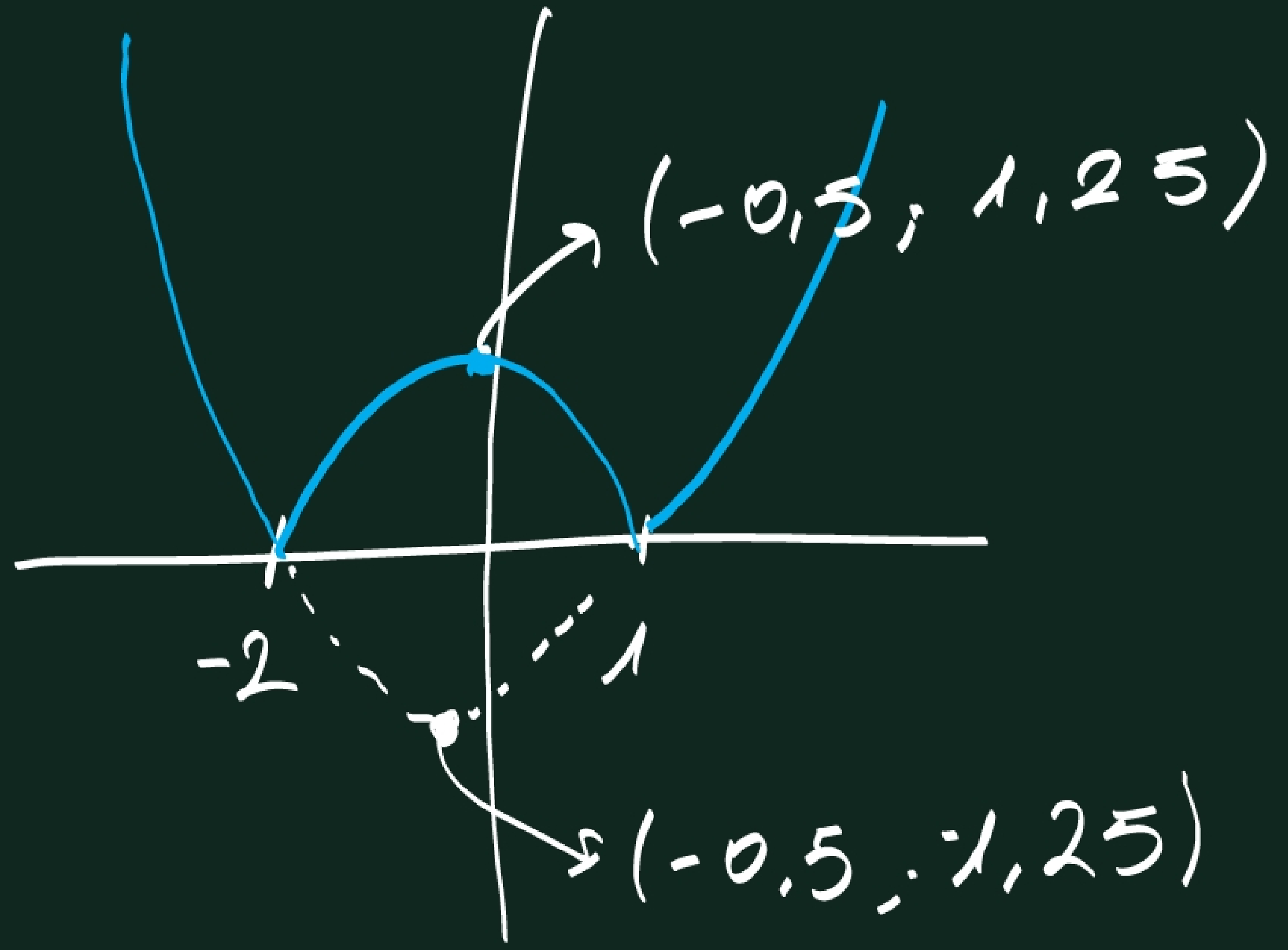
$$x = \frac{-1 \pm \sqrt{1+8}}{2} \begin{cases} 1 \\ -2 \end{cases}$$



