BOLETÍN DE INTEGRAIS

1.1 a) $\int \frac{2}{4+x^2}\,dx = 2\int \frac{1}{4+x^2}\,dx = 2\int \frac{1}{4(1+(\frac{x}{2})^2)}\,dx$

$\left[4+x^2 = 4\left(1+\frac{x^2}{4}\right) = 4\left(1+\left(\frac{x}{2}\right)^2\right) \right]$

$= \frac{2}{4}\int \frac{1}{1+(\frac{x}{2})^2}\,dx = \frac{2}{4}\cdot 2\int \frac{\frac{1}{2}}{1+(\frac{x}{2})^2}\,dx =$

$= \frac{4}{4}\int \frac{1/2}{1+(\frac{x}{2})^2}\,dx = \operatorname{arctag}\left(\frac{x}{2}\right) + K$

b) $\int \frac{-7}{5+7x^2}\,dx = -7\int \frac{1}{5+7x^2}\,dx =$

$\left[5+7x^2 = 5\left(1+\frac{7x^2}{5}\right) = 5\left(1+\frac{x^2}{5/7}\right) = 5\left(1+\left(\frac{x}{\sqrt{5/7}}\right)^2\right) \right]$

$= -7\int \frac{1}{5\left(1+\left(\frac{x}{\frac{\sqrt{5}}{\sqrt{7}}}\right)^2\right)}\,dx = -\frac{7}{5}\int \frac{1}{1+\left(\frac{\sqrt{7}x}{\sqrt{5}}\right)^2}\,dx =$

$= -\frac{7}{5}\cdot\frac{\sqrt{5}}{\sqrt{7}}\int \frac{\frac{\sqrt{7}}{\sqrt{5}}}{1+\left(\frac{\sqrt{7}x}{\sqrt{5}}\right)^2}\,dx = \frac{-7\sqrt{5}}{5\sqrt{7}}\operatorname{arctag}\left(\frac{\sqrt{7}}{\sqrt{5}}\right) + K$

$\shortparallel$

$-\frac{\sqrt{35}}{5}\operatorname{arctag}\frac{\sqrt{35}}{5}$

$\left[\frac{-7\sqrt{5}}{5\sqrt{7}} = \frac{-7\sqrt{5}\sqrt{7}}{5\cdot 7} = \frac{-\sqrt{35}}{5}, \quad \frac{\sqrt{7}}{\sqrt{5}} = \frac{\sqrt{7}\sqrt{5}}{5} = \frac{\sqrt{35}}{5} \right]$

2]

a) $\int x e^{4x}\,dx = \left[\begin{array}{l} u = x \Rightarrow du = dx \\ dv = e^{4x}dx \Rightarrow v = \int e^{4x}dx = \frac{1}{4}e^{4x} \end{array}\right]$

$= x \cdot \frac{1}{4}e^{4x} - \int \frac{1}{4}e^{4x}\,dx = \frac{xe^{4x}}{4} - \frac{1}{4}\cdot\frac{1}{4}\int 4\cdot e^{4x}\,dx$

$= \frac{xe^{4x}}{4} - \frac{e^{4x}}{16} + k$

b) $\int x^3 \ln x\,dx = \left[\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x}dx \\ dv = x^3 dx \Rightarrow v = \int x^3 dx = \frac{x^4}{4} \end{array}\right]$

$= \ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4}\cdot\frac{1}{x}\,dx = \frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3\,dx$

$= \frac{x^4 \ln x}{4} - \frac{1}{4}\,\frac{x^4}{4} = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + k$

c) $\int e^{5x}\cos x\,dx = \left[\begin{array}{l} u = \cos x \Rightarrow du = -\operatorname{sen} x\,dx \\ dv = e^{5x} \Rightarrow v = \int e^{5x} = \frac{1}{5}e^{5x} \end{array}\right]$

