Ficha I Ecuaciones y Sist. trigonométricas

[1] $\boxed{2\,\text{sen}x\cos x = \text{sen}x} \iff 2\,\text{sen}x\cos x - \text{sen}x = 0$

$\iff \text{sen}x\,(2\cos x - 1) = 0 \left\langle \begin{array}{l} \text{sen}x = 0 \\ \text{ó} \\ 2\cos x = 1 \end{array}\right.$

* Si $\text{sen}x = 0 \iff x = 0, \pi, 2\pi, 3\pi, \dots = 0°, 180°, 360°, \dots$

$S_1 = \boxed{\{k\pi \,/\, k\in\mathbb{Z}\}} = \boxed{\{k\cdot 180° \,/\, k\in\mathbb{Z}\}}$

* Si $2\cos x = 1$ (con $\text{sen}x \neq 0$) $\iff \cos x = \frac{1}{2} \iff x = \left\{ \begin{array}{l} 60°, 60°+360° \dots \\ -60°, -60°+360° \dots \end{array}\right\}$

$\boxed{S_2 = \{60° + k360° \,/\, k\in\mathbb{Z}\} \cup \cup\{-60° + k360°\}}$

en radianes

$x \in \boxed{\left\{\frac{\pi}{3} + 2k\pi \,/\, k\in\mathbb{Z}\right\} \cup \left\{-\frac{\pi}{3} + 2k\pi \,/\, k\in\mathbb{Z}\right\}}$

$\boxed{S = S_1 \cup S_2}$

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[2] $\boxed{\cos 3x + \cos x = \cos 2x}$ (lioso, quizás demasiado para este grupo (no poner en examen) además muy largo)

Relaciones ángulos sumas y dobles

$\text{sen}\,2\alpha = 2\,\text{sen}\alpha\cos\alpha$ | $\text{sen}(\alpha+\beta) = \text{sen}\alpha\cos\beta + \cos\alpha\,\text{sen}\beta$

$\cos 2\alpha = \cos^2\alpha - \text{sen}^2\alpha$ | $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \text{sen}\alpha\,\text{sen}\beta$

$\cos(3\alpha) = \cos(2\alpha+\alpha) = \cos 2\alpha\cdot\cos\alpha - \text{sen}2\alpha\,\text{sen}\alpha =$

$= (\cos^2\alpha - \text{sen}^2\alpha)\cos\alpha - 2\,\text{sen}\alpha\cos\alpha\,\text{sen}\alpha =$

$= \cos^3\alpha - \text{sen}^2\alpha\cos\alpha - 2\,\text{sen}^2\alpha\cos\alpha =$

$= \cos^3\alpha - 3\,\text{sen}^2\alpha\cos\alpha = \cos\alpha(\cos^2\alpha - 3\,\text{sen}^2\alpha)$

Usando esto en la igualdad del principio:

$\cos\alpha(\cos^2\alpha - 3\,\text{sen}^2\alpha) + \cos\alpha = \cos^2\alpha - \text{sen}^2\alpha$

$\cos\alpha(\cos^2\alpha - 3\,\text{sen}^2\alpha) + \cos\alpha - \cos^2\alpha + \text{sen}^2\alpha = 0$

(1) $\cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha + \cos\alpha - \cos^2\alpha + \text{sen}^2\alpha = 0$

Además sabemos que (2) $\cos^2\alpha + \text{sen}^2\alpha = 1$ siempre

llamo $a = \cos\alpha$, $b = \text{sen}\,\alpha$ $\Rightarrow$ sustituyendo en (1) y (2)

(1) $a^3 - 3ab^2 + a - a^2 + b^2 = 0$ $\quad a(a^2 \cancel{- 3b^2 + 1 - a}) + b^2 = 0$

(2) $a^2 + b^2 = 1$

$\Downarrow$

$b^2 = 1 - a^2$ sustituyo en (1)

$$a^3 - 3a(1-a^2) + a - a^2 + 1 - a^2 = 0$$

$$a^3 - 3a + 3a^3 + a - a^2 + 1 - a^2 = 0$$

$$4a^3 - 2a^2 - 2a + 1 = 0$$

Ruffini

posibles raíces $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$

	4	-2	-2	1
1/2		2	0	-1
	4	0	-2	$\boxed{0}$

$4a^2 - 2 = 0 \Leftrightarrow$ $\quad a = \pm\frac{\sqrt{2}}{2}$

$(4a^2 - 2 = (2a - \sqrt{2})(2a + \sqrt{2})$ $\quad a = \frac{\sqrt{2}}{2}$ $\quad a = -\frac{\sqrt{2}}{2}$