$$V_x = \frac{-b}{2a} = \frac{-1}{2} = -0,5$$

$$V_y = \left(\frac{1}{2}\right)^2 + \frac{1}{2} - 2 =$$

$$= \frac{1}{4} + \frac{1}{2} - 2 = \frac{-5}{4} = -1,25$$

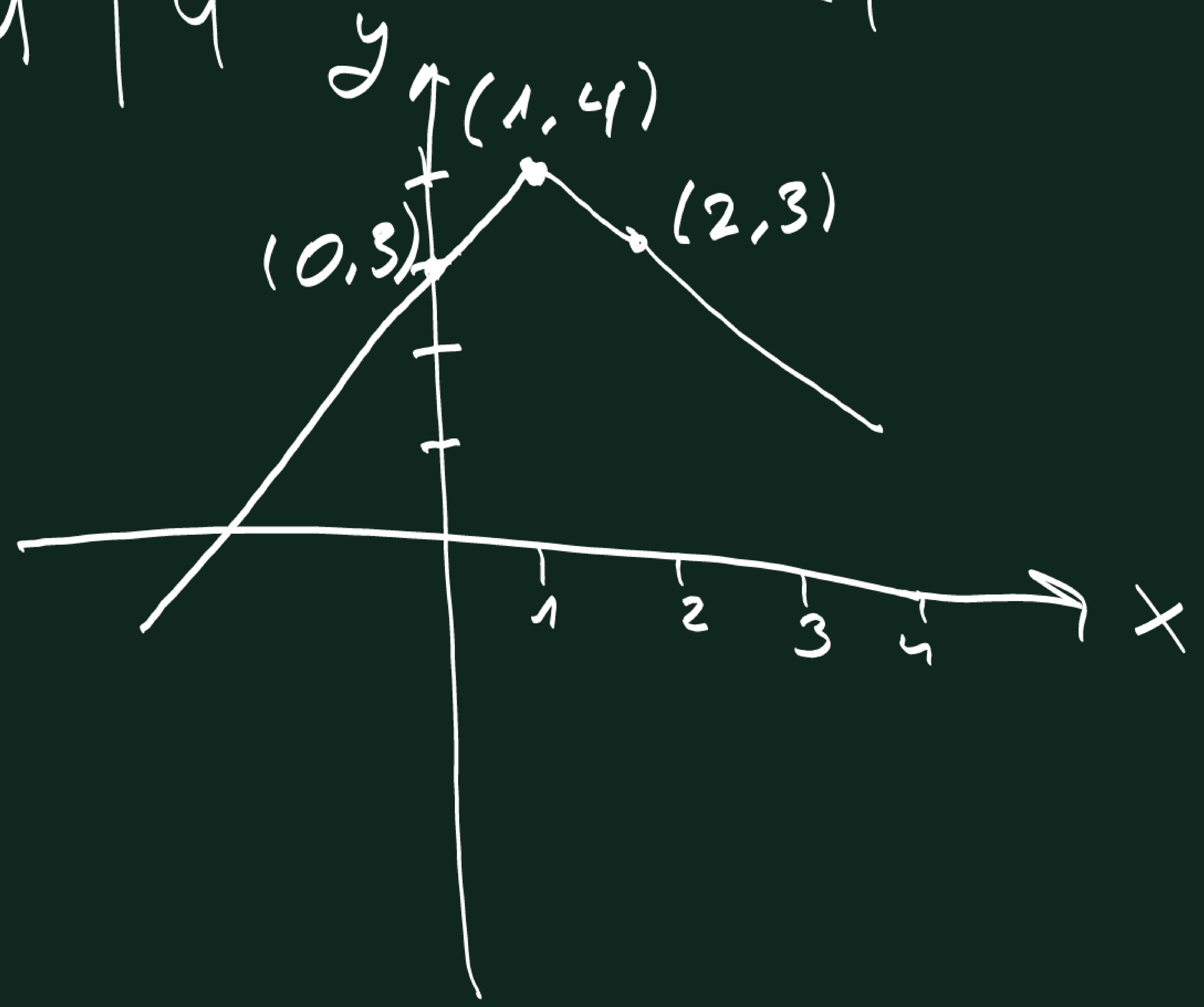


⑤

a) $f(x) = \begin{cases} x+3 & x<1 \\ 5-x & x \geq 1 \end{cases}$

X	Y
0	3
1	4

X	Y
1	4
2	3



b) $f(x) = \begin{cases} 2 & x \leq -2 \\ x^2 & -2 < x < 1 \\ x & x \geq 1 \end{cases}$

