1) a) $f(x) = \begin{cases} ax + bx^2, & 0 \leq x < 2 \\ c + \sqrt{x-1}, & 2 \leq x \leq 5 \end{cases}$

Calcular a, b para que a función sexa derivable en $(0,5)$, $f(0) = f(5)$

Primeiro estudamos a continuidade en $x = 2$:

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} ax + bx^2 = 2a + 4b$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} c + \sqrt{x-1} = c + \sqrt{2-1} = c + 1$

Entón a función é continua en $x = 2$ se $2a + 4b = c + 1$ 0'15

Estudamos a derivabilidade en $x = 2$:

$f'(x) = \begin{cases} a + 2bx, & 0 < x < 2 \\ \dfrac{1}{2\sqrt{x-1}}, & 2 < x < 5 \end{cases}$

$f'(2^-) = \lim_{x \to 2^-} f'(x) = \lim_{x \to 2} a + 2bx = a + 4b$

$f'(2^+) = \lim_{x \to 2^+} f'(x) = \lim_{x \to 2} \dfrac{1}{2\sqrt{x-1}} = \dfrac{1}{2\sqrt{2-1}} = \dfrac{1}{2}$ 0'10

Entón a función é derivable en $x = 2$ se $a + 4b = \frac{1}{2}$

Ademais $f(0) = f(5) \Rightarrow f(0) = 0$
$f(5) = c + \sqrt{5-1} = c + 2$ $\Rightarrow \boxed{c = -2}$ 0'25

$$\left.\begin{array}{l} 2a+4b=c+1 \\ a+4b=\frac{1}{2} \\ c=-2 \end{array}\right\} \Rightarrow \begin{array}{l} 2a+4b=-1 \\ a+4b=\frac{1}{2} \end{array} \Rightarrow$$

$$\begin{array}{l} 2a+4b=-1 \\ -a-4b=-\frac{1}{2} \\ \hline a=-1-\frac{1}{2}=-\frac{3}{2} \end{array} \Rightarrow \boxed{a=-\frac{3}{2}}$$

0'25

$$a+4b=\frac{1}{2} \Rightarrow -\frac{3}{2}+4b=\frac{1}{2}$$

$$4b=\frac{1}{2}+\frac{3}{2}$$

$$4b=\frac{4}{2} \Rightarrow 4b=2$$

$$\boxed{b=\frac{1}{2}}$$

0'25

$$a=-\frac{3}{2},\quad b=\frac{1}{2},\quad c=-2.$$

b) $$\lim_{x\to 0} \frac{2x\cos 2x-\operatorname{sen}2x}{x^3}=\left[\frac{0}{0}\right] \underset{L'H}{=}$$

$$=\lim_{x\to 0} \frac{\cancel{2\cos 2x}+2x(-\operatorname{sen}2x)\cdot 2-(\cancel{\cos 2x})\cdot 2}{3x^2}=$$

$$=\lim_{x\to 0} -\frac{4x\operatorname{sen}2x}{3x^2}=\lim_{x\to 0} \frac{-4\operatorname{sen}2x}{3x}=\left[\frac{0}{0}\right] \underset{L'H}{=}$$

$$=\lim_{x\to 0} \frac{-4\cos 2x\cdot 2}{3}=-\frac{4\cos 0\cdot 2}{3}=\boxed{-\frac{8}{3}}$$

0'5

$$\lim_{x\to\infty}\left(\frac{x+2}{x^2+x+2}\right)^{\frac{1}{x^2}} = \left[1^{\infty}\right] = L$$

$$\ln \lim_{x\to\infty}\left(\frac{x+2}{x^2+x+2}\right)^{\frac{1}{x^2}} = \ln L \Rightarrow$$

$$\ln\left(\lim_{x\to\infty}\frac{x+2}{x^2+x+2}\right)^{\frac{1}{x^2}} = \lim_{x\to\infty}\ln\left(\frac{x+2}{x^2+x+2}\right)^{\frac{1}{x^2}} =$$

