4 REGLAS PARA OBTENER LAS DERIVADAS DE ALGUNAS FUNCIONES

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1 Halla las funciones derivadas de estas funciones:

a) $f(x) = x^5$ **b)** $f(x) = \frac{1}{x^2}$ **c)** $f(x) = \sqrt[3]{x}$ **d)** $f(x) = \sqrt[3]{x^2}$ **e)** $f(x) = \frac{\sqrt{x^3} \cdot \sqrt[3]{x^4}}{x^2}$

a) $f'(x^5) = 5x^4$

b) $f'\left(\frac{1}{x^2}\right) = f'(x^{-2}) = -2x^{-3} = \frac{-2}{x^3}$

c) $f'\left(\sqrt[3]{x}\right) = f'(x^{1/3}) = \frac{1}{3}x^{(1/3)-1} = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

d) $f'\left(\sqrt[3]{x^2}\right) = f'(x^{2/3}) = \frac{2}{3}x^{(2/3)-1} = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

e) $f'\left(\frac{\sqrt{x^3} \cdot \sqrt[3]{x^4}}{x^2}\right) = f'\left(\frac{x^{3/2} \cdot x^{4/3}}{x^2}\right) = f'\left(x^{5/6}\right) = \frac{5}{6}x^{(5/6)-1} = \frac{5}{6\sqrt[6]{x}}$

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Hazlo tú

1 Halla la función derivada de las siguientes funciones:

a) $f(x) = 5x^4 - 2x^2 + 3x - 7$ **b)** $g(x) = \sqrt{5x} - \sqrt[3]{3x^4}$ **c)** $h(x) = \frac{3x}{x^2\sqrt[3]{x}}$

a) $f'(x) = 5 \cdot 4x^3 - 2 \cdot 2x + 3 = 20x^3 - 4x + 3$

b) $g(x) = \sqrt{5}\sqrt{x} - \sqrt[3]{3}\sqrt[3]{x^4} = \sqrt{5}\,x^{1/2} - \sqrt[3]{3}\,x^{4/3}$

$g'(x) = \sqrt{5} \cdot \frac{1}{2}x^{-1/2} - \sqrt[3]{3} \cdot \frac{4}{3}x^{1/3} = \frac{\sqrt{5}}{2\sqrt{x}} - \frac{4\sqrt[3]{3}}{3}\sqrt[3]{x}$

c) $h(x) = \frac{3x}{x^2\,x^{1/3}} = 3x^{-4/3}$

$h'(x) = 3 \cdot \left(-\frac{4}{3}\right)x^{-7/3} = -\frac{4}{x^{7/3}} = -\frac{4}{x^2\sqrt[3]{x}}$

Hazlo tú

2 Hazlo tú. Halla la función derivada de las siguientes funciones:

a) $f(x) = \frac{5^{4x}}{125}$ **b)** $g(x) = \frac{x^2 - 3x + 1}{x^2 + x - 3}$ **c)** $h(x) = \frac{x^3 - 5x^2 + 2x - 1}{x}$

a) $f(x) = \frac{1}{125}(5^4)^x = \frac{1}{125}625^x$

$f'(x) = \frac{1}{125}625^x \ln 625 = \frac{\ln 625}{125}625^x$

b) $g'(x) = \frac{(2x-3)\cdot(x^2+x-3) - (x^2-3x+1)\cdot(2x+1)}{(x^2+x-3)^2} = \frac{2x^3 - x^2 - 9x + 9 - (2x^3 - 5x^2 - x + 1)}{(x^2+x-3)^2} = \frac{4x^2 - 8x + 8}{(x^2+x-3)^2}$

c) $h'(x) = \frac{(3x^2 - 10x + 2)x - 1\cdot(x^3 - 5x^2 + 2x - 1)}{x^2} = \frac{3x^3 - 10x^2 + 2x - x^3 + 5x^2 - 2x - 1}{x^2} = \frac{2x^3 - 5x^2 + 1}{x^2}$

Para practicar

Halla la función derivada de las siguientes funciones:

2 $f(x) = 5x^2 + 7x - 2\sqrt{x}$

$$f'(x) = 5 \cdot 2x + 7 - 2 \cdot \frac{1}{2\sqrt{x}} = 10x + 7 - \frac{1}{\sqrt{x}}$$

3 $f(x) = \sqrt{3x^3} \cdot e^x$

$$f(x) = \sqrt{3}\,\sqrt{x^3}\,e^x = \sqrt{3}\,x^{3/2}\,e^x$$

$$f'(x) = \sqrt{3}\left(\frac{3}{2}x^{1/2}\,e^x + x^{3/2}\,e^x\right) = \sqrt{3}\left(\frac{3}{2}\sqrt{x}\,e^x + x\sqrt{x}\,e^x\right) = \sqrt{3x}\,e^x\left(\frac{3}{2} + x\right)$$