$4a^2 = 2$

$a^2 = \frac{2}{4}$

$a^2 = \frac{1}{2}$ $\longrightarrow$ Son 2 formas $\neq$ de llegar al mismo resultado elegir el q. mejor entendáis

$a = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$a = 1/2 \Rightarrow b^2 = 1 - a^2$; $b^2 = 1 - \frac{1}{4}$; $b^2 = \frac{3}{4}$; $\boxed{b = \pm\frac{\sqrt{3}}{2}}$

$a = \pm\frac{\sqrt{2}}{2} \Rightarrow b^2 = 1 - a^2$; $b^2 = 1 - \frac{2}{4}$; $b^2 = \frac{2}{4}$; $b^2 = \frac{1}{2}$; $b = \frac{1}{\pm\sqrt{2}}$

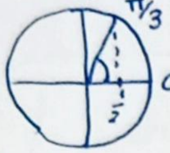
$\Rightarrow b = \pm\frac{\sqrt{2}}{2}$

Resumen de Soluciones $\quad a=\cos x,\ b=\operatorname{sen} x$

1ª) $a=\frac{1}{2},\ b=\frac{\sqrt{3}}{2}$ (1er cuadrante) $\Leftrightarrow \{x=60^\circ+k\,360^\circ,\ k\in\mathbb{Z}\}$

$S_1=\{x=\frac{\pi}{3}+2k\pi\ ,\ k\in\mathbb{Z}\}$

o lo que es lo mismo:
60° + nº entero de vueltas



2ª) $a=\frac{1}{2},\ b=-\frac{\sqrt{3}}{2}$ (4º cuadrante) $\Leftrightarrow \{x=-60^\circ+k\,360^\circ,\ k\in\mathbb{Z}\}$

o en radianes:

$S_2=\{-\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}\}$

3º) $a=\frac{\sqrt{2}}{2}\ ,\ b=\frac{\sqrt{2}}{2}$ (1º cuadrante) $\Leftrightarrow \{x=45^\circ+k\,360^\circ/k\in\mathbb{Z}\}$

en radianes:

$S_3=\{\frac{\pi}{4}+2k\pi/k\in\mathbb{Z}\}$

4ª) $a=\frac{\sqrt{2}}{2}\ ,\ b=-\frac{\sqrt{2}}{2}$ (4º cuadrante)

$S_4=\{x=45^\circ+k\cdot 360^\circ/k\in\mathbb{Z}\}$,

en radianes

$S_4=\{-\frac{\pi}{4}+2k\pi/k\in\mathbb{Z}\}$

5º) $a=-\frac{\sqrt{2}}{2}\ ,\ b=\frac{\sqrt{2}}{2}$ (2º cuadrante)

$S_5=\{135^\circ+k\,360^\circ/k\in\mathbb{Z}\}$

en radianes

$S_5=\{\frac{3\pi}{4}+2k\pi/k\in\mathbb{Z}\}$

$90^\circ+45^\circ=135^\circ$
$\parallel$
$\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

6º) $a=-\frac{\sqrt{2}}{2}\ ,\ b=-\frac{\sqrt{2}}{2}$ (3º cuadrante)

$S_6=\{225^\circ+k\cdot 360^\circ/k\in\mathbb{Z}\}$

en radianes

$S_6=\{\frac{5\pi}{4}+2k\pi\ /k\in\mathbb{Z}\}$

$180^\circ+45^\circ=225^\circ$

$\pi+\frac{\pi}{4}=\frac{5\pi}{4}$

Resumen las soluciones S_3, S_4, S_5, S_6 se pueden escribir como 45° + nº entero $\cdot 90^\circ$ i.e. $\{45^\circ+k\,90^\circ/k\in\mathbb{Z}\}=S_7$

radianes $\frac{\pi}{4}+k\frac{\pi}{2}=\frac{\pi+2k\pi}{4}=\frac{(2k+1)}{4}\pi=\{(2k+1)\frac{\pi}{4}\ |k\in\mathbb{Z}\}=S_7$