$= \cos x \cdot \left(\frac{1}{5}e^{5x}\right) - \int \frac{1}{5}e^{5x}\cdot(-\operatorname{sen} x)\,dx =$

$= \frac{e^{5x}\cos x}{5} + \frac{1}{5}\int e^{5x}\operatorname{sen} x\,dx =$

$\left\{\begin{array}{l} \int e^{5x}\operatorname{sen} x\,dx = \left[\begin{array}{l} u = \operatorname{sen} x \Rightarrow du = \cos x\,dx \\ dv = e^{5x}dx \Rightarrow v = \frac{1}{5}e^{5x} \end{array}\right] = \\ = \frac{1}{5}e^{5x}\operatorname{sen} x - \int \frac{1}{5}e^{5x}\cos x\,dx \end{array}\right.$

$$= \frac{e^{5x}\cos x}{5} + \frac{1}{5}\left(1/5\, e^{5x} \operatorname{sen} x - 1/5 \int e^{5x} \cos x \, dx \right) =$$

$$= \frac{e^{5x}\cos x}{5} + \frac{e^{5x}\operatorname{sen} x}{25} - 1/25 \int e^{5x} \cos x \, dx$$

$$I = \frac{e^{5x}\cos x}{5} + \frac{e^{5x}\operatorname{sen} x}{25} - 1/25\, I \Rightarrow$$

$$I + 1/25\, I = \frac{e^{5x}\cos x}{5} + \frac{e^{5x}\operatorname{sen} x}{25} = \frac{5e^{5x}\cos x + e^{5x}\operatorname{sen} x}{25}$$

$$25I + I = 5e^{5x}\cos x + e^{5x}\operatorname{sen} x$$

$$\boxed{I = \frac{5e^{5x}\cos x + e^{5x}\operatorname{sen} x}{26}}$$

d) $\int 2^x \cos x \, dx = \left[\begin{array}{l} u = 2^x \Rightarrow du = 2^x \ln 2 \, dx \\ dv = \cos x \, dx \Rightarrow v = \int \cos x \, dx = \operatorname{sen} x \end{array} \right]$

$= 2^x \operatorname{sen} x - \int \operatorname{sen} x \cdot 2^x \ln 2 \, dx = 2^x \operatorname{sen} x - \ln 2 \int \operatorname{sen} x \cdot 2^x dx \circledast$

$$\left[\begin{array}{l} \int \operatorname{sen} x \cdot 2^x dx = \left[\begin{array}{l} 2^x = u \Rightarrow du = 2^x \ln 2 dx \\ dv = \operatorname{sen} x \, dx \Rightarrow v = -\cos x \end{array} \right. \\ = 2^x(-\cos x) - \int -\cos x \cdot 2^x \ln 2 \, dx = \\ = -2^x \cos x + \ln 2 \int 2^x \cos x \, dx \end{array} \right.$$

$\int 2^x \cos x \, dx = 2^x \operatorname{sen} x - \ln 2 \left(-2^x \cos x + \ln 2 \int 2^x \cos x \, dx \right)$

$\int 2^x \cos x \, dx = 2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x - (\ln 2)^2 \int 2^x \cos x \, dx$

$I = 2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x - (\ln 2)^2 \cdot I$

$I + (\ln 2)^2 I = 2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x$

$$\boxed{I = \frac{2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x}{1 + (\ln 2)^2}}$$

e) $\int \left(1-\frac{1}{x^2}\right) \ln x \, dx = \left[\begin{array}{l} u=\ln x \to du=\frac{1}{x}dx \\ dv=\left(1-\frac{1}{x^2}\right)dx \to v=\int\left(1-\frac{1}{x^2}\right)dx \\ \quad =x-\frac{x^{-1}}{-1}=x+\frac{1}{x} \end{array} \right.$

$\ln x \cdot \left(x+\frac{1}{x}\right) - \int \left(x+\frac{1}{x}\right) \cdot \frac{1}{x} \, dx =$

