Soluciones (1)

1 $$\left(\frac{\cos^2 x}{\text{sen}^2 x}\right)' = \frac{(\cos^2 x)'\,\text{sen}^2 x - \cos^2 x\,(\text{sen}^2 x)'}{\text{sen}^4 x} =$$

$$= \frac{2\cdot\cos x\,(\cos x)'\cdot\text{sen}^2 x - \cos^2 x\,(2\cdot\text{sen}\,x\cdot(\text{sen}\,x)')}{\text{sen}^4 x} =$$

$$= \frac{2\cos x\,(-\text{sen}\,x)\,\text{sen}^2 x - \cos^2 x\cdot 2\cdot\text{sen}\,x\cdot\cos x}{\text{sen}^4 x} =$$

$$= \frac{-2\cos x\,\text{sen}\,x\;\text{sen}^2 x - 2\,\text{sen}\,x\cos x\;\cos^2 x}{\text{sen}^4 x} \underset{\uparrow}{=}$$

$\text{sen}\,2x = 2\,\text{sen}\,x\cos x$

$$= \frac{-\text{sen}\,2x\cdot\text{sen}^2 x - \text{sen}\,2x\cos^2 x}{\text{sen}^4 x} = \frac{-\text{sen}\,2x\,(\text{sen}^2 x + \cos^2 x)}{\text{sen}^4 x} \underset{\uparrow}{=}$$

$\text{sen}^2 + \cos^2 = 1$

$$= \boxed{\frac{-\text{sen}\,2x}{\text{sen}^4 x}} \text{ ó también } \boxed{\frac{-2\cos x}{\text{sen}^3 x}}$$ ✓ (revisado Photomath)

2 $$\left(e^{2\cos x}\cdot \text{Ln}(x^2+1)\right)' = (e^{2\cos x})'\,\text{Ln}(x^2+1) + e^{2\cos x}\cdot\left(\text{Ln}(x^2+1)\right)' =$$

$$= e^{2\cos x}\cdot(2\cos x)'\cdot\text{Ln}(x^2+1) + e^{2\cos x}\cdot\frac{(x^2+1)'}{x^2+1} =$$

$$= e^{2\cos x}\cdot 2\cdot(-\text{sen}\,x)\,\text{Ln}(x^2+1) + e^{2\cos x}\,\frac{2x}{x^2+1} =$$

$$= e^{2\cos x}\left[-2\,\text{sen}\,x\,\text{Ln}(x^2+1) + \frac{2x}{x^2+1}\right]$$ ✓ verif. Photomath

3 $$\left(\left(x-\sqrt{1-x^2}\right)^2\right)' =$$

$$2\cdot\left(x-\sqrt{1-x^2}\right)\cdot\left(x-\sqrt{1-x^2}\right)' =$$

$$= 2\left(x-\sqrt{1-x^2}\right)\left(1-\left(\sqrt{1-x^2}\right)'\right) =$$

$$= 2\left(x-\sqrt{1-x^2}\right)\cdot\left(1-\frac{(1-x^2)'}{2\sqrt{1-x^2}}\right) = 2\left(x-\sqrt{1-x^2}\right)\left(1-\frac{-2x}{2\sqrt{1-x^2}}\right) =$$

$$= 2\left(x-\sqrt{1-x^2}\right)\left(1+\frac{\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}\right) = \frac{2\left(x-\sqrt{1-x^2}\right)\left(\sqrt{1-x^2}+x\right)}{\sqrt{1-x^2}} =$$

$$= \frac{2\left(x-\sqrt{1-x^2}\right)\left(x+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} = \frac{2\left(x^2-\left(\sqrt{1-x^2}\right)^2\right)}{\sqrt{1-x^2}} =$$