$$V_x = -\frac{b}{2a} = \frac{0}{2} \Rightarrow V_x = 0$$

$$V_y = 0^2 = 0$$

x	y
-2	4
0	0
1	1

x	y
1	1
2	2

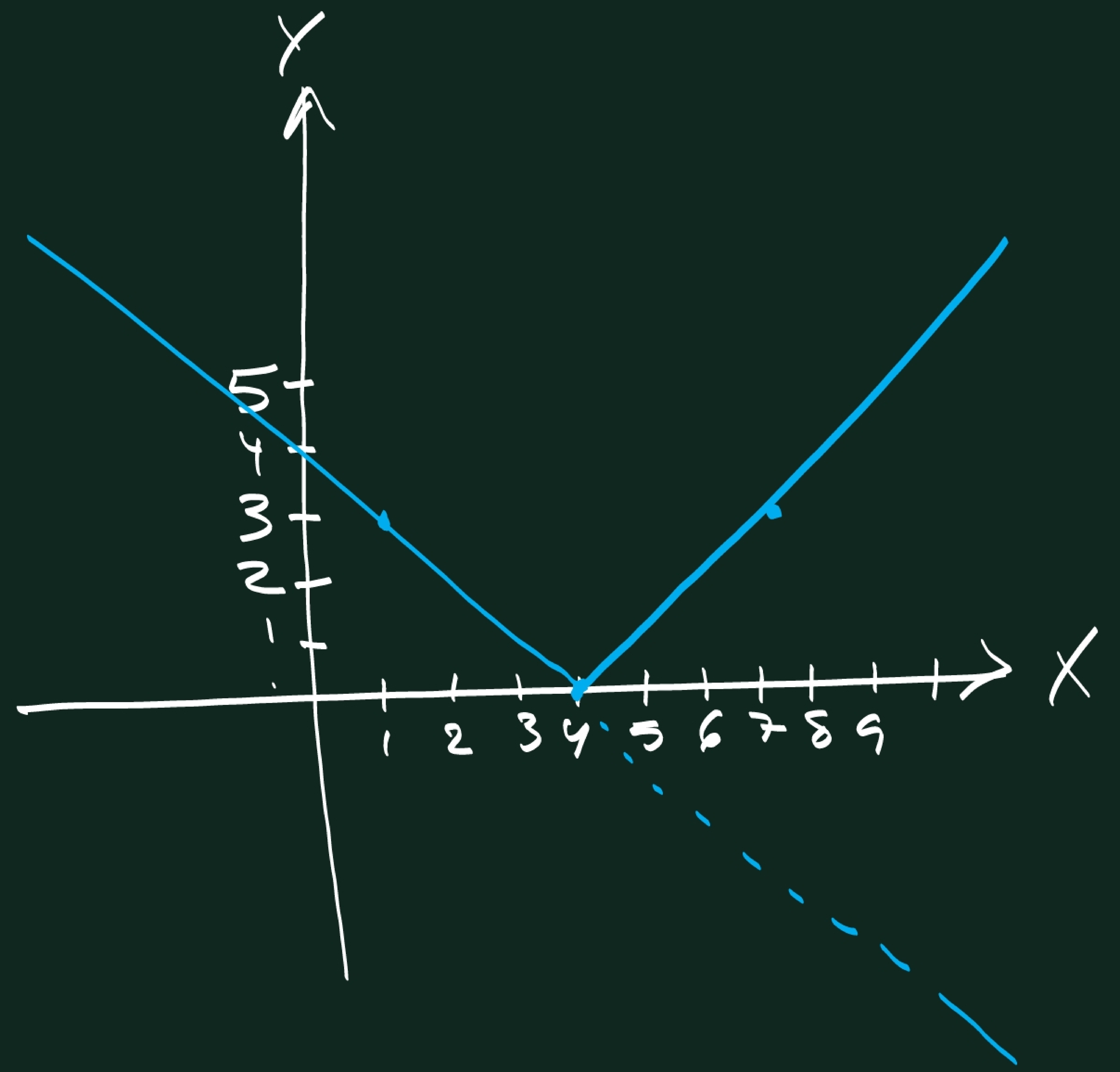
⑥ a) $f(x) = |4-x|$ $\begin{cases} 4-x, & x \le 4 \\ -(4-x); & x > 4 \end{cases}$