Soluciones $=S_1\cup S_2\cup S_7$

Comprobación (No es necesaria pero ayuda a comprobar los resultados y a entenderlos)

1) si $x = 60°$ $\Rightarrow$ $\cos(3 \cdot 60°) + \cos 60° = -1 + \frac{1}{2} = -\frac{1}{2} = \cos 120°$

sí//

2) si $x = -60°$ $\Rightarrow$ $\cos(3 \cdot (-60°)) + \cos(-60°) = -1 + \frac{1}{2} = -\frac{1}{2} = -\cos 60°$

sí//

3) si $x = 45°$ $\Rightarrow$ $\cos(3 \cdot 45°) + \cos 45° = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0 = \cos 90°$

4) si $x = -45°$ $\Rightarrow$ $\cos(3(-45°)) + \cos(-45°) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0$ sí//

5) si $x = 135°$ $\Rightarrow$ $\cos(3 \cdot 135°) + \cos(135°) =$

$= \cos(45°) + \cos(135°) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0 =$

$= \cos 270°$

sí//

$$\begin{array}{r} 135 \\ \times 3 \\ \hline 405° \end{array} \qquad \begin{array}{r} 405° \\ -360° \\ \hline 45° \end{array}$$

6) si $x = 225°$ $\Rightarrow$ $\cos(3 \cdot 225°) + \cos(225°) =$

$= \cos(315°) + \cos(225°) =$

$= \cos(-45°) + \cos(225°) =$

$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0 = \cos(90°) = \cos(450°)$

sí//

$$\begin{array}{r} 225 \\ \times 3 \\ \hline 675° \end{array} \qquad \begin{array}{r} 675° \\ -360° \\ \hline 315° \end{array}$$

$$\begin{array}{r} -360° \\ +315° \\ \hline -45° \end{array}$$

$$\begin{array}{r} 225° \\ \times 2 \\ \hline 450° \end{array} \qquad \begin{array}{r} +450° \\ -360° \\ \hline 90° \end{array}$$

—— • ——

Ej $\boxed{3}$ $\cos^2 x + 2\,\text{sen}\,x = 2$

añadimos $\cos^2 x + \text{sen}^2 x = 1$ que siempre se cumple } formamos un sistema

cambiamos la variable para q. sea más fácil de ver

$a = \cos x$
$b = \text{sen}\,x$ $\Rightarrow$ $\left.\begin{array}{l} a^2 + 2b = 2 \\ a^2 + b^2 = 1 \end{array}\right\}$ resolvemos el sistema sustitución $a^2 = 1 - b^2$

$1 - b^2 + 2b = 2$; $-b^2 + 2b + 1 - 2 = 0$; $-b^2 + 2b - 1 = 0$;

$b^2 - 2b + 1 = 0$ ec. 2° grado

$b = \dfrac{2 \pm \sqrt{4-4}}{2} = \boxed{1}$ solc. doble

como $a^2 = 1 - b^2$
$a^2 = 1 - 1 = 0$
$\Rightarrow a = 0$.

Solución: $a = 0$, $b = 1$

como $a = \cos x$
$b = \text{sen}\,x$

$\cos x = 0$
$\text{sen}\,x = 1$

$90° = \frac{\pi}{2}$

$\boxed{x = 90° + \text{n}^{\text{o}}\ \text{entero de vueltas}} =$

$= \boxed{\{90° + k \cdot 360° \mid k \in \mathbb{Z}\}}$

en radianes $= \left\{\frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z}\right\} =$

$= \left\{\frac{\pi + 4k\pi}{2} \mid k \in \mathbb{Z}\right\}$ $= \boxed{\left\{\frac{4k+1}{2}\pi \mid k \in \mathbb{Z}\right\}}$

Se puede escribir de las 3 formas y puntúa lo mismo

——— o ———

Ej $\boxed{4}$ $\text{sen}\,x + \cos x = 1$

añadimos $\cos^2 x + \text{sen}^2 x = 1$ q. siempre se cumple y formamos el sistema $\left.\begin{array}{l} \text{sen}\,x + \cos x = 1 \\ \text{sen}^2 x + \cos^2 x = 1 \end{array}\right\}$

cambiamos la variable para q. resulte más cómodo

$\left.\begin{array}{l} a = \cos x \\ b = \text{sen}\,x \end{array}\right\}$ $\Rightarrow$ $\left.\begin{array}{l} b + a = 1 \\ b^2 + a^2 = 1 \end{array}\right\}$ sustitución $b = 1 - a$

