

1. $\int 3x^5 dx$

Solución:

$$\int 3x^5 dx = 3 \int x^5 dx = 3 \frac{x^{5+1}}{5+1} + C = 3 \frac{x^6}{6} + C = \frac{x^6}{2} + C$$

2. $\int \frac{2}{x^2} dx$

Solución:

$$\int \frac{2}{x^2} dx = 2 \int x^{-2} dx = 2 \frac{x^{-2+1}}{-2+1} + C = 2 \frac{x^{-1}}{-1} + C = -\frac{2}{x} + C$$

3. $\int (3x^2 - 2x + 1) dx$

Solución:

$$\begin{aligned} \int (3x^2 - 2x + 1) dx &= 3 \int x^2 dx - 2 \int x dx + \int dx = 3 \cdot \frac{x^3}{3} - 2 \cdot \frac{x^2}{2} + x + C = \\ &= x^3 - x^2 + x + C \end{aligned}$$

4. $\int \frac{2x^4 - 3x^2 - 1}{x^2} dx$

Solución:

$$\begin{aligned} \int \frac{2x^4 - 3x^2 - 1}{x^2} dx &= \int \left(2x^2 - 3 - \frac{1}{x^2} \right) dx = \frac{2x^3}{3} - 3x + \frac{1}{x} + C = \\ &= \frac{2x^4 - 9x^2 + 3}{3x} + C \end{aligned}$$

5. $\int x(x+a)(x+b) dx$

Solución:

$$\begin{aligned}
 \int x(x+a)(x+b)dx &= \int (x^2+ax)(x+b)dx = \\
 &= \int (x^3+bx^2+ax^2+abx)dx = \frac{x^4}{4} + b\frac{x^3}{3} + a\frac{x^3}{3} + ab\frac{x^2}{2} + C = \\
 &= \frac{3x^4 + 4(a+b)x^3 + 6abx^2}{12} + C
 \end{aligned}$$

6. $\int \sqrt{3x^3}dx$

Solución:

$$\int \sqrt{3x^3}dx = \int \sqrt{3}\sqrt{x^3}dx = \sqrt{3} \int x^{3/2}dx = \frac{2\sqrt{3}x^{5/2}}{5} + C$$

7. $\int \frac{-2}{\sqrt[3]{x^2}}dx$

Solución:

$$\int \frac{-2}{\sqrt[3]{x^2}}dx = -2 \int x^{-2/3}dx = -6\sqrt[3]{x} + C$$

8. $\int \frac{dx}{\sqrt[n]{x}}$

Solución:

$$\int \frac{dx}{\sqrt[n]{x}} = \frac{x^{-1/n+1}}{-1/n+1} + C = \frac{n}{n-1} \sqrt[n]{x^{n-1}} + C$$

9. $\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx$

Solución:

$$\begin{aligned}
 \int (\sqrt{x}+1)(x-\sqrt{x}+1)dx &= \int (x\sqrt{x}+1)dx = \int (x \cdot x^{1/2}+1)dx = \\
 &= \int (x^{3/2}+1)dx = \frac{2}{5}\sqrt{x^5}+x+C
 \end{aligned}$$

$$10. \int \frac{(x^2 + 1)(x^2 - 2)}{\sqrt[3]{x^2}} dx$$

Solución:

$$\begin{aligned} \int \frac{(x^2 + 1)(x^2 - 2)}{\sqrt[3]{x^2}} dx &= \int \frac{x^4 - x^2 - 2}{x^{2/3}} dx = \int \left(\frac{x^4}{x^{2/3}} - \frac{x^2}{x^{2/3}} - \frac{2}{x^{2/3}} \right) dx = \\ &= \int (x^{10/3} - x^{4/3} - 2x^{-2/3}) dx = \frac{3}{13} \sqrt[3]{x^{13}} - \frac{3}{7} \sqrt[3]{x^7} - \frac{3}{2} \sqrt[3]{x} + C \end{aligned}$$

$$11. \int \frac{x + \sqrt{x}}{x^2} dx$$

Solución:

$$\int \frac{x + \sqrt{x}}{x^2} dx = \int \left(\frac{x}{x^2} + \frac{x^{1/2}}{x^2} \right) dx = \ln |x| - \frac{2}{\sqrt{x}} + C$$

$$12. \int (e^x + 3^x - \sin x + 4) dx$$

Solución:

$$\int (e^x + 3^x - \sin x + 4) dx = e^x + \frac{3^x}{\ln 3} + \cos x + 4x + C$$

$$13. \int \frac{4^x + 6^x}{2^{x-1}} dx$$

Solución:

