EX. REPASO

① $A(1,-2,2)$

$$s: \begin{cases} 2x-y+z=8 \\ x-y+2z=9 \end{cases} \qquad r \parallel s$$

$$r \begin{cases} A \\ \vec{v_r} = \vec{v_s} \end{cases}$$

$$\vec{v_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = (-1,3,-1)$$

Pb que $r: \dfrac{x-1}{-1} = \dfrac{y+2}{3} = \dfrac{z-2}{-1}$

ó $r: \begin{cases} x = 1-t \\ y = -2+3t \\ z = 2-t \end{cases}$

②

$$\pi \begin{cases} A(-1,0,2) \\ \vec{v_r} \\ \vec{n_\alpha} \end{cases} \qquad \alpha: 2x+y+z-5=0$$

$$\vec{n_\alpha} = (2,1,1)$$

$$r: \frac{x-1}{2} = y+1 = \frac{z}{0}$$

$$\vec{v_r} = (2,1,0)$$

$$\begin{vmatrix} x+1 & y & z-2 \\ 2 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 0$$

$$\pi \equiv -x+2y-1=0$$

$$\pi \begin{cases} \vec{n_\alpha}(2,1,1) \\ \vec{v_r}(2,1,0) \\ A(-1,0,2) \end{cases}$$

(3) Coplanarios

$A(+1,-1,0)$ $B(2,2,1)$, $C(1,-2,1)$, $D(0,-1,2)$?

A,B,C,D coplanarios $\Leftrightarrow$ $\vec{AB}, \vec{AC}, \vec{AD}$ coplanarios

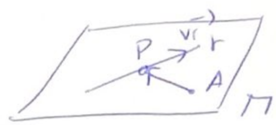
$\Leftrightarrow \det[\vec{AB}, \vec{AC}, \vec{AD}] = 0$

$\vec{AB} = (1,3,1)$
$\vec{AC} = (0,-1,1)$
$\vec{AD} = (-1,0,2)$

$\rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 2 \end{vmatrix} \neq 0 \rightarrow$ Non son coplanarios

(4)

$r: \begin{cases} 2x+y+z=0 \\ y-z=0 \end{cases}$



$A(1,1,0)$

$\pi \begin{cases} A(1,1,0) \\ \vec{v_r} \\ \vec{AP} \end{cases}$

$A \in r$? $1-0 \neq 0$ Non
$A \notin r$

Busco un pto de r
fago $x=0$ e queda:

$$\begin{aligned} y+z&=0 \\ y-z&=0 \\ \hline 2y&=0 \rightarrow y=0 \\ &\rightarrow z=0 \end{aligned}$$

$P_r = (0,0,0)$

Entón π queda determinado por $\begin{cases} A(1,1,0) \\ \vec{AP_r} = (-1,-1,0) \\ \vec{v_r} = (-1,1,1) \end{cases}$

$$\vec{v_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = (-2,2,2) \rightarrow \vec{v_r} = (-1,1,1)$$

$$\pi: \begin{vmatrix} x-1 & y-1 & z \\ -1 & -1 & 0 \\ -1 & 1 & 1 \end{vmatrix} = 0$$

$-(x-1) - z - z + (y-1) = 0$

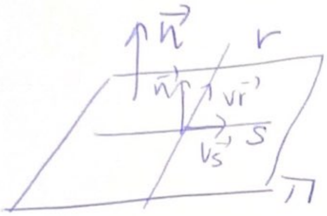
$-x + y - 2z = 0$

$\underline{\pi: x - y + 2z = 0}$

⑤ calcule Ec recta s tq

$s \subset \pi: x - 2y + z = 4$

$s \perp r$,, $r: \begin{cases} x - y - z + 1 = 0 \\ 3x - y + z - 3 = 0 \end{cases}$



$\vec{v_s} = \vec{v_r} \times \vec{n}$

$\pi: x - 2y + z - 4 = 0$

$\vec{n_\pi} = (1, -2, 1)$

$r: \begin{cases} x - y - z + 1 = 0 \\ 3x - y + z - 3 = 0 \end{cases}$

$\vec{v_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ 3 & -1 & 1 \end{vmatrix} = (-2, -4, 2)$ ou

$\vec{v_r} = (-1, -2, 1)$

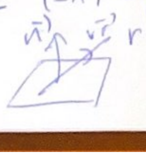
Então

$\vec{v_s} = \vec{v_r} \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -2 & 1 \\ 1 & -2 & 1 \end{vmatrix} = (0, -2, -4)$

ou tamén $\vec{v_s} = (0, +1, +2)$

Falta pto. Como $s \subset \pi$ e $s \perp r$ → s pasa polo pto de intersección de r con π. Calculo pto intersección

