Soluciones

1

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 1 & 2 & -1 & 4 \\ 1 & -8 & 5 & -6 \end{array}\right) \xrightarrow[F_1 \leftrightarrow F_2]{} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 1 & 3 \\ 1 & -8 & 5 & -6 \end{array}\right) \xrightarrow[\substack{F_2 - 2F_1 \\ F_3 - F_1}]{}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 3 & -5 \\ 0 & -10 & 6 & -10 \end{array}\right) \xrightarrow[F_3 - 2F_2]{} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 3 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

rango $A = 2 \rightarrow$ SCIndet , ∞ soluciones

$$\left.\begin{array}{l} x + 2y - z = 4 \\ -5y + 3z = -5 \\ \boxed{z = \lambda} \end{array}\right\} \quad \begin{array}{l} -5y = -5 - 3z \\ -5y = -5 - 3\lambda \\ y = \dfrac{-5 - 3\lambda}{-5} = \boxed{1 + \dfrac{3}{5}\lambda} \end{array}$$

$x + 2y - z = 4$; $x = 4 - 2y + z$; $x = 4 - 2 - \frac{6}{5}\lambda + \lambda$

$x = 2 - \frac{6-5}{5}\lambda$; $\boxed{x = 2 - \frac{1}{5}\lambda}$

Soluciones $\left\{ \left(2 - \frac{1}{5}\lambda,\ 1 + \frac{3}{5}\lambda,\ \lambda\right) \mid \lambda \in \mathbb{Z} \right\}$

2 A) $\log_{5x^2-6x} 2^3 = \log_x 2 = y \rightarrow x^y = 2 \rightarrow x^{3y} = 2^3$

$\Rightarrow$ ~~$(5x^2 - 6x)$~~

$(5x^2 - 6x)^y = 2^3 = (x^y)^3$

$5x^2 - 6x = x^3$; $x^3 - 5x^2 + 6x = 0$; $x(x^2 - 5x + 6) = 0$

Soluciones posibles $x = 0,\ x = 2,\ x = 3$

$\cancel{x=0}$ No vale $\nexists \log_0$

$x = 2 \rightarrow \log_{5\cdot 4 - 12} 8 = \log_8 8 = 1 = \log_2 2$ Sí vale

$x = 3 \rightarrow \log_{5\cdot 9 - 18} 8 = \log_{27} 8 = \log_{3^3} 2^3 = \log_3 2$

$\log_3 2$ Sí, vale

$\boxed{2B}$ $\log(x-1) - \left[\log\sqrt{5+x} + \log\sqrt{5-x}\right] = 0$

$\log(x-1) - \log\sqrt{25-x^2} = 0$

$\log\dfrac{x-1}{\sqrt{25-x^2}} = 0$; $\dfrac{x-1}{\sqrt{25-x^2}} = 1$

$x-1 = \sqrt{25-x^2}$; $(x-1)^2 = 25-x^2$; $x^2-2x+1 = 25-x^2$

$2x^2-2x-24=0$; $x^2-x-12=0$;

$x = \dfrac{1\pm\sqrt{1+48}}{2} = \dfrac{1\pm 7}{2}$ $\begin{cases} 4 \\ -3 \end{cases}$

<u>Comp</u> $x=4 \to \log(4-1) - \log(\sqrt{5+4}) - \log(\sqrt{5-4}) =$

$= \log 3 - \log\sqrt{9} - \underbrace{\log\sqrt{1}}_{= 0} = \log 3 - \log 3 = 0$ sí//

$x=-3 \to \log(x-1) = \log(-4)$ $\nexists$ No vale

Sol $\boxed{x=4}$

$\boxed{3A}$ $4\,\text{sen}\,\dfrac{x}{2} + 2\cos x = 3$

$4\left(\pm\sqrt{\dfrac{1-\cos x}{2}}\right) + 2\cos x = 3$

$\left(\pm\sqrt{\dfrac{1-\cos x}{2}}\right)^2 = \left(\dfrac{3-2\cos x}{4}\right)^2$

