Función Constante:

La derivada de una función constante es siempre igual a 0.

y= f(x) = 4 **sol**: $y' = 0$
y= f(x) = -1500 **sol**: $y' = 0$
y= f(x) = 0 **sol**: $y' = 0$

Función Potencial:

$y = f^n(x) \rightarrow y' = n \cdot f(x)^{n-1} \cdot f'(x)$

$y = x$ **sol**: $y' = 1$
$y = 4x^7$ **sol**: $y' = 28x^6 \cdot 1 = 28x^6$
$y = x^3$ **sol**: $y' = 3x^2 \cdot 1 = 3x^2$
$y = (2x+1)^3$ **sol**: $y' = 3(2x+1)^2 \cdot 2 = 6(2x+1)^2 \cdot$
$y = (3x^2 - 4x)^5$ **sol**: $y' = 5(3x^2 - 4x)^4 \cdot (6x - 4)$

Función Irracional:

$y = \sqrt[n]{f(x)} \rightarrow y' = \frac{f'(x)}{n\sqrt[n]{f^{n-1}(x)}}$

$y = \sqrt{x}$ **sol**: $y' = \frac{1}{2\sqrt{x}}$
$y = \sqrt[3]{x^2}$ **sol**: $y' = \frac{2x}{3\sqrt[3]{(x^2)^2}} = \frac{2x}{3\sqrt[3]{x^4}}$
$y = \sqrt[5]{(x^3-2)^4}$ **sol**: $y' = \frac{4(x^3-2)^3 \cdot (3x^2-0)}{5\sqrt[5]{((x^3-2)^4)^4}} = \frac{4(x^3-2)^3 \cdot (3x^2)}{5\sqrt[5]{(x^3-2)^{16}}} = \frac{12x^2}{5\sqrt[5]{x^3-2}}$

Puedes realizar también las derivadas transformando la función irracional en potencial

$\mathbf{y = \sqrt[5]{(x^3-2)^4} \rightarrow y = (x^3-2)^{\frac{4}{5}}}$

$\mathbf{y' = \frac{4}{5}(x^3-2)^{\frac{4}{5}-1} \cdot (3x^2) = \frac{4}{5}(x^3-2)^{\frac{-1}{5}} \cdot (3x^2) = \frac{12x^2}{5\sqrt[5]{x^3-2}}}$

Función Exponencial:

$y = a^{f(x)} \rightarrow y' = a^{f(x)} \cdot \mathrm{Ln}a \cdot f'(x)$

$\mathbf{y = e^x}$ **sol: $y' = e^x \cdot \ln e \cdot 1 = e^x$**
$\mathbf{y = 3^x}$ **sol: $y' = 3^x \cdot \ln 3 \cdot 1 = 3^x \cdot \ln 3$**
$\mathbf{y = 5^{x^3-2x}}$ **sol: $y' = 5^{x^3-2x} \cdot (\ln 5) \cdot (3x^2 - 2)$**
$\mathbf{y = e^{5x^3-7}}$ **sol: $y' = e^{5x^3-7} \cdot (\ln e) \cdot (15x^2 - 0) = e^{5x^3-7} \cdot (15x^2)$**

Función Logarítmica:

$y = \log_a f(x) \rightarrow y' = \frac{f'(x)}{f(x)\cdot Ln a}$

$y = \ln x = \log_e x$ **Sol**: $y' = \frac{1}{x\cdot ln e} = \frac{1}{x}$

$y = \ln(3x^2 - 6) = \log_e(3x^2 - 6)$ **Sol**: $y' = \frac{6x-0}{(3x^2-6)\cdot ln e} = \frac{6x}{3x^2-6}$

$y = \log_3(5x^4 - 2)$ **Sol**: $y' = \frac{20x^3-0}{(5x^4-2)\cdot ln 3} = \frac{20x^3}{(5x^4-2)\cdot ln 3}$

$y = \log_7(7x^3 - 5x)$ **Sol**: $y' = \frac{21x^2-5}{(7x^3-5x)\cdot ln 7}$

$y = \log_5(3x^2 - 8x)^6$ **Sol**: $y' = \frac{6(3x^2-8x)^5\cdot(6x-8)}{(3x^2-8x)^6\cdot ln 5} = \frac{36x-48}{(3x^2-8x)\cdot ln 5}$

Operaciones con derivadas

- Producto: $[f(x)\cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$y = 3^{x^3+2}\cdot (3x^4 - 5x^2)$ **Sol**: $y' = 3^{x^3+2}\cdot (3x^2)\cdot (ln 3)\cdot (3x^4 - 5x^2) + 3^{x^3+2}\cdot (12x^3 - 10x)$