$\left\{ \begin{array}{l} * \text{ r e } \pi \text{ son secantes} \\ \vec{v_r} \cdot \vec{n} = (-1, -2, 1) \cdot (1, -2, 1) = -1 + 4 + 1 = 4 \neq 0 \end{array} \right\}$



Paso r a paramétricas:

$$r: \begin{cases} x-y = z-1 \\ 3x-y = 3-z \end{cases}$$

$$\begin{array}{l} x-y=\alpha-1 \\ -3x+y=-3+\alpha \\ \hline -2x=-4+2\alpha \\ \underline{x=2-\alpha} \end{array} \rightarrow \begin{array}{l} y=x-\alpha+1 \\ y=2-\alpha-\alpha+1 \\ y=3-2\alpha \end{array}$$

$$r: \begin{cases} x=2-t \\ y=3-2t \\ z=t \end{cases} \qquad \pi: x-2y+z-4=0$$

$\underline{r \cap \pi}$

$$(2-t)-2(3-2t)+t-4=0$$
$$2-t-6+4t+t-4=0$$
$$4t-8=0 \rightarrow \underline{\underline{t=2}} \Rightarrow$$

$\Rightarrow \underline{I=(0,-1,2)}$

Por lo tanto, $s \begin{cases} I(0,-1,2) \\ \vec{v_s}(0,1,2) \end{cases} \rightarrow s: \dfrac{x}{0}=\dfrac{y+1}{1}=\dfrac{z-2}{2}$

ov

$$s: \begin{cases} x=0 \\ y=-1+t \\ z=2+2t \end{cases}$$

(6)

$$M^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & a-1 \\ 2 & 1 & a & a \\ 1 & a & 1 & 1 \end{array}\right), \quad M = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & a \\ 1 & a & 1 \end{pmatrix}}_{M}$$

$$|M| = -a^2 + 3a - 2 = 0 \Leftrightarrow \begin{cases} a = 2 \\ a = 1 \end{cases}$$

- Se $a \neq 2, 1 \Rightarrow |M| \neq 0 \Rightarrow rg\,M = 3 = rg\,M^* \Rightarrow$ SCD
  $\Rightarrow$ Córtanse nun punto

- Se $a = 2$
  $|M| = 0$
  Como $\exists \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \neq 0 \Rightarrow rg\,M = 2$
  Calculo $rg\,M^*$

$$M^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 2 \\ 1 & 2 & 1 & 1 \end{array}\right)$$

$c_1 = c_3 = c_4$

$\Rightarrow rg\,M^* = 2$

- Como $rg\,M = 2 = rg\,M^* <$ nº inc $\Rightarrow$ SCI inf sol.

Córtanse nunha recta
- ou ben 2 son = e o 3º córtaos
- ou 3 distintos e córtanse nunha recta

Como:

$\pi_1: \ x + y + z = 1$

$\pi_2: \ 2x + y + 2z = 2$

$\pi_3: \ x + 2y + z = 1$

Son os 3 distintos e córtanse nunha recta

Se $a=1$

$|M|=0$

$$M^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right)$$

Como $\exists \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \neq 0$

$C_2 = C_3$.

$rg\, M = 2$

$rg\, M^*$?

$$\exists \begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \neq 0 \Rightarrow rg\, M^* = 3$$

Como $rg\, M = 2 \neq 3 = rg\, M^* \Rightarrow$ S.I $\Rightarrow$

$\Rightarrow \begin{cases} \text{os 2 son // e o 3º é secante con eles} \\ \text{os 3 non // e secantes 2 a 2} \end{cases}$

$$\left.\begin{array}{l} \pi_1: x+y+z=0 \\ \pi_2: 2x+y+z=1 \\ \pi_3 = x+y+z=1 \end{array}\right] \quad \pi_1 // \pi_3$$

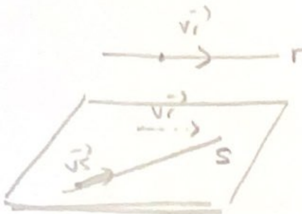
Polo tto 2 son paralelos e o 3º, π_2, córtaos

En cq caso, os 3 non se cortan en nada común.