$4 - x = 0$

$\boxed{x = 4}$

$x = 4$

x	y
1	3
4	0
7	3



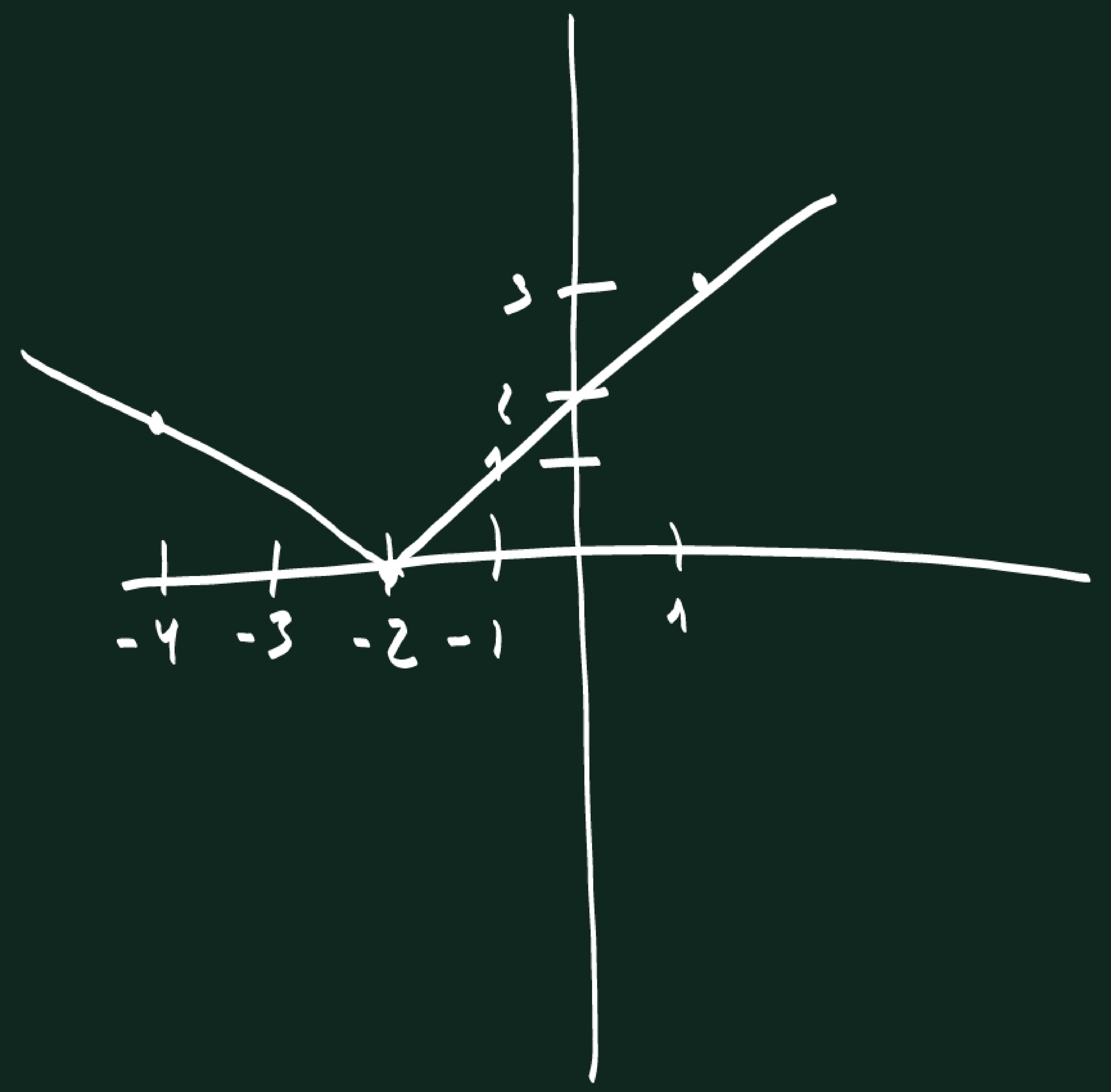
⑥ b)

$$f(x) = |x+2| \begin{cases} -(x+2) & x > -2 \\ x+2 & x \le -2 \end{cases}$$

$x + 2 = 0$

$\boxed{x = -2}$

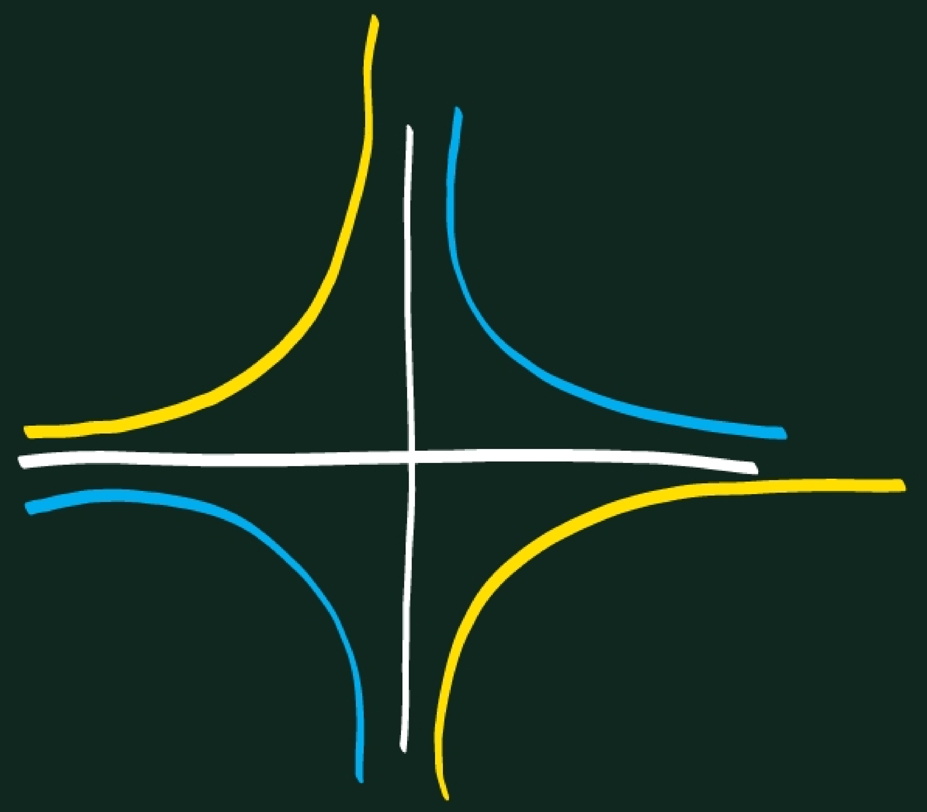
x	y
-4	2
-2	0
1	3



④ a) b)