$= \ln x \left(x+\frac{1}{x}\right) - \int 1+\frac{1}{x^2} \, dx = \ln x \left(x+\frac{1}{x}\right) - x - \frac{x^{-1}}{-1} + k$

$= \ln x \left(x+\frac{1}{x}\right) - x + \frac{1}{x} + K.$

3) a) $\int \frac{2x^2+x}{x^2-1} \, dx = \int 2 \, dx + \int \frac{x+2}{x^2-1} \, dx =$

$$\begin{array}{l|l} 2x^2+x & x^2-1 \\ \hline -2x^2+2 & 2 \\ \hline x+2 & \end{array}$$

$\frac{2x^2+x}{x^2-1} = 2+\frac{x+2}{x^2-1}$

$\int \frac{x+2}{x^2-1} \, dx = \int \frac{-1/2}{x+1} \, dx + \int \frac{3/2}{x-1} \, dx \Rightarrow$

$\frac{x+2}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} \to x+2 = A(x-1)+B(x+1)$

$x=1 \to 3 = 2B \to B=3/2$

$x=-1 \to 1=-2A \to A=-1/2$

$$I = 2x + \left(-\tfrac{1}{2}\right) \ln|x+1| + \tfrac{3}{2} \ln|x-1| + k$$

$$I = 2\left(2x - \frac{\ln|x+1|}{2} + 3\frac{\ln|x-1|}{2}\right) + k.$$

b) $$\int \frac{x^2}{x+1}\,dx = \int (x-1)\,dx + \int \frac{1}{x+1}\,dx = \frac{x^2}{2} - x + \ln|x+1| + K$$

$$\begin{array}{l|l} x^2 & x+1 \\ \hline -x^2-x & x-1 \\ \hline -x & \\ +x+1 & \\ \hline 1 & \end{array}$$

$$\frac{x^2}{x+1} = x + \frac{(-x)}{x+1}$$

$$\frac{x^2}{x+1} = x-1 + \frac{1}{x+1}$$

c) $$\int \frac{x^2-3}{x^4-x^3}\,dx$$

$$x^4 - x^3 = x^3(x-1)$$

$$\frac{x^2-3}{x^4-x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} = \frac{\qquad}{(x-1)^3 x^3}$$

$$x^2-3 = A x^2(x-1) + Bx(x-1) + Dx^3 + C(x-1)$$

$x=0 \Rightarrow -3 = -C \Rightarrow \boxed{C=3}$

$x=1 \Rightarrow \boxed{-2 = D}$

$x=-1 \Rightarrow -2 = A(-2) + B\cdot 2 + C(-2) + (-2)(-1)$

$-2 = -2A + 2B - 6 + 2 \Rightarrow -2A + 2B = 2$

$\boxed{-A + B = 1}$

$x=2, \quad 1 = 4A + 2B + 8D + C$

$1 = 4A + 2B - 16 + 3$

$4A + 2B = 13 + 1$

$4A + 2B = 14$

$\boxed{2A + B = 7}$

$$\begin{array}{r} -A + \cancel{B} = 1 \\ -2A - \cancel{B} = -7 \\ \hline -3A = -6 \\ \boxed{A = 2} \end{array} \qquad \begin{array}{l} 2A + B = 7 \\ 4 + B = 7 \\ \boxed{B = 3} \end{array}$$

$$\int \frac{x^3 - 3}{x^4 - x^3} dx = \int \frac{2}{x} dx + \int \frac{3}{x^2} dx + \int \frac{3}{x^3} dx + \int \frac{-2}{x-1} dx$$

$$= 2\ln|x| + 3 \cdot \frac{x^{-1}}{-1} + 3 \cdot \frac{x^{-2}}{-2} - 2\ln|x-1|$$