⑦ Calcula o plano $\pi \parallel r$ e que contén a s, sendo

$$r: \frac{x}{2} = \frac{y-2}{-1} = \frac{z-1}{3} \quad \text{e}$$

$$s: \begin{cases} x = 2-2\lambda \\ y = 2\lambda \\ z = 5-6\lambda \end{cases}$$



1º) comprobamos que r e s teñen distinta dirección

$\vec{v_r}\,(2,-1,3)$
$\vec{v_s}\,(-2,2,-6)$

Non son proporcionais polo que ou córtanse ou crúzanse, teñen distinta dirección.

Como $\pi \supset s$, collemos un pto $P_s \in s$, que tamén $P_s \in \pi$. $P_s = (2,0,5)$

π queda determinado por $\begin{cases} P_s = (2,0,5) \\ \vec{v_r} = (2,-1,3) \\ \vec{v_s} = (-2,2,-6) \end{cases}$

$$\pi: \begin{vmatrix} x-2 & y & z-5 \\ 2 & -1 & 3 \\ -2 & 2 & -6 \end{vmatrix} = 0 \Rightarrow \underline{\pi: 3y + z - 5 = 0}$$

⑧ Sabendo que $r \parallel s$, calcula o plano que as contén, sendo:

$$r: \frac{x+5}{3} = \frac{y-1}{2} = \frac{z-2}{-4}$$

$$s: \begin{cases} 2x+y+2z+5=0 \\ 2x-y+z+11=0 \end{cases}$$

$\vec{v_r} = (3,2,-4)$

$P_r = (-5,1,2)$

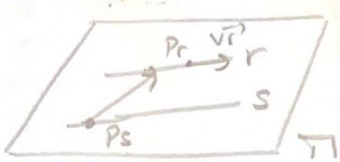
$$\vec{v_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = (3,2,-4)$$

Calculamos un pto $P_S \in s$ facendo $z=0$

$$\begin{cases} 2x+y=-5 \\ 2x-y=-11 \end{cases}$$

$\Rightarrow x=-4, y=3, z=0$

$P_S(-4,3,0)$



$\vec{P_rP_S}\,(+1,2,-2)$

π queda determinado por $\begin{cases} \vec{v_r} \\ \vec{P_rP_S} \\ P_r \end{cases}$

$$\pi \equiv \begin{vmatrix} x+5 & y-1 & z-2 \\ 3 & 2 & -4 \\ 1 & 2 & -2 \end{vmatrix} = 0$$

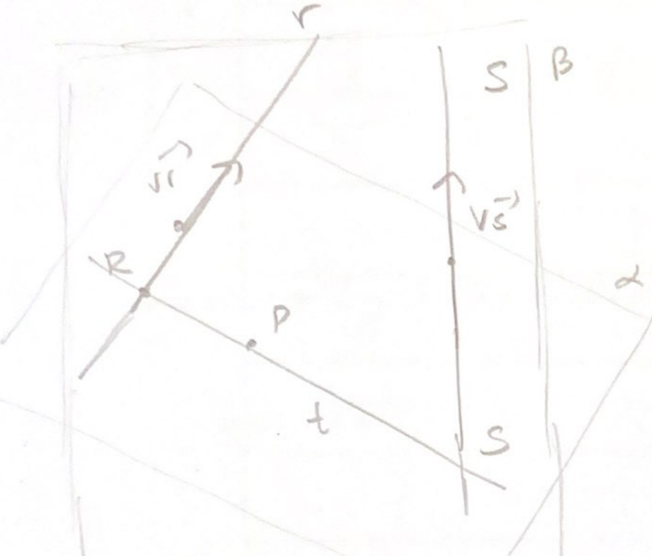
$\Rightarrow \pi: 2x+y+2z+5=0$

9. Calcula a recta que pasa por $P(1,0,-1)$ e corta a r e a s, sendo:

$$r: \begin{cases} 3x+2y-z+1=0 \\ 2x-y+z+4=0 \end{cases} \qquad s: \begin{cases} x=3+t \\ y=t \\ z=1+t \end{cases}$$

$S(3,0,1)$

$\vec{v_s}(1,1,1)$



Paso r a paramétricas tomando $x=t$

$2y-z=-1-3t$

$-y+z=-4-2t$

$y=-5-5t$

$z=-9-7t$

$$r: \begin{cases} x=t \\ y=-5-5t \\ z=-9-7t \end{cases}$$

$R(0,-5,-9)$

$\vec{v_r}(1,-5,-7)$

A rectas r e s

Posición relativa?

