1) a) $f(x) = x^3 - 5x^2 + 7x + a$.

$y = f(x)$ é continua e derivable en todo $\mathbb{R}$ por ser unha función polinómica $\Rightarrow y = f(x)$ continua e derivable en $[1, 3]$.

$$\left.\begin{aligned} f(1) &= 1 - 5 + 7 + a = 3 + a \\ f(3) &= 27 - 45 + 21 + a = 3 + a \end{aligned}\right\} \Rightarrow f(1) = f(3) \ \forall a.$$

Entón verifica as hipótesis do teorema de Rolle.

b) Polo tanto, existe $c \in (1,3) \,/\, f'(c) = 0$

$f'(x) = 3x^2 - 10x + 7$

$3x^2 - 10x + 7 = 0$

$$x = \frac{10 \pm \sqrt{100 - 12 \cdot 7}}{6} = \frac{10 \pm \sqrt{100 - 84}}{6} = \frac{10 \pm \sqrt{16}}{6}$$

$$x = \frac{10 \pm 4}{6} \begin{cases} x = \frac{14}{6} = \frac{7}{3} \in (1,3) \Rightarrow \\ x = \frac{6}{6} = 1 \notin (1,3) \end{cases}$$

$\Rightarrow$ O punto é $x = \frac{7}{3}$.

2) $f(x) = \dfrac{ax^2+bx+c}{x^2-4}$, a, b, c ?

Asintota horizontal en $y=-1$, mínimo no punto (0, 1). $\Rightarrow$

Se $y=-1$ é asintota horizontal $\lim\limits_{x\to\infty} \dfrac{ax^2+bx+c}{x^2-4} = -1$

$$\Rightarrow \boxed{a=-1} \Rightarrow f(x) = \frac{-x^2+bx+c}{x^2-4}$$

Mínimo en (0, 1) $\Rightarrow$ $f(0)=1$
$f'(0)=0$

Se $f(0)=1 \Rightarrow \dfrac{-0+0+c}{-4} = \dfrac{c}{-4} = 1 \Rightarrow \boxed{c=-4}$

$$f(x) = \frac{-x^2+bx-4}{x^2-4}$$

$$f'(x) = \frac{(-2x+b)(x^2-4)-(-x^2+bx-4)\,2x}{(x^2-4)^2} =$$

$$= \frac{\cancel{-2x^3}+\cancel{8x}+bx^2-4b-(\cancel{-2x^3}+2bx^2-\cancel{8x})}{(x^2-4)^2}$$

$$= \frac{-bx^2+16x-4b}{(x^2-4)^2} \Rightarrow f'(0)=0 \Rightarrow \frac{-4b}{(-4)^2}=0$$

$\Downarrow$

$\boxed{b=0}$

$$f(x) = \frac{-x^2+0x-4}{x^2-4} \Rightarrow \boxed{f(x) = \frac{-x^2-4}{x^2-4}}$$

3] $f(x) = x e^{-x}$

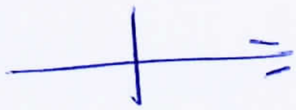
a) $\mathrm{Dom}(f) = \mathbb{R}$

<u>Asíntotas</u>: A.V. non ten por ser o $\mathrm{dom}(f) = \mathbb{R}$.

A.H. $\lim\limits_{x \to \infty} x e^{-x} = \left[\infty e^{-\infty} = \infty \cdot \frac{1}{e^{\infty}} = \infty \cdot 0\right]$

$= \lim\limits_{x \to \infty} \frac{x}{e^x} = \left[\frac{\infty}{\infty}\right] \underset{L'H}{=}$

$= \lim\limits_{x \to \infty} \frac{1}{e^x} = \frac{1}{\infty} = 0 \Rightarrow$



$\lim\limits_{x \to -\infty} x e^{-x} = (-\infty) e^{\infty} = -\infty \cdot \infty = -\infty$

b) $y = x e^{-x} \Rightarrow y' = e^{-x} + x \cdot (-1) e^{-x}$

$y' = e^{-x}(1-x) \Rightarrow y' = 0 \Leftrightarrow x = 1$ posible extremo.

$\nearrow$ 1 $\searrow$

$y'(0) = e^{-0} \cdot 1 > 0$

$y'(2) = e^{-2}(1-2) < 0$

Crecente $(-\infty, 1)$

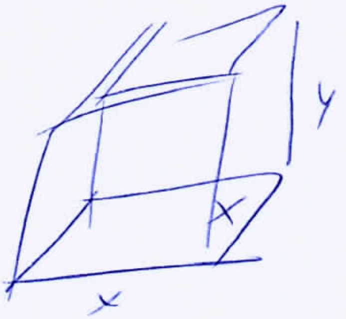
Decrecente $(1, +\infty)$

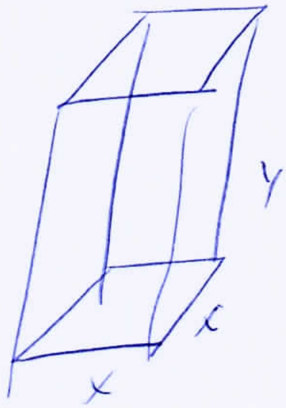
$(1, f(1))$ máximo relativo $\hookrightarrow (1, 1/e)$ máximo relativo.