$\dfrac{1-\cos x}{2} = \dfrac{9-12\cos x+4\cos^2 x}{16}$

$8-8\cos x = 9-12\cos x+4\cos^2 x$

$4\cos^2 x - 12\cos x + 8\cos x + 9 - 8 = 0$

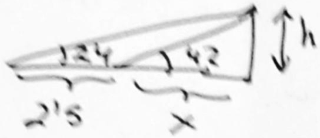
$4\cos^2 x - 4\cos x + 1 = 0$ $\quad t = \cos x$

$4t^2-4t+1=0$

$t = \dfrac{4\pm\sqrt{16-16}}{2\cdot 4} = \dfrac{4}{8} = \dfrac{1}{2} \to \cos x = \dfrac{1}{2}$ $\begin{cases} 60^\circ + k\,360^\circ \\ 300^\circ + k\,360^\circ \end{cases}$

$k \in \mathbb{Z}$

5B ②



$$\left.\begin{array}{l} tg\,42 = \dfrac{h}{x} \\ tg\,24 = \dfrac{h}{x+2'5} \end{array}\right\}$$

$$x = \frac{h}{tg\,42} \; ; \quad (x+2'5)\,tg\,24 = h$$

$$\left(\frac{h}{tg\,42} + 2'5\right) tg\,24 = h ; \quad h\,\frac{tg\,24}{tg\,42} + 2'5\,tg\,24 = h$$

$$\left(\frac{tg\,24}{tg\,42} - 1\right) h = -2'5\,tg\,24 ; \quad h = \frac{2'5\,tg\,24}{1 - \dfrac{tg\,24}{tg\,42}}$$

4

$$\lim_{x\to-\infty}\left(\frac{3x^2-x}{3x^2+1}\right)^{2x+1} = \lim_{x\to+\infty}\left(\frac{3x^2+x}{3x^2+1}\right)^{-2x+1} =$$

$$= \lim_{\infty}\left(1 + \frac{3x^2+x}{3x^2+1} - 1\right)^{-2x+1} = \lim_{\infty}\left(1 + \frac{\cancel{3x^2}+x-\cancel{3x^2}-1}{3x^2+1}\right)^{-2x+1} =$$

$$= \lim_{\infty}\left(1 + \frac{x-1}{3x^2+1}\right)^{-2x+1} = \lim_{\infty}\left(1 + \frac{1}{\frac{3x^2+1}{x-1}}\right)^{-2x+1} =$$

$$= \lim_{\infty}\left[\left(1 + \frac{1}{\frac{3x^2+1}{x-1}}\right)^{\frac{3x^2+1}{x-1}}\right]^{\frac{x-1}{3x^2+1}\cdot(-2x+1)} =$$

$$= e^{\lim_{\infty}\frac{(x-1)(-2x+1)}{3x^2+1}} = e^{\lim_{\infty}\frac{-2x^2+2x+x-1}{3x^2+1}} =$$

$\partial P = \partial Q = 1$

$$\boxed{e^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{e^2}}}$$

$\boxed{5A}$ a) $f(x) = \dfrac{x}{x^2-4}$ dom $f = \mathbb{R} - \{2, -2\}$

en 2 y −2 discontinuidad

$\lim\limits_{x\to 2} \dfrac{x}{x^2-4} = \dfrac{2}{0}$ IND $\lim\limits_{x\to 2^-} f(x) = -\infty$

$\dfrac{1'9}{1'9^2-4} < 0$

$\lim\limits_{x\to 2^+} f(x) = +\infty$ $\quad \nexists \lim\limits_{x\to 2} f(x)$

$\dfrac{2'1}{2'1^2-4} > 0$

$\boxed{x=2}$ discontinuidad inevitable de salto ∞

lo mismo para $x=-2$ $\lim\limits_{x\to -2} f = \dfrac{-2}{0}$ IND

$\lim\limits_{x\to -2^-} f = -\infty$ $\quad \lim\limits_{x\to -2^+} f = +\infty$