$$= 2\ln|x| - \frac{3}{x} - \frac{3}{2x^2} - 2\ln|x-1| + K$$

d) $\displaystyle\int \frac{dx}{x(x-1)(x+2)} = \frac{-1}{2} \ln|x| + \frac{1}{3} \ln|x-1| + \frac{1}{6} \ln|x+2| + K$

$$\frac{1}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$$

$$1 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

$x=1, \quad 1 = 3B \Rightarrow B = 1/3$

$x=0, \quad 1 = -2A \Rightarrow A = -1/2$

$x=-2, \quad 1 = C(-2)(-3) \Rightarrow C = 1/6$

4) a) $\displaystyle\int \frac{e^x+1}{e^x}\,dx = \left[\begin{matrix} e^x = t \\ e^x dx = dt \\ dx = \frac{dt}{e^x} \\ dx = \frac{dt}{t} \end{matrix}\right] = \int \frac{t+1}{t}\,\frac{dt}{t} =$

$$= \int \frac{t+1}{t^2}\,dt = \int \frac{t}{t^2}\,dt + \int \frac{1}{t^2}\,dt =$$

$$= \int \frac{1}{t}\,dt + \int \frac{1}{t^2}\,dt$$

$$= \ln|t| + \frac{t^{-1}}{-1} = \ln|e^x| - \frac{1}{e^x} + K.$$

b) $\int x\sqrt{2x+1}\,dx = \left[\begin{array}{c} 2x+1=t^2 \\ 2\,dx = 2t\,dt \\ dx = \frac{2t\,dt}{2} \\ dx = t\,dt \end{array}\right] \to x = \frac{t^2-1}{2}$

$= \int \frac{t^2-1}{2} \cdot t \underbrace{t\,dt} = \frac{1}{2}\int (t^4 - t^2)\,dt$

$= \frac{1}{2}\left(\frac{t^5}{5} - \frac{t^3}{3}\right) = \frac{1}{2}\left[\frac{(\sqrt{2x+1})^5}{5} - \frac{(\sqrt{2x+1})^3}{3}\right] + K$

c) $\int \frac{2x^2}{\sqrt{x^3+2}}\,dx = \left[\begin{array}{c} x^3+2 = t^2 \\ 3x^2\,dx = 2t\,dt \\ x^2\,dx = \frac{2}{3}t\,dt \end{array}\right] =$

$2\int \frac{1}{t} \cdot \frac{2}{3} t\,dt = \frac{4}{3}\int dt = \frac{4}{3} \cdot t = \frac{4}{3}\sqrt{x^3+2} + K$

d) $\int \frac{x^3}{\sqrt{x^2+1}}\,dx = \left[\begin{array}{c} x^2+1 = t^2 \\ 2x\,dx = 2t\,dt \end{array}\right] = \to x^2 = t^2 - 1$

$\int \frac{x^2}{\sqrt{x^2+1}} \cdot x\,dx = \int \frac{t^2-1}{t} \cdot t\,dt = \int (t^2-1)\,dt$

$= \frac{t^3}{3} - t = \frac{\sqrt{x^2+1}}{3} - \sqrt{x^2+1} + K.$

(5) a) $\displaystyle\int \frac{x+3}{\sqrt{x+2}}\,dx = \left[\begin{array}{l} x+2=t^2 \to x=t^2-2 \\ dx = 2t\,dt \end{array}\right] = \int \frac{t^2-2+3}{t}\cdot 2t\,dt$

$$= \int \frac{t^2-2+3}{t}\cdot 2t\,dt = 2\int t^2+1\,dt = 2\left(\frac{t^3}{3}+t\right) =$$

$$= 2\left(\frac{(\sqrt{x+2})^3}{3} + \sqrt{x+2}\right) + k.$$

b) $\displaystyle\int x^2\cdot e^{-x}\,dx = \left[\begin{array}{l} u=x^2 \to du = 2x\,dx \\ dv = e^{-x}\,dx \to v=\int e^{-x}\,dx = -e^{-x} \end{array}\right]$