(4) a) $f(x) = \dfrac{2}{x}$

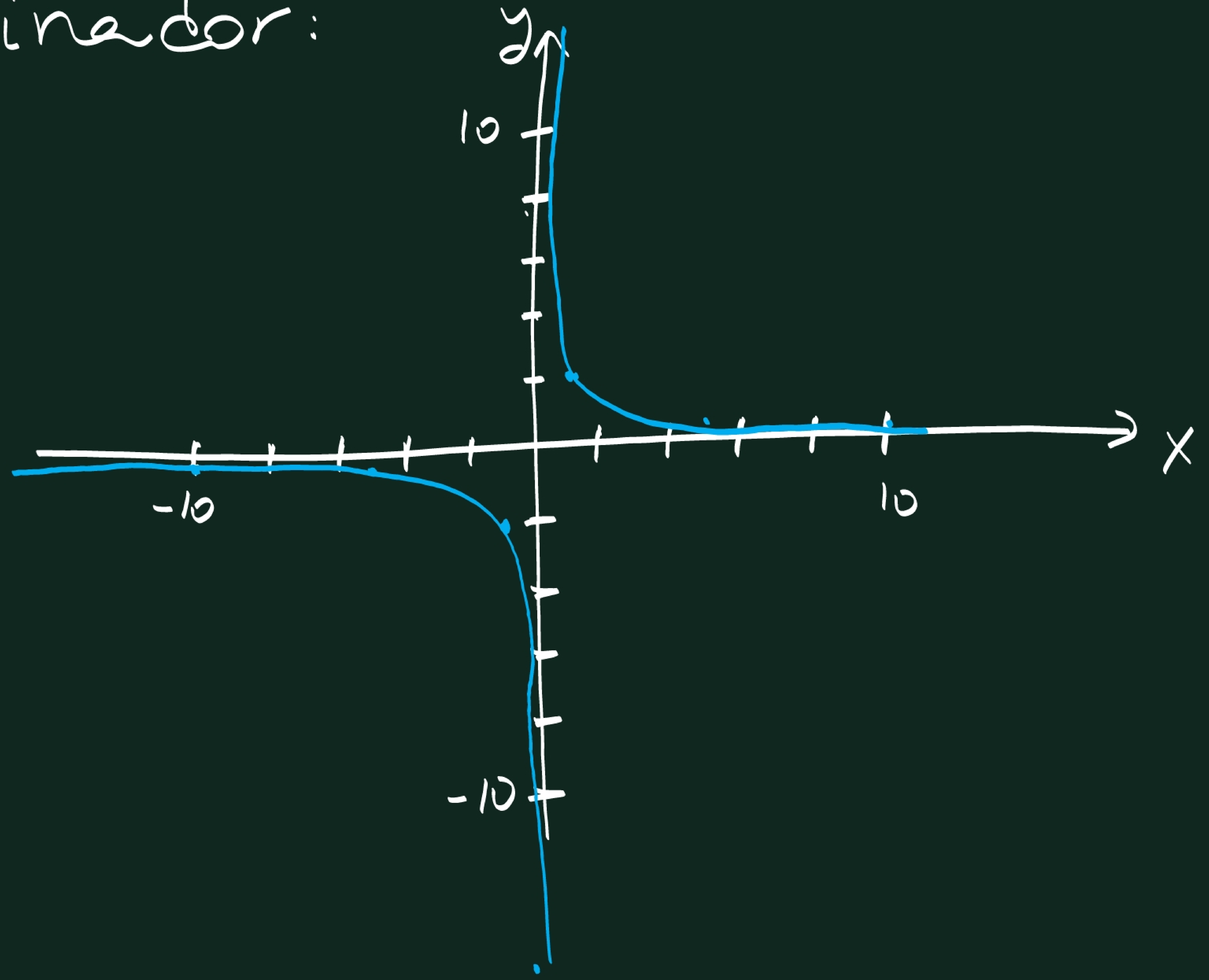
Función de Proporcionalidade Inversa



Valor que anula o denominador:

$\boxed{x = 0}$

x	y
-10	-0,2
-5	-0,4
-1	-2
-0,1	-20
0,1	20
1	2
5	0,4
10	0,2

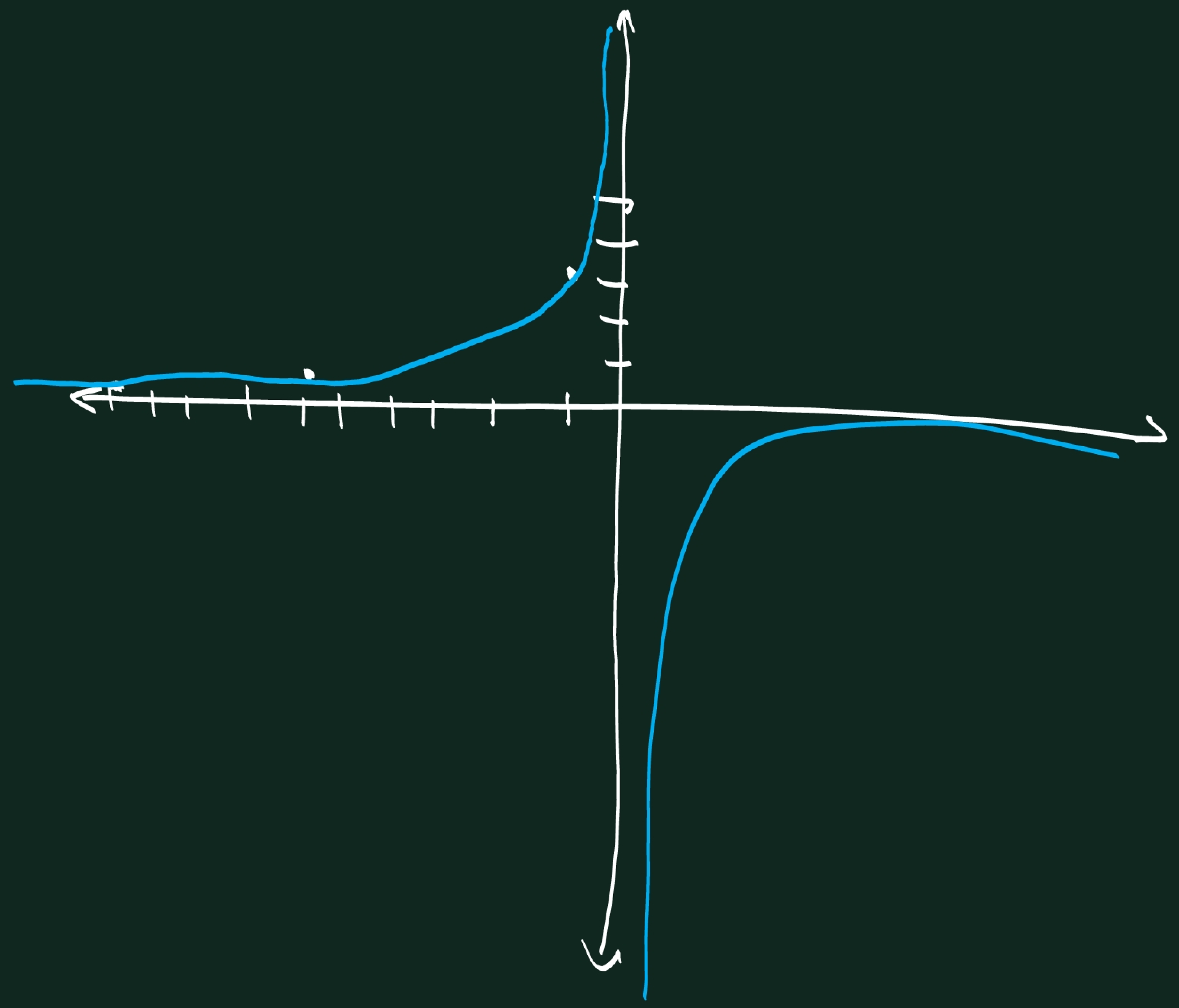


b) $f(x) = \frac{-3}{x}$

Anula denominador

$\boxed{x = 0}$

x	y
-0'1	30
-1	3
-6	½
-10	0'3



⑧ A : Parábola positiva $\rightarrow$ e) $y = x^2 + 2x - 2$

~~B~~ : ~~Prop.~~ Inv ; $\frac{+}{x}$ ou $\frac{-}{x}$ $\rightarrow$ f) $y = \frac{-4}{x}$

C : Radical $\rightarrow$ a) $y = \sqrt{x-4}$

D : Exponencial ; a^x ou $0,\ldots^x$ $\rightarrow$ d) $y = 3^{-x} = \frac{1}{3^x} = \left(\frac{1}{3}\right)^x$

E : Cte. $\rightarrow$ g) $y = \frac{7}{2}$

$\cancel{\frac{2}{3^x} = \left(\frac{2}{3}\right)^x}$

F : Lineal, $m < 0$; $n = 0$ $\rightarrow$ b) $y = -\frac{2}{3}x$
$-mx$

G : Lineal, $m > 0$, $n < 0$ $\rightarrow$ h) $y = \frac{2x-3}{5} = \frac{2}{5}x - \frac{3}{5}$