3, continuación

$$= \frac{2\left(x^2-(1-x^2)\right)}{\sqrt{1-x^2}} = \frac{2(x^2-1+x^2)}{\sqrt{1-x^2}} = \boxed{\frac{2(2x^2-1)}{\sqrt{1-x^2}}}\ \checkmark$$

verif. Photomath

4 $$\left(\ln\left(\sqrt{\frac{1+x}{1-x}}\right)\right)' = \left(\frac{1}{2}\left(\ln(1+x)-\ln(1-x)\right)\right)' =$$

PRIMERO ¡¡PROPIEDADES DE LOGARITMOS ANTES DE DERIVAR!!

$$= \frac{1}{2}\left(\frac{1}{1+x} - \frac{-1}{1-x}\right) = \frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1-x}\right) =$$

$$= \frac{1}{2}\left(\frac{1-\cancel{x}+1+\cancel{x}}{1-x^2}\right) = \frac{1}{2}\,\frac{2}{1-x^2} = \boxed{\frac{1}{1-x^2}}\ \checkmark$$

verific. Photomath

5 $$\left(\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)\right)' = \left(\ln\left(\sqrt{1+e^x}-1\right)-\ln\left(\sqrt{1+e^x}+1\right)\right)' =$$

¡PRIMERO propiedades de logaritmos! derivar al final

$$= \frac{\left(\sqrt{1+e^x}-1\right)'}{\sqrt{1+e^x}-1} - \frac{\left(\sqrt{1+e^x}+1\right)'}{\sqrt{1+e^x}+1} = \frac{\left(\sqrt{1+e^x}\right)'}{\sqrt{1+e^x}-1} - \frac{\left(\sqrt{1+e^x}\right)'}{\sqrt{1+e^x}+1} =$$

$$= \frac{\frac{(1+e^x)'}{2\sqrt{1+e^x}}}{\sqrt{1+e^x}-1} - \frac{\frac{(1+e^x)'}{2\sqrt{1+e^x}}}{\sqrt{1+e^x}+1} = \frac{\frac{e^x}{2\sqrt{1+e^x}}}{\sqrt{1+e^x}-1} - \frac{\frac{e^x}{2\sqrt{1+e^x}}}{\sqrt{1+e^x}+1} =$$

$$= \frac{e^x}{2\sqrt{1+e^x}\left(\sqrt{1+e^x}-1\right)} - \frac{e^x}{2\sqrt{1+e^x}\left(\sqrt{1+e^x}+1\right)} =$$

continuación del $\boxed{5}$

$$\overset{\uparrow}{=} \frac{e^x\left(\sqrt{1+e^x}+1\right)-e^x\left(\sqrt{1+e^x}-1\right)}{2\sqrt{1+e^x}\cdot e^x} = (*) =$$

$$\left[\begin{array}{l} \text{mcm } (\sqrt{1+e^x}-1) \text{ y } (\sqrt{1+e^x}+1) = \\ = (\sqrt{1+e^x}-1)(\sqrt{1+e^x}+1) = (\sqrt{1+e^x})^2 - 1 = 1+e^x-1 = \underline{e^x} \end{array}\right]$$

$$= (*) = \frac{e^x\left[\cancel{\sqrt{1+e^x}}+1-\cancel{\sqrt{1+e^x}}+1\right]}{2e^x\sqrt{1+e^x}} = \frac{2e^x}{2e^x\sqrt{1+e^x}} \overset{\uparrow}{=}$$

$e^x \neq 0$ siempre

$$= \boxed{\frac{1}{\sqrt{1+e^x}}}$$ ✓ verificado con Photomath

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$\boxed{6}$ $$\left(\operatorname{Ln}(\operatorname{Ln}x)\right)' = \frac{(\operatorname{Ln}x)'}{\operatorname{Ln}x} = \frac{\frac{1}{x}}{\operatorname{Ln}x} = \boxed{\frac{1}{x\operatorname{Ln}x}}$$