$f(1) = 1 \cdot e^{-1} = 1/e$

4)

$V = 32\, m^3$





$32 = x^2 y$

$y = \frac{32}{x^2}$

$C = 2x^2 + 4xy$ mínima

$C = \cancel{2}x^2 + 4x \cdot \frac{32}{x^2} = \cancel{2}x^2 + \frac{128}{x}$

$c' = 2x + 128\left(\frac{-1}{x^2}\right) = 4x - \frac{128}{x^2}$

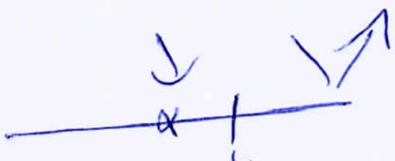
$c' = 0 \Leftrightarrow 2x = \frac{128}{x^2} \Rightarrow x^3 = \frac{128}{2}$

$x = 64$

$x = \sqrt[3]{64}$

$x = \sqrt[3]{4^3}$

$\boxed{x = 4m}$ $y = \frac{32}{16} = 2$ met



$c'(1) = 1 + \frac{128}{1} \gtrless 0$ mínimo.

$c'(5) = 20 - \frac{128}{25} = 20 - > 0$

5) a) $f(x) = (x+10)e^{2x}$

$$\int f(x)\,dx = \int \underbrace{(x+10)}_{u} \underbrace{e^{2x}\,dx}_{dv} = \left[\begin{array}{l} u = x+10 \Rightarrow du = dx \\ dv = e^{2x}dx \Rightarrow v = \frac{1}{2}e^{2x} \end{array}\right.$$

$$= (x+10)\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}\,dx = \frac{(x+10)e^{2x}}{2} - \frac{1}{4}e^{2x} + K$$

$$F(x) = \frac{(x+10)e^{2x}}{2} - \frac{e^{2x}}{4} + K$$

$$F(x) = e^{2x}\left[\frac{x+10}{2} - \frac{1}{4}\right] + K \quad 0'15$$

$$F(0) = 0 \Rightarrow e^{0}\left[\frac{10}{2} - \frac{1}{4}\right] + K = 0$$

$$5 - \frac{1}{4} + K = 0$$

$$K = \frac{1}{4} - 5$$

$$K = \frac{-19}{4} \quad 0'10$$

$$F(x) = e^{2x}\left[\frac{x+10}{2} - \frac{1}{4}\right] - \frac{19}{4}$$

Comprobación: $F'(x) = e^{2x}\cdot 2\left[\frac{x+10}{2} - \frac{1}{4}\right] + e^{2x}\cdot\frac{1}{2} =$

$$= e^{2x}\left[x+10 - \frac{1}{2}\right] + e^{2x}\cdot\frac{1}{2}$$

$$= e^{2x}(x+10). \quad 0'25$$

b) $\int \underbrace{\ln x}_{u} \cdot \underbrace{x^2\,dx}_{dv} = \left[\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x}\,dx \\ dv = x^2\,dx \Rightarrow v = \frac{x^3}{3} \end{array} \right]$

$= (\ln x)\,\frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx = \frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2\,dx$

$= \frac{x^3 \ln x}{3} - \frac{1}{3}\,\frac{x^3}{3} = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + K.$

c) $\int \left(\tan x + \frac{1}{\tan x} \right) dx = \int \tan x\,dx + \int \frac{1}{\tan x}\,dx$

$= \int \frac{\operatorname{sen} x}{\cos x}\,dx + \int \frac{\cos x}{\operatorname{sen} x}\,dx =$

$= -\ln|\cos x| + \ln|\operatorname{sen} x| + K.$

d) $\int \left(\cos x \cdot e^{\operatorname{sen} x} + \frac{1}{x+1} + \sqrt{3x+2} \right) dx =$

$= \int \cos x \cdot e^{\operatorname{sen} x}\,dx + \int \frac{1}{x+1}\,dx + \int \sqrt{3x+2}\,dx$

$= e^{\operatorname{sen} x} + \ln|x+1| + \frac{1}{3} \cdot \frac{(3x+2)^{3/2}}{3/2} =$

$= e^{\operatorname{sen} x} + \ln|x+1| + \frac{2}{9}\sqrt{(3x+2)^3} + K.$