$$\begin{aligned} \int \frac{4^x + 6^x}{2^{x-1}} dx &= \int \frac{4^x}{2^{-1} \cdot 2^x} dx + \int \frac{6^x}{2^{-1} \cdot 2^x} dx = \\ &= \frac{1}{2^{-1}} \int \left(\frac{4}{2} \right)^x dx + \frac{1}{2^{-1}} \int \left(\frac{6}{2} \right)^x dx = 2 \int 2^x dx + 2 \int 3^x dx = \\ &= 2 \frac{2^x}{\ln 2} + 2 \frac{3^x}{\ln 3} + C = 2 \left(\frac{2^x}{\ln 2} + \frac{3^x}{\ln 3} \right) + C \end{aligned}$$

$$14. \int \frac{x^2 + 2}{(x + 1)^2 - 2x} dx$$

Solución:

$$\int \frac{x^2 + 2}{(x+1)^2 - 2x} dx = \int \frac{x^2 + 2}{x^2 + 2x + 1 - 2x} dx = \int \frac{x^2 + 2}{x^2 + 1} dx$$

Cando chegamos a integrais de fraccións alxebraicas con polinomio no numerador de grado igual ou maior que o grado do polinomio do denominador, temos que facer a división de polinomios.

$$\begin{array}{r|l} x^2 + 2 & x^2 + 1 \\ -x^2 - 1 & 1 \\ \hline & 1 \end{array}$$

Cando facemos a división sabemos que Dividendo = divisor · cociente + resto, se dividimos toda a expresión entre o divisor:

$$\frac{\text{Dividendo}}{\text{divisor}} = \text{cociente} + \frac{\text{resto}}{\text{divisor}}$$

Logo no noso caso:

$$\frac{x^2 + 2}{x^2 + 1} = 1 + \frac{1}{x^2 + 1}$$

Continuamos coa integral:

$$\int \frac{x^2 + 2}{(x+1)^2 - 2x} dx = \int \frac{x^2 + 2}{x^2 + 1} dx = \int \left(1 + \frac{1}{x^2 + 1}\right) dx = x + \arctan x + C$$

15. $\int \operatorname{tg}^2 x dx$

Solución:

$$\begin{aligned} \int \operatorname{tg}^2 x dx &= \int (1 + \operatorname{tg}^2 x - 1) dx = \int (1 + \operatorname{tg}^2 x) dx - \int dx = \\ &= \operatorname{tg} x - x + C \end{aligned}$$

16. $\int \cos 2x dx$

Solución:

$$\int \cos 2x dx = \int 2 \cdot \frac{1}{2} \cos 2x dx = \frac{1}{2} \int 2 \cos 2x dx = \frac{1}{2} \sin 2x + C$$

17. $\int \sin^2 x dx$

Solución:

$$\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{1}{2}x - \frac{1}{4} \sin 2x + C = \frac{1}{2}x - \frac{1}{2} \sin x \cos x + C$$

18. $\int \sin 3x \cos(2x + 1) dx$

Solución:

$$\begin{aligned} \int \sin 3x \cos(2x + 1) dx &= \int \frac{\sin(5x + 1) + \sin(x - 1)}{2} dx = \\ &= \frac{1}{2} \left(\int \sin(5x + 1) dx + \int \sin(x - 1) dx \right) = \\ &= \frac{1}{2} \left(\int \frac{1}{5} \cdot 5 \cdot \sin(5x + 1) dx + \int \sin(x - 1) dx \right) = \\ &= \frac{1}{2} \left(\frac{1}{5} \int 5 \cdot \sin(5x + 1) dx + \int \sin(x - 1) dx \right) = \\ &= \frac{1}{2} \left(-\frac{\cos(5x + 1)}{5} - \cos(x - 1) \right) + C \\ &= -\frac{1}{2} \left(\frac{\cos(5x + 1)}{5} + \cos(x - 1) \right) + C \end{aligned}$$

19. $\int x e^{x^2+1} dx$

Solución:

$$\int x e^{x^2+1} dx = \int \frac{1}{2} 2x e^{x^2+1} dx = \frac{1}{2} \int 2x e^{x^2+1} dx = \frac{1}{2} e^{x^2+1} + C$$