4 $f(x) = \dfrac{e^x \cdot \cos x}{2^{x+4}}$

$$f(x) = \frac{1}{2^4} \cdot \frac{e^x \cos x}{2^x}$$

$$f'(x) = \frac{1}{2^4} \cdot \frac{(e^x \cos x - e^x \operatorname{sen} x)\,2^x - e^x \cos x\,2^x \ln 2}{(2^x)^2} = \frac{1}{2^4} \cdot \frac{e^x \cos x - e^x \operatorname{sen} x - e^x \cos x \ln 2}{2^x} =$$

$$= \frac{1}{2^4} \cdot \frac{e^x(\cos x - \operatorname{sen} x - \ln 2 \cos x)}{2^x}$$

5 $f(x) = x \cdot 3^x \cdot \operatorname{tg} x$

$$f'(x) = 3^x \cdot \operatorname{tg} x + x \cdot 3^x \ln 3 \cdot \operatorname{tg} x + \frac{x \cdot 3^x}{\cos^2 x}$$

6 $f(x) = \dfrac{\log_2 x}{x}$

$$f'(x) = \frac{\frac{1}{x} \cdot \frac{1}{\ln 2} \cdot x - \log_2 x \cdot 1}{x^2} = \frac{\frac{1}{\ln 2} - \log_2 x}{x^2} = \frac{1 - \ln 2 \log_2 x}{x^2 \ln 2}$$

7 $f(x) = \dfrac{2x^3 - 5x + 3}{x^2}$

$$f(x) = 2x - \frac{5}{x} + \frac{3}{x^2} = 2x - 5 \cdot \frac{1}{x} + 3 \cdot x^{-2}$$

$$f'(x) = 2 + \frac{5}{x^2} + 3 \cdot (-2)\,x^{-3} = 2 + \frac{5}{x^2} - \frac{6}{x^3}$$

8 $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$

$$f'(x) = \frac{2x(x^2 - 1) - (x^2 + 1)\,2x}{(x^2 - 1)^2} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$$

9 $f(x) = (\operatorname{sen} x)\left(x^2 + \dfrac{\pi}{2}\right)$

$$f'(x) = (\cos x)\left(x^2 + \frac{\pi}{2}\right) + (\operatorname{sen} x)\,(2x)$$

10 $\boldsymbol{f(x) = \dfrac{2x}{\cos x}}$

$$f'(x) = \frac{2\cos x + 2x\,sen\,x}{\cos^2 x} = \frac{2}{\cos x} + 2x\,tg\,x$$

11 $\boldsymbol{f(x) = \dfrac{x^2 \cdot 5^x}{x^3}}$

$$f(x) = \frac{1}{x}5^x$$

$$f'(x) = -\frac{1}{x^2}5^x + \frac{1}{x}5^x \ln 5 = 5^x\,\frac{x\ln 5 - 1}{x^2}$$

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Halla la función derivada de las siguientes funciones:

12 $\boldsymbol{f(x) = sen\,(x^2 - 5x + 7)}$

$$f'(x) = (2x - 5)\cos(x^2 - 5x + 7)$$

13 $\boldsymbol{f(x) = \sqrt[3]{(5x+3)^2} = (5x+3)^{2/3}}$

$$f'(x) = \frac{2}{3}(5x+3)^{-1/3}\cdot 5 = \frac{10}{3\sqrt[3]{5x+3}}$$

14 $\boldsymbol{f(x) = sen^2\left(3x + \dfrac{\pi}{2}\right)}$

$$f'(x) = 2sen\left(3x + \frac{\pi}{2}\right)\cos\left(3x + \frac{\pi}{2}\right)\cdot 3 = 6sen\left(3x + \frac{\pi}{2}\right)\cos\left(3x + \frac{\pi}{2}\right)$$

También, usando la fórmula del seno del ángulo doble, podríamos dar el resultado de esta otra manera:

$$f'(x) = 2sen\left(3x + \frac{\pi}{2}\right)\cos\left(3x + \frac{\pi}{2}\right)\cdot 3 = 3sen\,(6x + \pi) = -3\,sen\,6x$$

15 $\boldsymbol{f(x) = \dfrac{\log x^2}{x}}$

$$f(x) = \frac{2\log x}{x} \to f'(x) = \frac{2(1 - \ln 10 \log x)}{x^2 \ln 10}$$

16 $\boldsymbol{f(x) = \cos(3x - \pi)}$

$$f'(x) = -3\,sen\,(3x - \pi)$$

17 $\boldsymbol{f(x) = \sqrt{1 + 2x}}$

$$f'(x) = \frac{1}{\sqrt{1+2x}}$$

18 $\boldsymbol{f(x) = x\,e^{2x+1}}$

$$f'(x) = e^{2x+1} + x\,e^{2x+1}\cdot 2 = e^{2x+1}(1 + 2x)$$

19 $\boldsymbol{f(x) = \dfrac{sen\,(x^2+1)}{\sqrt{1-x^2}}}$

$$f'(x) = \frac{2x\sqrt{1-x^2}\,cos\,(x^2+1) + [x\,sen\,(x^2+1)]/\sqrt{1-x^2}}{1-x^2} = \frac{2x\,(1-x^2)\,cos\,(x^2+1) + x\,sen\,(x^2+1)}{\sqrt{(1-x^2)^3}}$$