$$= \lim_{x\to\infty}\frac{1}{x^2}\cdot\ln\left(\frac{x+2}{x^2+x+2}\right) = \lim_{x\to\infty}\frac{\ln\left(\frac{x+2}{x^2+x+2}\right)}{x^2} =$$

$$= \left[\frac{\ln 1}{0} = \frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{\frac{x+2}{x^2+x+2}}\cdot\frac{(x^2+x+2)-(x+2)\cdot(2x+1)}{(x^2+x+2)^2}}{2x} =$$

$$= \lim_{x\to\infty}\frac{\frac{\cancel{x^2+x+2}}{x+2}\cdot\frac{x^2+x+2-(2x^2+x+4x+2)}{(x^2+x+2)^{\cancel{2}}}}{2x} =$$

$$= \lim_{x\to\infty}\frac{\frac{x^2+x+2-2x^2-5x-2}{(x+2)(x^2+x+2)}}{2x} = \lim_{x\to\infty}\frac{-x^2-4x}{(x+2)(x^2+x+2)2x}$$

$$= \lim_{x\to\infty}\frac{x(-x-4)}{(x+2)(x^2+x+2)2x} = \lim_{x\to\infty}\frac{-x-4}{(x+2)(x^2+x+2)\cdot 2} =$$

$$= \frac{-4}{2\cdot 2\cdot 2} = \frac{-4}{8} = -\frac{1}{2} \Rightarrow -\frac{1}{2} = \ln L \Rightarrow e^{-1/2} = L$$

0'5 $\boxed{L = 1/\sqrt{e}}$ $\sqrt{3}$

2] a) $f(x) = x^3 + 3x^2 + ax - 6$, a?

x_0 punto de inflexión $\Rightarrow f'(x_0) = -3$.

Primeiro calculamos o punto de inflexión:

$f'(x) = 3x^2 + 6x + a$

$f''(x) = 6x + 6 \Rightarrow f''(x) = 0 \Leftrightarrow 6x + 6 = 0$

$x = -1$ posible punto de inflexión 0'25

$f'''(x) = 6 \neq 0 \Rightarrow (-1, f(-1))$ punto de inflexión.

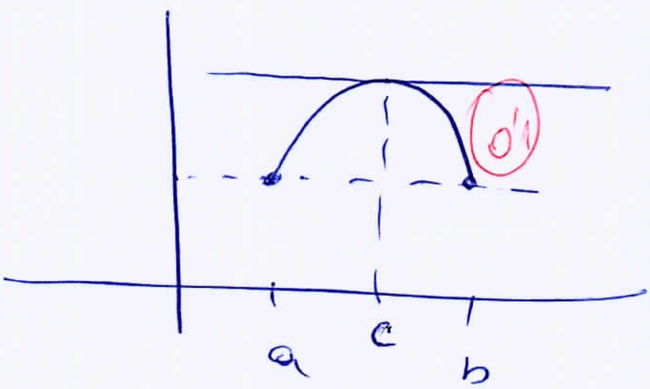
$f'(-1) = -3 \Leftrightarrow f'(x) = 3x^2 + 6x + a$ 0'25 $\Big\} \Rightarrow$

$f'(-1) = 3 - 6 + a = -3 + a$ 0'25

$\Rightarrow -3 + a = -3$

$\boxed{a = 0}$ 0'25

b) $y = f(x)$ continua en $[a, b]$ 0'2
$y = f(x)$ derivable en (a, b) 0'2
$f(a) = f(b)$
$\Big\} \Rightarrow \exists c \in (a, b) / f'(c) = 0$. 0'2 0'2



Se unha función verifica as hipótesis do teorema de Rolle $\Rightarrow$ existe polo menos un punto, no cal a tanxente é horizontal (un punto no intervalo aberto) 0'1

3) $f(x) = \dfrac{\ln x}{x}$

a) $Dom(f) = (0, +\infty)$ 0'25

$x > 0$

$x \neq 0$

Puntos de corte cos eixes de coordenadas:

$x = 0 \Rightarrow \nexists$

$y = 0 \Rightarrow 0 = \dfrac{\ln x}{x} \Rightarrow \ln x = 0$

$e^0 = x \Rightarrow x = 1 \Rightarrow$

$\Rightarrow (1, 0)$ 0'25

ASÍNTOTAS

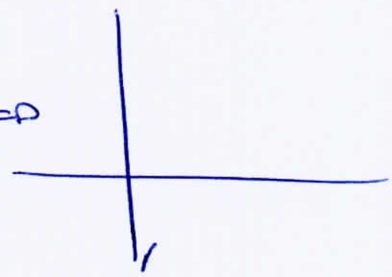
ASÍNTOTA VERTICAL

$0 \notin Dom(f) \Rightarrow \lim\limits_{x \to 0^+} \dfrac{\ln x}{x} = \left[\dfrac{\ln 0^+}{0^+} = \dfrac{-\infty}{0^+}\right] =$

$= \lim\limits_{x \to 0^+} \dfrac{1}{x} \cdot \ln x = \left[\dfrac{1}{0^+} \cdot \ln 0^+ =\right.$

$\left. = +\infty \cdot -\infty\right] = -\infty \Rightarrow$

0'25

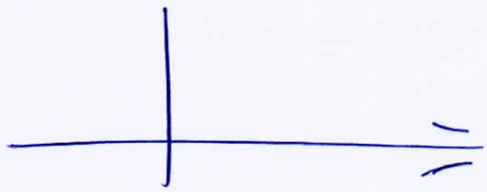


$\Rightarrow x = 0$ Non é asíntota vertical

ASÍNTOTA HORIZONTAL

$$\lim_{x \to \infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty}\right] \underset{L'H}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = 0.$$

$\Rightarrow y = 0$ asíntota horizontal en $+\infty$. 0'25

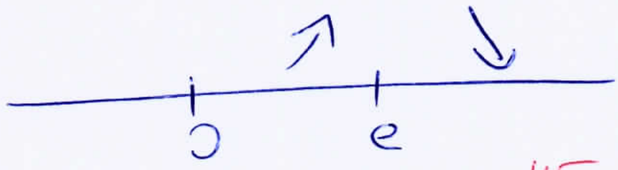


b) $x_0 \,/\, f'(x_0) = 0$

$$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$ 0'25

$$f'(x) = 0 \Leftrightarrow 1 - \ln x = 0$$
$$1 = \ln x \Rightarrow e^1 = x \Rightarrow x = e$$

posible extremo. 0'25

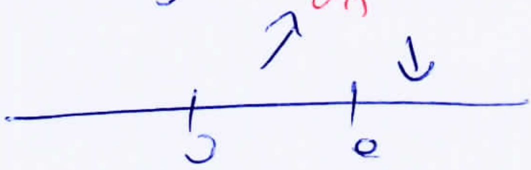


0'15

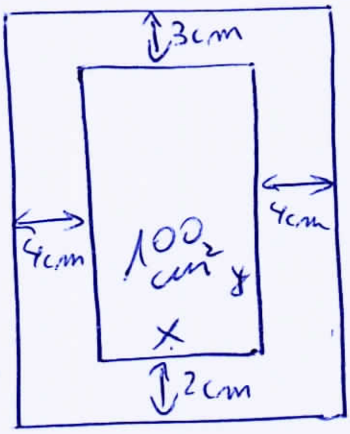
$$f'(1) = \frac{1 - \ln 1}{1} = 1 > 0 \Rightarrow$$ Creciente en $(0, e)$ 0'10

$$f'(3) = \frac{1 - \ln 3}{3^2} = \frac{-0'98}{3^2} < 0 \Rightarrow$$ Decreciente en $(e, +\infty)$ 0'10

0'15



4]



menor cantidade posible de papel.