$$= x^2(-e^{-x}) - \int (-e^{-x})\,2x\,dx = -x^2e^{-x} + 2\int x e^{-x}\,dx =$$

$$\int x e^{-x}\,dx = \left[\begin{array}{ll} u=x \to & du = dx \\ dv = e^{-x}\,dx & v=-e^{-x} \end{array}\right]$$

$$= -xe^{-x} - \int -e^{-x}\,dx = -xe^{-x} - e^{-x}$$

$$= -x^2e^{-x} + 2\cdot\left(-xe^{-x} - e^{-x}\right) = -x^2e^{x} - 2xe^{-x} - 2e^{-x} + k.$$

d) $\displaystyle\int \sqrt{x}\,\ln x = \left[\begin{array}{ll} u=\ln x \to & du = \frac{1}{x}\,dx \\ dv = \sqrt{x}\,dx & v=\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2} \end{array}\right]$

$$= \ln x\left(\tfrac{2}{3}x^{3/2}\right) - \int \tfrac{2}{3}x^{3/2}\cdot\frac{1}{x}\,dx = \frac{2}{3}\sqrt{x^3}\,\ln x - \frac{2}{3}\int x^{\frac{3}{2}-1}\,dx$$

$$= \frac{2}{3}x\sqrt{x}\,\ln x - \frac{2}{3}\int x^{1/2}\,dx = \frac{2}{3}x\sqrt{x}\,\ln x - \frac{2}{3}\cdot\frac{x^{3/2}}{3/2} = \frac{2}{3}x\sqrt{x}\,\ln x - \frac{4}{9}x\sqrt{x} + k$$

c) $\int x\sqrt{x+1}\,dx = \begin{bmatrix} x+1 = t^2 \\ dx = 2t\,dt \end{bmatrix}$

$= \int (t^2-1) \cdot t \cdot 2t\,dt = 2\int t^2(t^2-1)\,dt$

$= 2\int t^4 - t^2\,dt = 2\left(\frac{t^5}{5} - \frac{t^3}{3}\right)$

$= 2\left(\frac{(\sqrt{x+1})^5}{5} - \frac{(\sqrt{x+1})^3}{3}\right) + k.$

e) $\int x \ln(1+x)\,dx = \left[\begin{array}{l} u = \ln(1+x) \Rightarrow du = \frac{1}{1+x} \\ dv = x\,dx \Rightarrow v = \frac{x^2}{2} \end{array}\right]$

$\frac{x^2}{2}\ln(1+x) - \int \frac{x^2}{2}\cdot\frac{1}{x+1}\,dx = \frac{x^2}{2}\ln(1+x) - \frac{1}{2}\int \frac{x^2}{x+1}\,dx =$

$\int \frac{x^2}{x+1}\,dx = \int (x-1)\,dx + \int \frac{1}{x+1}\,dx$

$$\begin{array}{r|l} x^2 & x+1 \\ \hline -x^2-x & x-1 \\ \hline -x & \\ +x+1 & \\ \hline 1 & \end{array}$$

$\frac{x^2}{x+1} = x-1+\frac{1}{x+1}$

$= \frac{x^2}{2}\ln(1+x) - \frac{1}{2}\left(\frac{x^2}{2} - x + \ln|x+1|\right) + K$

$= \frac{x^2}{2}\ln(1+x) - \frac{x^2}{4} + \frac{x}{2} - \frac{\ln|x+1|}{2} + K.$

f) $\int \frac{e^x}{e^{2x}+3e^x+2}\,dx = \left[\begin{array}{l} e^x = t \\ e^x\,dx = dt \end{array}\right] = \int \frac{dt}{t^2+3t+2} =$

$t^2+3t+2=0$

$t = \frac{-3\pm\sqrt{9-8}}{2} = \frac{-3\pm 1}{2} \begin{array}{l} -2 \\ -1 \end{array}$