$(1-a)^2 + a^2 = 1 \Leftrightarrow$

$\Leftrightarrow 1 - 2a + a^2 + a^2 = 1 \Leftrightarrow 2a^2 - 2a = 0 \Leftrightarrow 2a(a-1) = 0$

Ec. 2° incompleta

soluciones: $a = 0 \Rightarrow b = 1$
o
$a = 1 \Rightarrow b = 0$

pág 6 **Soluciones:**

$\boxed{90° + \text{n° entero de vueltas}}$

1º) $a = 0$ y $b = 1$

$\cos x = 0$ y $\operatorname{sen} x = 1$

Sí

también se puede escribir como:

$$\boxed{S_1 = \{90° + k\,360° / k \in \mathbb{Z}\}}$$

o en radianes:

$$S_1 = \left\{ \frac{\pi}{2} + 2k\pi / k \in \mathbb{Z} \right\} = \left\{ \frac{\pi + 4k\pi}{2} / k \in \mathbb{Z} \right\} =$$

$$= \boxed{\left\{ \frac{4k+1}{2}\pi / k \in \mathbb{Z} \right\}}$$

Cualquiera de las 3 formas vale

2º) $a = 1$ y $b = 0$

$\cos x = 1$ y $\operatorname{sen} x = 0$

$$S_2 = \{0° + \text{n° entero de vueltas}\} = \{k \cdot 360° / k \in \mathbb{Z}\} =$$

$$= \{2k\pi / k \in \mathbb{Z}\}$$

Soluciones $= \boxed{S_1 \cup S_2}$

$\boxed{5}$ $\cos x + \sqrt{3}\operatorname{sen} x = 0$

añadimos como siempre $\operatorname{sen}^2 x + \cos^2 x = 1$ y hacemos el cambio de variable $a = \cos x$, $b = \operatorname{sen} x$. Nos queda el sistema:

$$\left.\begin{array}{l} a + \sqrt{3}\,b = 0 \\ a^2 + b^2 = 1 \end{array}\right\} \quad \text{Sustitución: } a = -\sqrt{3}\,b$$

$$(-\sqrt{3}b)^2 + b^2 = 1; \quad 3b^2 + b^2 = 1;$$

$$4b^2 = 1 \quad ; \quad b^2 = \frac{1}{4} \quad ; \quad b = \pm\frac{1}{2}$$

ec. 2° incompleta

$90° + 60° = 150° = \frac{\pi}{2} + \frac{\pi}{3} = \frac{3\pi + 2\pi}{6} = \frac{5\pi}{6}$

1º) si $b = \frac{1}{2} \Rightarrow a = -\frac{\sqrt{3}}{2}$;

2º cuadrante

2º) si $b = -\frac{1}{2} \Rightarrow a = \frac{\sqrt{3}}{2}$;

$-30° = -\frac{\pi}{6}$ 4º cuadrante

$$\text{Soluciones} = \{150° + k \cdot 360° / k \in \mathbb{Z}\} \cup \{-30° + k\,360° / k \in \mathbb{Z}\} \quad \text{en grados}$$

$$\left\{ \frac{5\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\} \quad \text{en radianes}$$

pág 7 Ej $\boxed{6}$ $\quad 4 \operatorname{sen}\left(x-\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)=\sqrt{3}$

(para este curso y este grupo se deja hacer en grados) $\frac{\pi}{6}=30^\circ$

$$4 \operatorname{sen}(x-30^\circ)\cos(x-30^\circ)=\sqrt{3}$$

Cambio de variable: $\boxed{\alpha = x-30^\circ}$ i.e. $x=\alpha+30^\circ$

$4 \operatorname{sen}\alpha \cos\alpha = \sqrt{3}$; usamos ángulo doble

$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$

$\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha$ ← ésta

$2(2\operatorname{sen}\alpha\cdot\cos\alpha)=\sqrt{3}$

$2\cdot\operatorname{sen}2\alpha=\sqrt{3}$; $\operatorname{sen}2\alpha=\frac{\sqrt{3}}{2}$ tenemos dos posibilidades (esto exigirá comprobación para ver cuál es la correcta)

$$\operatorname{sen}2\alpha=\frac{\sqrt{3}}{2} \begin{cases} \cos 2\alpha = \frac{1}{2} & (1^{er} \text{ cuadrante}) \\ \text{ó} & \\ \cos 2\alpha = -\frac{1}{2} & (2^{\circ} \text{ cuadrante}) \end{cases}$$