$y = \log_3(x^2 - 3)\cdot \left(\sqrt[3]{2x}\right)$ **Sol:** $y' = \frac{2x}{(x^2-3)\cdot ln 3}\cdot \left(\sqrt[3]{2x}\right) + \log_3(x^2 - 3)\cdot \frac{2}{3\sqrt[3]{(2x)^2}}$

$y = \log_3(x^3 - 2x)\cdot 3^{2x-1}$ **Sol**: $y' = \left(\frac{3x^2-2}{(x^3-2x)\cdot ln 3}\right)\cdot 3^{2x-1} + \log_3(x^3 - 2x)\cdot (3^{2x-1}\cdot 2\cdot ln 3)$

- Cociente: $\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$

$y = \frac{5^{3x}}{(3x^2+4)}$ **Sol:** $y' = \frac{(5^{3x}\cdot 3\cdot ln 5)\cdot(3x^2+4) - (6x)\cdot 5^{3x}}{(3x^2+4)^2}$

$y = \frac{\ln(x^3-7x)}{(x^5-2)}$ **Sol:** $y' = \frac{\frac{3x^2-7}{(x^3-7x)}\cdot(x^5-2) - (5x^4)\cdot\ln(x^3-7x)}{(x^5-2)^2}$

$y = \frac{3^{2x-1}}{\log_3(x^3-2x)\cdot}$ **Sol**: $y' = \frac{(3^{2x-1}\cdot 2\cdot ln 3)\cdot\left(\log_3(x^3-2x) - \left(\frac{3x^2-2}{(x^3-2x)\cdot ln 3}\right)\cdot 3^{2x-1}\right)}{(\log_3(x^3-2x)\cdot)^2}$

jercicios

1. $y=\left(4x^3-5\right)^4+3x$ — Sol: $y'=4(4x^3-5)^3\cdot(12x^2)+3$

2. $y=\sqrt[3]{x}(3x^2+6x)$ — Sol: $y'=\frac{1}{3\sqrt[3]{x^2}}\cdot(3x^2+6x)+\sqrt[3]{x}\cdot(6x+6)$

3. $y=\sqrt{\frac{3x^2+2}{x}}$ — Sol: $y'=\frac{1}{2\sqrt{\frac{3x^2+2}{x}}}\cdot\frac{6x\cdot x-1\cdot(3x^2+2)}{x^2}$

4. $y=\ln\left(x^3-2x\right)^3$ — Sol: $y'=\frac{1}{(x^3-2x)^3\cdot Lne}\cdot 3(x^3-2x)^2\cdot(3x^2-2)$

5. $y=\ln^2\left(x^3-2x\right)^4$ — Sol: $y'=2(ln\,(x^3-2x)^4)\cdot\frac{4(x^3-2x)^3\cdot(3x^2-2)}{(x^3-2x)^4\cdot Lne}$

6. $y=\left(6x^3+\sqrt{x^3}\right)\ln x$ — Sol: $y'=\left(18x^2+\frac{3x^2}{2\sqrt{x^3}}\right)\cdot(\ln x)+\left(6x^3+\sqrt{x^3}\right)\cdot\frac{1}{x}$

7. $y=\left(x^3-5x+2\right)^4\cdot\ln x$ — Sol: $y'=4(x^3-5x+2)^3\cdot(3x^2-5)\cdot(lnx)+(x^3-5x+2)^4\cdot\frac{1}{x}$

8. $y=\left(4x^3-\sqrt{x}\right)^3\ln x^4$ — Sol: $y'=3\left(4x^3-\sqrt{x}\right)^2\cdot\left(12x^2-\frac{1}{2\sqrt{x}}\right)\cdot(lnx^4)+\left(4x^3-\sqrt{x}\right)^3\cdot\frac{4x^3}{x^4}$

9. $y=\ln^3(x-3)$ — Sol: $y'=3(ln(x-3))^2\cdot\frac{1}{x-3}$

10. $y=\frac{3x^2-2x}{x+1}$ — Sol: $y'=\frac{(6x-2)(x+1)-1(3x^2-2x)}{(x+1)^2}$

11. $y=2^{x^2-1}\ln\left(x^3-2x\right)$ — Sol: $y'=\left(2^{x^2-1}\cdot 2x\cdot ln2\right)\cdot(ln(x^3-2x))+2^{x^2-1}\cdot\frac{(3x^2-2)}{x^3-2x}$

12. $y=\ln\left(\frac{3x+2}{x^2}\right)^2$ — Sol: $y'=\frac{1}{\left(\frac{3x+2}{x^2}\right)^2\cdot(lne)}\cdot 2\left(\frac{3x+2}{x^2}\right)\cdot\frac{3\cdot x^2-2x\cdot(3x+2)}{(x^2)^2}$