$\frac{1}{t^2+3t+2} = \frac{A}{t+2} + \frac{B}{t+1}$

$1 = A(t+1) + B(t+2)$

$t=-1 \Rightarrow 1 = B$

$t=-2 \Rightarrow 1 = -A \Rightarrow A = -1$

$$\int \frac{-1}{t+2}\,dt + \int \frac{1}{t+1}\,dt = -\ln|t+2| + \ln|t+1| + K$$

$$= -\ln|e^x+2| + \ln|e^x+1| + K.$$

g) $\displaystyle\int \frac{2}{3+3e^x}\,dx = \left[\begin{array}{l} e^x = t \\ e^x dx = dt \\ dx = \frac{dt}{e^x} = \frac{dt}{t} \end{array}\right] =$

$$= \int \frac{2}{3(1+e^x)}\,dx = \frac{2}{3}\int \frac{1}{1+t}\cdot\frac{dt}{t} =$$

$$= \frac{2}{3}\int \frac{1}{(1+t)t}\,dt = \frac{2}{3}\left(\int \frac{1}{t+1}\,dt + \int \frac{-1}{t+1}\,dt\right) =$$

$$\frac{1}{(1+t)\cdot t} = \frac{A}{t} + \frac{B}{t+1}$$

$$1 = A(t+1) + Bt$$

$t=-1$, $\quad 1 = -B \Rightarrow B = -1$

$t=0$, $\quad \boxed{1 = A}$

$$= \frac{2}{3}\left(\ln|t| - \ln|t+1|\right) + K$$

$$= \frac{2}{3}\left(\ln|e^x| - \ln|e^x+1|\right) + K.$$

h) $\int \frac{x^3+3}{x^2-x}\,dx = \int (x+1)\,dx + \int \frac{x+3}{x^2-x}\,dx =$

$$\begin{array}{r|l} x^3+3 & x^2-x \\ \hline -x^3+x^2 & x+1 \\ \hline x^2+3 & \\ -x^2+x & \\ \hline x+3 & \end{array}$$

$$\frac{x^3+3}{x^2-x} = x+1+\frac{x+3}{x^2-x}$$

$$\frac{x+3}{x^2-x} = \frac{A}{x} + \frac{B}{x-1} \Rightarrow x+3 = A(x-1) + Bx$$

$x^2-x = x(x-1)$

$x=0 \Rightarrow 3 = -A \Rightarrow \boxed{A=-3}$

$x=1 \Rightarrow \boxed{4=B}$

$$= \int (x+1)\,dx + \int \frac{-3}{x}\,dx + \int \frac{4}{x-1}\,dx =$$

$$= \frac{x^2}{2} + x - 3\ln|x| + 4\ln|x-1| + K.$$

i) $\int \frac{x^3+2}{x^2-1}\,dx = \int x\,dx + \int \frac{x+2}{x^2-1}\,dx$ $=$

$$\begin{array}{l|l} x^3+2 & x^2-1 \\ \hline -x^3+x & x \\ \hline x+2 & \end{array}$$

$$\frac{x^3+2}{x^2-1} = x + \frac{x+2}{x^2-1}$$

$$\int \frac{x+2}{x^2-1}\,dx \underset{\downarrow}{=} \int \frac{-1/2}{x+1}\,dx + \int \frac{3/2}{x-1}\,dx = \frac{-1}{2}\ln|x+1| + \frac{3}{2}\ln|x-1|$$

$$\frac{x+2}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$x+2 = A(x-1) + B(x+1)$$

$x=1$, $\quad 3 = 2B \Rightarrow B = 3/2$

$x=-1$, $\quad 1 = -2A \Rightarrow A = -1/2$

$$= \frac{x^2}{2} + \left(-\tfrac{1}{2}\right)\ln|x+1| + \tfrac{3}{2}\ln|x-1| + K.$$