$$xy = 100 \quad 0'25 \quad \Rightarrow \quad y = \frac{100}{x} \quad 0'25$$

$$F = (x+8)(y+5) \quad 0'25$$

$$F = (x+8)\cdot\left(\frac{100}{x}+5\right)$$

$$F = 100 + 5x + \frac{800}{x} + 40 \quad 0'25$$

$$F = 5x + \frac{800}{x} + 140 \quad 0'25$$

$$F'(x) = 5 - \frac{800}{x^2} \Rightarrow \quad 0'10$$

$$F'(x) = 0 \Leftrightarrow 5 - \frac{800}{x^2} = 0 \quad 0'25$$

$$5 = \frac{800}{x^2}$$

$$x^2 = \frac{800}{5}$$

$$x^2 = 160$$

$$x = \sqrt{160} = 4\sqrt{10} \cong 12'65 \text{ cm}$$

posible extremo. 0'15

Comprobemos que se $x = 4\sqrt{10}$ a función ten un mínimo:

$$F'(10) = 5 - \frac{800}{10^2} = 5 - \frac{800}{100} = 5 - 8 < 0 \quad 0'15 \Rightarrow \quad \searrow \; 4\sqrt{10} \; \nearrow$$

$$F'(15) = 5 - \frac{800}{15^2} = \frac{5\cdot 15^2 - 800}{15^2} > 0$$

Entón a función ten un mínimo en $(4\sqrt{10}, F(4\sqrt{10}))$

As dimensión serán $x = 4\sqrt{10}$

$$y = \frac{100}{4\sqrt{10}} = \frac{100\sqrt{10}}{4\cdot 10} = \frac{10\sqrt{10}}{4} =$$

$$y = \frac{5\sqrt{10}}{2}$$ 0'10

Conclusión as dimensións serán $x = 4\sqrt{10}$ cm para $y = \frac{5\sqrt{10}}{2}$ cm.

que se utilice a mínima cantidade de papel.

5] a) $\int \left(\frac{2}{\sqrt{1-x^2}} + \frac{3}{\cos^2 x} + sen^3 x \cdot \cos x + \frac{\sqrt{x}}{x^2} \right) dx$

$$= \underbrace{2 \cdot arc\,sen\,x + 3\,tag\,x}_{0'15} + \underbrace{\frac{sen^4 x}{4} + \frac{x^{\frac{1}{2}-2+1}}{-\frac{1}{2}}}_{0'10} + K$$

$$= 2 arcsen\,x + 3\,tag\,x + \frac{sen^4 x}{4} - 2x^{-\frac{1}{2}} + K$$

$$= 2\,arsen\,x + 3\,tag\,x + \frac{sen^4 x}{4} - \frac{2}{\sqrt{x}} + K$$ 0'25

b) $\int (2x+1)e^{-x} dx = \left[\begin{array}{l} u = 2x+1 \\ dv = e^{-x} dx \end{array} \Rightarrow \begin{array}{l} du = 2\,dx \\ v = -e^{-x} \end{array} \right] =$ 0'25

$= (2x+1)(-e^{-x}) - \int (-e^{-x})\,2\,dx = -(2x+1)e^{-x} - 2e^{-x} + K$

0'25

c) $\int x\sqrt[3]{4+x^2}\,dx = \int x(4+x^2)^{1/3}\,dx = \frac{1}{2}\int 2x(4+x^2)^{1/3}\,dx$

$= \frac{1}{2}\cdot\frac{(4+x^2)^{1/3+1}}{1/3+1} = \frac{1}{2}\,\frac{(4+x^2)^{4/3}}{4/3} = \frac{1}{2}\cdot\frac{3}{4}\sqrt[3]{(4+x^2)^4}$ 0'25

$= \frac{3}{8}\sqrt[3]{(4+x^2)^4} + K.$ 0'25

d) $\int \frac{2}{4+x^2}\,dx = 2\int\frac{1}{4+x^2}\,dx = 2\int\frac{1}{4(1+(\frac{x}{2})^2)}\,dx =$ 0'25

$4+x^2 = 4\left(1+\frac{x^2}{4}\right) = 4\left(1+\left(\frac{x}{2}\right)^2\right)$

$= \frac{2}{4}\int\frac{1}{1+(\frac{x}{2})^2}\,dx = 2\cdot\frac{2}{4}\int\frac{\frac{1}{2}}{1+(\frac{x}{2})^2}\,dx = \operatorname{arctag}\frac{x}{2} + K.$

0'25