13. $y=e^{4x^2+2x}$ — Sol: $y'=e^{4x^2+2x}\cdot(8x+2)\cdot(lne)$

14. $y=\frac{e^{2x}+1}{e^{2x}-1}$ — Sol: $y'=\frac{(e^{2x}\cdot 2\cdot lne)\cdot(e^{2x}-1)-(e^{2x}\cdot 2\cdot lne)\cdot(e^{2x}+1)}{(e^{2x}-1)^2}$

15. $y=\left(3x^2+5x\right)\ln(x-1)$ — Sol: $y'=(6x+5)\cdot(\ln(x-1))+(3x^2+5x)\cdot\frac{1}{x-1}$

16. $y=\sqrt[4]{3x^3+2x}$ — Sol: $y'=\frac{9x^2+2}{4\sqrt[4]{(3x^3+2x)^3}}$

17. $y=\frac{2}{x^3+2x^4}$ — Sol: $y'=\frac{0\cdot(x^3+2x^4)-2\cdot(3x^2+8x^3)}{(x^3+2x^4)^2}=\frac{-2(3x^2+8x^3)}{(x^3+2x^4)^2}$

18. $y = 3x^4\left(x^3+2\right)^5$ **Sol:** $\mathbf{y' = 12x^3 \cdot (x^3+2)^5 + 5(x^3+2)^4 \cdot (3x^2) \cdot 3x^4}$

19. $y = 7^{\ln(x^2-5x)}$ **Sol:** $\mathbf{y' = 7^{\ln(x^2-5x)} \cdot \frac{(2x-5)}{(x^2-5x)\cdot(\ln e)} \cdot (\ln 7)}$

20. $y = \frac{4x+3}{x-3}$ **Sol:** $\mathbf{y' = \frac{4\cdot(x-3)-1\cdot(4x+3)}{(x-3)^2} = \frac{-6}{(x-3)^2}}$

21. $y = \left(x^4+3\right)\ln(2x+1)$ **Sol:** $\mathbf{y' = (4x^3)\cdot\left(\ln(2x+1)\right) + (x^4+3)\cdot\frac{2}{(2x+1)}}$

22. $y = \frac{4}{\sqrt{x^3+5}}$ **Sol:** $\mathbf{y' = \frac{0\cdot\sqrt{x^3+5} - 4\cdot\frac{3x^2}{2\sqrt{x^3+5}}}{\left(\sqrt{x^3+5}\right)^2} = \frac{-6x^2}{\sqrt{(x^3+5)^3}}}$

23. $y = \frac{2^x+1}{2^x}$ **Sol:** $\mathbf{y' = \frac{(2^x\cdot 1\cdot(\ln 2))\cdot 2^x - (2^x\cdot 1\cdot(\ln 2))\cdot(2^x+1)}{(2^x)^2}}$

24. $y = \frac{2}{e^{3x}}$ **Sol:** $\mathbf{y' = \frac{0\cdot e^{3x} - (e^{3x}\cdot 3\cdot \ln e)\cdot 2}{(e^{3x})^2} = \frac{-6e^{3x}}{(e^{3x})^2} = \frac{-6}{e^{3x}}}$

25. $y = \ln\sqrt{\frac{x+2}{x-1}}$ **Sol:** $\mathbf{y' = \frac{\frac{1\cdot(x-1)-1\cdot(x+2)}{(x-1)^2}}{\frac{2\sqrt{\frac{x+2}{x-1}}}{\sqrt{\frac{x+2}{x-1}}\cdot \ln e}} = \frac{\frac{-3}{(x-1)^2}}{2\cdot\left(\frac{x+2}{x-1}\right)} = \frac{-3}{2(x-1)(x+2)} = \frac{-3}{2x^2+2x-4}}$

26. $y = \log_3\left(x^3-2x-3\right)^4$ **Sol:** $\mathbf{y' = \frac{4(x^3-2x-3)^3\cdot(3x^2-2)}{(x^3-2x-3)^4\cdot(\ln 3)} = \frac{12x^2-8}{(x^3-2x-3)\cdot(\ln 3)}}$

27. $y = e^{3x^2-2}\sqrt{x^2-6}$ **Sol:** $\mathbf{y' = \left(e^{3x^2-2}\cdot(6x)\right)\cdot\left(\sqrt{x^2-6}\right) + \left(\frac{2x}{2\sqrt{x^2-6}}\right)\cdot\left(e^{3x^2-2}\right)}$

28. $y = (\log_3(3x^3-2x-1)^4)^5$ **Sol:** $\mathbf{y' = 5(\log_3(3x^3-2x-1)^4)^4 \cdot \frac{4(3x^3-2x-1)^3\cdot(9x^2-2)}{(3x^3-2x-1)^4\cdot(\ln 3)}}$