$\vec{RS}(3,5,10)$

$M=\begin{pmatrix} 1 & 1 & 1 \\ 1 & -5 & -7 \end{pmatrix}$ $rg\,M=2$

$M^*=\begin{pmatrix} 1 & 1 & 1 \\ 1 & -5 & -7 \\ 3 & 5 & 10 \end{pmatrix}$ $rg\,M^*=3$

r e s crúzanse

A recta t conséguese como a intersección de 2 planos: α que contén a r e pasa por P, e β que contén a s e pasa por P.

$\alpha: \begin{cases} \vec{v_r} = (1,-5,-7) \\ P = (1,0,-1) \\ \vec{PR} = (1,5,8) \end{cases}$ $\beta: \begin{cases} \vec{v_s} = (1,1,1) \\ P = (1,0,-1) \\ \vec{PS} = (2,0,2) \end{cases}$

$\alpha \equiv \begin{vmatrix} x-1 & y & z+1 \\ 1 & -5 & -7 \\ 1 & 5 & 8 \end{vmatrix} = 0$ $\beta \equiv \begin{vmatrix} x-1 & y & z+1 \\ 1 & 1 & 1 \\ 2 & 0 & 2 \end{vmatrix} = 0$

$\alpha: -x - 3y + 2z + 3 = 0$ $\beta: x - z - 2 = 0$

Polo tanto:

$t: \begin{cases} x + 3y - 2z - 3 = 0 \\ x - z - 2 = 0 \end{cases}$

Outra forma

Se t corta a r, entón $\vec{v_r}$, $\vec{v_t}$ e $\vec{PR}$ serán coplanarios. Análogamente, se t corta a s, $\vec{v_s}$, $\vec{v_t}$ e $\vec{PS}$ coplanarios.

$\vec{v_t} = (x,y,z)$

Como $\vec{v_r}$, $\vec{v_t}$ e $\vec{PR}$ coplanarios: $\begin{vmatrix} x & y & z \\ 1 & -5 & -7 \\ 1 & 5 & 8 \end{vmatrix} = 0$

$\Rightarrow 5x + 15y - 10z = 0$

Como $\vec{v_s}$, $\vec{v_t}$ e $\vec{PS}$ coplanarios: $\begin{vmatrix} x & y & z \\ 2 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow -2x + 2z = 0$

Resolvendo sistema: $\begin{matrix} x = z \\ x = 3y \end{matrix}$ $\Rightarrow \vec{v_t}\ (3,1,3)$

Con $P(1,0,-1)$ e $\vec{v_t}(3,1,3) \Rightarrow t: \frac{x-1}{3} = \frac{y}{1} = \frac{z+1}{3}$

(10) Ecuación do plano que contén á recta $r: \begin{cases} x=1 \\ y=3 \end{cases}$ e corta ao plano $\alpha: x+y+z=1$ unha recta paralela ao plano XY.

$\alpha: x+y+z-1=0$

$s \parallel \pi_{xy}$

plano XY $\Rightarrow \pi_{xy}: z=0$

$\vec{n}_{xy} = (0,0,1)$

$r: \begin{cases} x=1 \\ y=3 \end{cases}$

$P_r(1,3,0)$

$\vec{v_r}(0,0,1)$

O plano pedido contén a r, polo tanto, $r \subset \pi$,

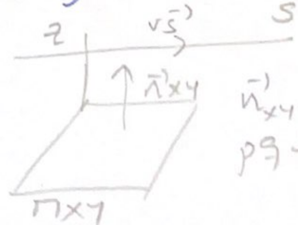
$P_r = (1,3,0) \in \pi$, $\vec{v_r} = (0,0,1)$ será un dos seus vectores directores.

Como $s \subset \pi$, o outro vector director será $\vec{v_s}$.

Como:

$$\left.\begin{array}{l} \vec{v_s} \perp \vec{n_\alpha} \\ \vec{v_s} \perp \vec{n}_{xy} \end{array}\right\} \Rightarrow \begin{array}{l} \vec{v_s} \perp (1,1,1) \\ \vec{v_s} \perp (0,0,1) \end{array} \Rightarrow$$

$$\Rightarrow \vec{v_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = (1,-1,0)$$



$\vec{n}_{xy} \perp \vec{v_s}$ pq $s \parallel \pi_{xy}$

Polo tanto:

$$\pi \begin{cases} P(1,3,0) \\ \vec{v_r}(0,0,1) \\ \vec{v_s}(1,-1,0) \end{cases}$$

$$\pi \equiv \begin{vmatrix} x-1 & y-3 & z \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 0 \Rightarrow \pi: x+y-4=0$$