20. $\int \frac{4x + 4}{x^2 + 2x + 1} dx$

Solución:

$$\int \frac{4x+4}{x^2+2x+1} dx = \int \frac{2(2x+2)}{x^2+2x+1} dx = 2 \int \frac{2x+2}{x^2+2x+1} dx =$$

$$= 2 \ln |x^2+x+1| + C.$$

$$21. \int \frac{\sqrt{1+\operatorname{tg} x}}{\cos^2 x} dx$$

Solución:

$$\int \frac{\sqrt{1+\operatorname{tg} x}}{\cos^2 x} dx = \int (1+\operatorname{tg} x)^{1/2} \cdot \frac{1}{\cos^2 x} dx = \frac{(1+\operatorname{tg} x)^{3/2}}{3/2} + C =$$

$$= \frac{2\sqrt{(1+\operatorname{tg} x)^3}}{3} + C$$

$$22. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

Solución:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int 2 \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx = 2 \int \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx = 2e^{\sqrt{x}} + C$$

$$23. \int \sin^5 x \cdot \cos x dx$$

Solución:

$$\int \sin^5 x \cdot \cos x dx = \frac{(\sin x)^6}{6} + C = \frac{1}{6} \sin^6 x + C$$

$$24. \int \frac{x}{1+(x^2-3)^2} dx$$

Solución:

$$\int \frac{x}{1+(x^2-3)^2} dx = \frac{1}{2} \int \frac{2x}{1+(x^2-3)^2} dx = \frac{1}{2} \operatorname{arctg}(x^2-3) + C$$

$$25. \int \frac{x+1}{\sqrt{x^2+2x+3}} dx$$

Solución:

$$\int \frac{x+1}{\sqrt{x^2+2x+3}} dx = \int \frac{1}{2} \frac{2(x+1)}{\sqrt{x^2+2x+3}} dx = \int \frac{2x+2}{2\sqrt{x^2+2x+3}} dx = \sqrt{x^2+2x+3} + C$$

26. $\int (x^2 + 1)^{20} \cdot 5x dx$

Solución:

$$\int (x^2 + 1)^{20} \cdot 5x dx = \int \frac{1}{2} \cdot 2 \cdot (x^2 + 1)^{20} \cdot 5x dx = \frac{5}{2} \int 2x(x^2 + 1)^{20} dx = \frac{5}{2} \frac{(x^2 + 1)^{21}}{21} + C = \frac{5(x^2 + 1)^{21}}{42} + C$$

Observa o gran parecido que, en principio, teñen as dúas integrais seguintes. Con todo, os resultados son completamente diferentes. Mentres unha é de tipo logarítmico, a outra é de tipo arcotangente.

27. $\int \frac{e^{2x}}{1 + e^{2x}} dx$

Solución:

$$\int \frac{e^{2x}}{1 + e^{2x}} dx = \int \frac{1}{2} \cdot \frac{2e^{2x}}{1 + e^{2x}} dx = \frac{1}{2} \ln |1 + e^{2x}| + C$$

28. $\int \frac{e^x}{1 + e^{2x}} dx$

Solución:

$$\int \frac{e^x}{1 + e^{2x}} dx = \int \frac{e^x}{1 + (e^x)^2} dx = \arctg e^x + C$$

29. $\int \frac{\cos(\ln x)}{x} dx$

Solución:

$$\int \frac{\cos(\ln x)}{x} dx = \int \frac{1}{x} \cos(\ln x) dx = \sin(\ln x) + C$$

$$30. \int \frac{2x}{\sqrt{1-x^4}} dx$$

Solución:

$$\int \frac{2x}{\sqrt{1-x^4}} dx = \int \frac{2x}{\sqrt{1-(x^2)^2}} dx = \arcsen x^2 + C$$

$$31. \int \operatorname{tg}(3x+2) dx$$

Solución:

$$\begin{aligned} \int \operatorname{tg}(3x+2) dx &= \int \frac{\operatorname{sen}(3x+2)}{\cos(3x+2)} dx = \int -\frac{1}{3} \cdot \frac{-3 \operatorname{sen}(3x+2)}{\cos(3x+2)} dx = \\ &= -\frac{1}{3} \ln |\cos(3x+2)| + C \end{aligned}$$

$$32. \int 2x(\operatorname{sen} x^2)(\cos^4 x^2) dx$$

Solución:

$$\begin{aligned} \int 2x(\operatorname{sen} x^2)(\cos^4 x^2) dx &= \int 2x(\operatorname{sen} x^2)(\cos x^2)^4 dx = \\ &= -\int -2x(\operatorname{sen} x^2)(\cos x^2)^4 dx = -\frac{(\cos x^2)^5}{5} + C \end{aligned}$$