✓ verif. Photomath

$\boxed{7}$ $$\left(\operatorname{Ln}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)\right)' \overset{\uparrow}{=} \left[\frac{1}{2}\left(\operatorname{Ln}(1-\cos x)-\operatorname{Ln}(1+\cos x)\right)\right]' =$$

primero PROP LOGARITMOS antes de derivar

$$= \frac{1}{2}\left(\frac{(1-\cos x)'}{1-\cos x} - \frac{(1+\cos x)'}{(1+\cos x)}\right) \overset{\uparrow}{=} \frac{1}{2}\,\frac{\operatorname{sen}x(1+\cos x)-(-\operatorname{sen}x)(1-\cos x)}{1-\cos^2 x}$$

mcm $(1-\cos x)$ y $(1+\cos x) = (1-\cos x)(1+\cos x)$
$= \underline{1-\cos^2 x}$

$$= \frac{\operatorname{sen}x + \cancel{\operatorname{sen}x\cos x} + \operatorname{sen}x - \cancel{\operatorname{sen}x\cos x}}{2(1-\cos^2 x)} = \frac{2\operatorname{sen}x}{2(1-\cos^2 x)} =$$

$$= \frac{\operatorname{sen}x}{1-\cos^2 x} = \frac{\operatorname{sen}x}{\operatorname{sen}^2 x} \overset{\uparrow}{=} \boxed{\frac{1}{\operatorname{sen}x}}$$ ✓ verific. PhotoMath

siempre que $\operatorname{sen}x \neq 0$

$$\boxed{8} \quad \left(x^{\cos x}\right)' = \cos x \; x^{\cos x - 1} \cdot (x)' + x^{\cos x} (\cos x)' \cdot \ln(x) =$$

$$= \cos x \cdot x^{\cos x - 1} + x^{\cos x} (-\operatorname{sen} x) \ln x =$$

$$= \cos x \; x^{\cos x - 1} - \operatorname{sen} x \; x^{\cos x} \ln x =$$

$$= \boxed{x^{\cos x - 1} \left(\cos x - \operatorname{sen} x \cdot x \cdot \ln x\right)}$$ ✓ verif. Photomath

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$$\boxed{9} \quad \left(\cos x^{\operatorname{sen} x}\right)' = \operatorname{sen} x (\cos x)^{\operatorname{sen} x - 1} \cdot (\cos x)' + (\cos x)^{\operatorname{sen} x} \cdot (\operatorname{sen} x)' \ln \cos x$$

$$= \operatorname{sen} x (\cos x)^{\operatorname{sen} x - 1} (-\operatorname{sen} x) + (\cos x)^{\operatorname{sen} x} \cos x \ln \cos x =$$

$$= -\operatorname{sen}^2 x (\cos x)^{\operatorname{sen} x - 1} + (\cos x)^{\operatorname{sen} x + 1} \ln \cos x =$$

$$= -(\cos x)^{\operatorname{sen} x - 1} \operatorname{sen}^2 x + (\cos x)^{\operatorname{sen} x + 1} \ln(\cos x) =$$

$$= (\cos x)^{\operatorname{sen} x - 1} \left[-\operatorname{sen}^2 x + (\cos x)^2 \ln(\cos x) \right) =$$

$$= \boxed{(\cos x)^{\operatorname{sen} x - 1} \left(-\operatorname{sen}^2 x + \cos^2 x \ln(\cos x) \right)}$$ ✓ verif. Photomath

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$$\boxed{10} \quad f'(3) = \lim_{h \to 0} \frac{f(h+3) - f(3)}{h} = \lim_{h \to 0} \frac{\frac{2}{h+3+1} - \frac{2}{3+1}}{h} =$$

$$= \lim_{h \to 0} \frac{\frac{2}{h+4} - \frac{2}{4}}{h} = \lim_{h \to 0} \frac{\frac{8 - 2(h+4)}{4(h+4)}}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{8} - 2h - \cancel{8}}{4h(h+4)} = \lim_{h \to 0} \frac{-1}{2(h+4)} = \boxed{-\frac{1}{8}}$$