H : Parábola negativa $\rightarrow$ c) $y = -x^2 - 3x + 1$

I : Prop. Inversa $\rightarrow$ i) $y = \frac{3}{x}$
$\frac{+a}{x}$

Transformación de Funcións

$$f(x) = x^2$$

Desprazamentos Verticais:

A función "sube" se lle sumo un número: $f(x)+3 = x^2+3$

" " "baixa" " " resto " " : $f(x)-5 = x^2-5$

Desprazamentos Horizontais:

A función móvese cara a dereita se lle resto un número á "x":

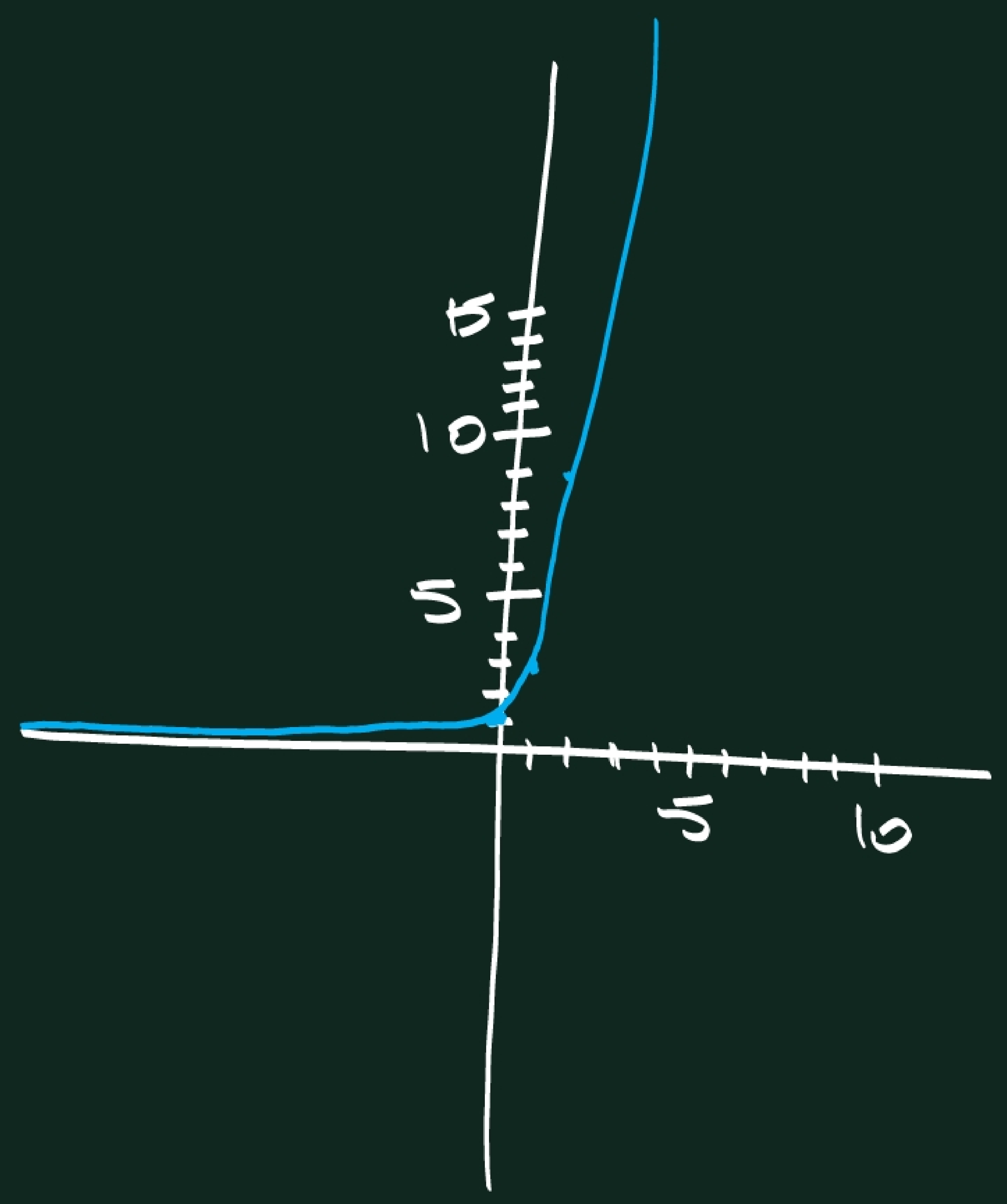
$$f(x-3) = (x-3)^2$$

A función móvese cara a esquerda se lle sumo un número á "x":