$\dfrac{(-1'9)}{(-1'9)^2-4} > 0$

$\dfrac{-2'1}{(-2'1)^2-4} < 0$

$\nexists \lim\limits_{x\to -2} f$ $\quad x=-2$ discont. inev. de salto ∞.

en el resto (su dominio) obviamente es continua

b) $\lim\limits_{x\to -2} f(x) =$ mirar apto a) A.V $x=-2$ y $x=+2$

$\lim\limits_{x\to 2} f(x)$ idem

A. H. $\lim\limits_{x\to \infty} f(x) = 0$ pasa lo mismo en $-\infty$

$\partial P = 1 < \partial Q = 2$ $\quad \lim\limits_{-\infty} f = \lim\limits_{\infty} \dfrac{-x}{x^2-4} = 0$

$\partial P < \partial Q$

$y=0$ AH tanto en $+\infty$ como $-\infty$

c) $\dfrac{1}{f(x)} = \dfrac{x^2-4}{x}$; $m = \lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{\infty} \dfrac{x^2-4}{x^2} = 1$

$\partial P = \partial Q = 2$

$n = \lim\limits_{\infty} f(x) - 1x = \lim\limits_{\infty} \dfrac{x^2-4}{x} - x = \lim\limits_{\infty} \dfrac{\cancel{x^2}-4-\cancel{x^2}}{x} =$

$\lim\limits_{\infty} -\dfrac{4}{x} = \dfrac{-4}{\infty} = \boxed{0}$

$\boxed{y=x}$ A.O. en $+\infty$; por ser F. Alg es simétrica y también es $y=x$ A.O. en $-\infty$

$\boxed{5A}$ d) $h(x) = (f\circ g)(x) = f(g(x)) = f\left(\sqrt{x-1}\right) =$ (3)

$$= \frac{\sqrt{x-1}}{\left(\sqrt{x-1}\right)^2 - 4} = \frac{\sqrt{x-1}}{x-1-4} = \boxed{\frac{\sqrt{x-1}}{x-5}}$$

$$\text{dom } h(x) \rightarrow \begin{cases} x-1 \geq 0 \rightarrow x \geq 1 \\ y \\ x-5 \neq 0 \rightarrow x \neq 5 \end{cases}$$

$\text{dom } h = \underline{[1, 5) \cup (5, +\infty)}$

$\boxed{5B}$ f func. trozos f_1, f_2, f_3 func elementales $\Rightarrow$ continuas en su dominio

$\text{dom } f_1 = \mathbb{R}$

$\text{dom } f_3 = \mathbb{R}$

$\text{dom } f_2 = ?$

$$\begin{array}{c|cc|c} & 1 & 3 & -4 \\ 1 & & 1 & 4 \\ \hline & 1 & 4 & \boxed{0} \end{array} \qquad \begin{array}{c|cc|c} & 1 & -4 & 3 \\ 1 & & 1 & -3 \\ \hline & 1 & -3 & \boxed{0} \end{array}$$

$$\frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}(x-3)} = f_2(x)$$

$f_2(x) = \dfrac{x+4}{x-3}$

$\text{dom } f_2 = \mathbb{R} - \{3\}$

como f_2 está definida para $1 < x < 2$ no hay problema

Ptos de unión

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} ax+1 = a+1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} \frac{x+4}{x-3} = \frac{5}{-2}$$

$a+1 = -\frac{5}{2}$

$a = -\frac{5}{2} - 1 = \frac{-5-2}{2}$

$a = -\frac{7}{2}$

$$\lim_{x\to 2^-} f = \lim_{x\to 2} \frac{x+4}{x-3} = \frac{6}{-1} = -6$$

$$\lim_{x\to 2^+} f = \lim_{x\to 2} x^2 + b = 4 + b$$

$-6 = 4+b$

$b = -6-4 = -10$

FIN

de 23:14 a 23:55 $\rightarrow$ $\underline{41'}$ para todas las opciones

$\rightarrow$ tiene que darles tiempo en 50'