$$f(x+5) = (x+5)^2$$

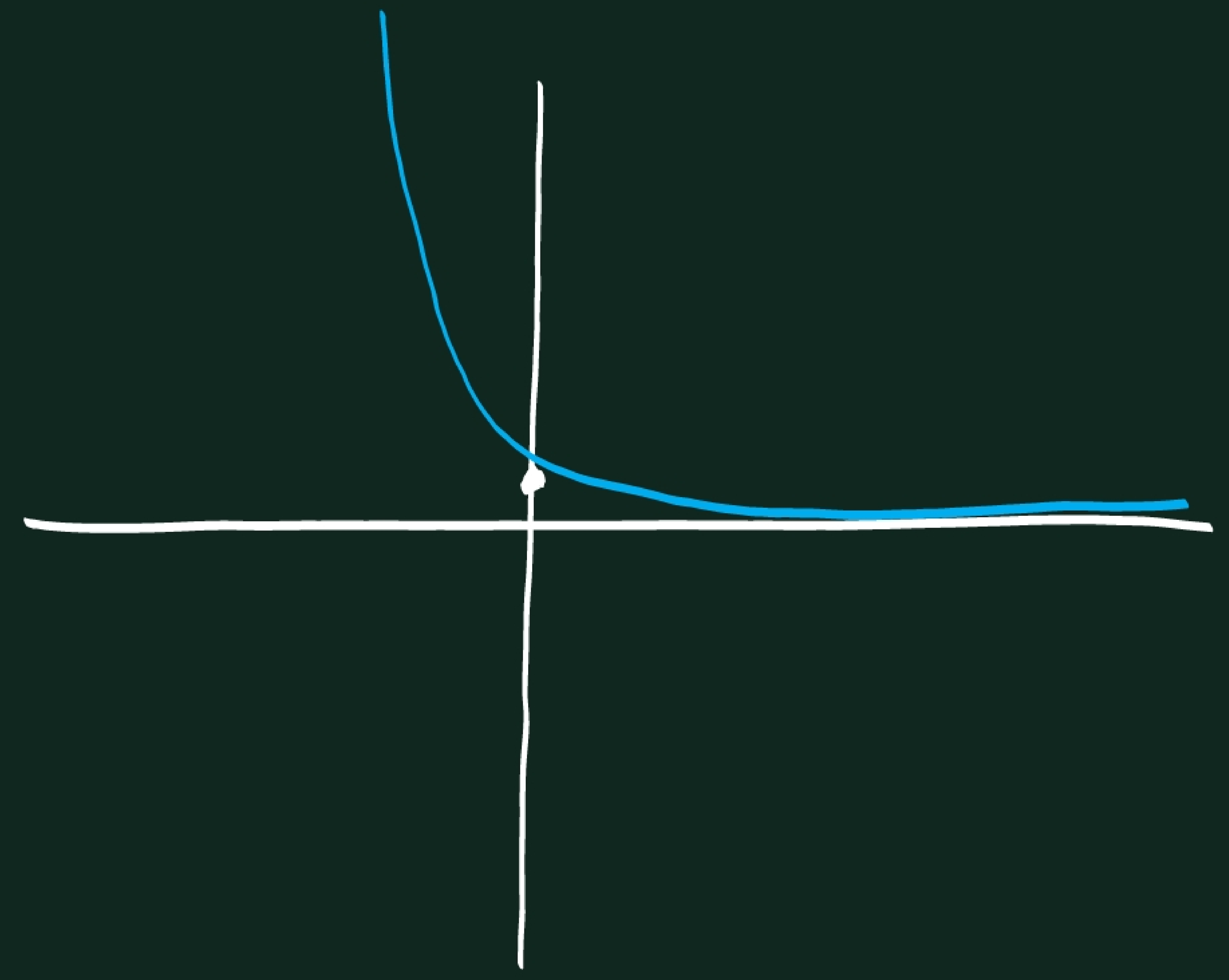
⑦ a) $y = 3^x$

x	y
-100	$0{,}0\ldots$
$\to -1$	$1/3 \approx 0{,}33$
$\to 0$	1
$\to 1$	3
$\to 2$	9
3	27



b) $y = 3^{-x} = \frac{1}{3^x} = \left(\frac{1}{3}\right)^x$

x	y
-2	9
-1	3
0	1
1	$1/3$
2	$1/9$



② g) $f(x) = \log \frac{x^2+x}{x-2} - 1$

Ptos. Corte Eixo X

$y = 0$

$$\log \frac{x^2+x}{x-2} = 1 \Leftrightarrow 10^1 = \frac{x^2+x}{x-2}$$

$$10x - 20 = x^2 + x$$

$$0 = x^2 - 9x + 20 \longrightarrow x = \frac{9 \pm \sqrt{81-80}}{2} = \frac{9 \pm 1}{2} \begin{cases} 5 \\ 4 \end{cases}$$