1º) si $\operatorname{sen}2\alpha=\frac{\sqrt{3}}{2}$ y $\cos 2\alpha=\frac{1}{2}$ $\Rightarrow 2\alpha=60^\circ \Rightarrow \alpha=30^\circ$

$\Rightarrow x = 30^\circ+30^\circ=\boxed{60^\circ}$

Comprobamos: $4(\operatorname{sen}(60^\circ-30^\circ)\cos(60^\circ-30^\circ)$

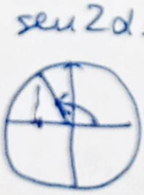
$=4\operatorname{sen}30^\circ\cdot\cos30^\circ=$

$4\cdot\frac{1}{2}\frac{\sqrt{3}}{2}=\sqrt{3}$

sí // VALE

2º) si $\operatorname{sen}2\alpha=\frac{\sqrt{3}}{2}$ y $\cos 2\alpha=-\frac{1}{2}$ $\Rightarrow 2\alpha=120^\circ \Leftrightarrow \alpha=60^\circ$

$\Leftrightarrow x=30^\circ+60^\circ=\boxed{90^\circ}$

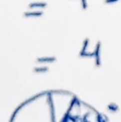


Comprobamos:

$4\operatorname{sen}(90^\circ-30^\circ)\cos(90^\circ-30^\circ)=$

$=4\operatorname{sen}60^\circ\cos60^\circ=$

$=4\frac{\sqrt{3}}{2}\cdot\frac{1}{2}=\sqrt{3}$ sí // VALE



pág 8 Las soluciones serán :

$S_1 = \boxed{\{60° + \text{n° entero de vueltas}\}} = \boxed{\{60° + k\,360° \,/\, k \in \mathbb{Z}\}} =$

$= \{\frac{\pi}{3} + 2k\pi \,/\, k \in \mathbb{Z}\} = \{\frac{\pi + 6k\pi}{3} \,/ k \in \mathbb{Z}\} =$

$60° = \frac{\pi}{3}$ $\qquad = \boxed{\{\frac{6k+1}{3}\pi \,/\, k \in \mathbb{Z}\}}$

(Se pueden escribir en grados o en radianes, pero el 10 sólo se pone si se escribe en radianes)

$S_2 = \boxed{\{90° + \text{n° entero de vueltas}\}} = \boxed{\{90° + k360° \,/ k \in \mathbb{Z}\}}$

$= \{\frac{\pi}{2} + 2k\pi \,/ k \in \mathbb{Z}\} = \{\frac{\pi + 4k\pi}{2} \,/ k \in \mathbb{Z}\} =$

$90° = \frac{\pi}{2}$ rad $\qquad = \boxed{\{\frac{4k+1}{2}\pi \,|\, k \in \mathbb{Z}\}}$

Soluciones $= \boxed{S_1 \cup S_2}$

Ej 7

$\cos 2x + \operatorname{sen} x = 4 \operatorname{sen}^2 x$

razones ángulo doble $\quad \cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$

sustituyendo : $\quad \cos^2 x - \operatorname{sen}^2 x + \operatorname{sen} x = 4 \operatorname{sen}^2 x$

$\cos^2 x - \operatorname{sen}^2 x + \operatorname{sen} x - 4 \operatorname{sen}^2 x = 0$

$\boxed{\cos^2 x - 5 \operatorname{sen}^2 x + \operatorname{sen} x = 0}$

añado $\cos^2 x + \operatorname{sen}^2 x = 1$ y cambio variable $a = \cos x$
$b = \operatorname{sen} x$

queda el sistema :

$$\left.\begin{array}{r} a^2 - 5b^2 + b = 0 \\ a^2 + b^2 = 1 \end{array}\right\}$$

por sustitución : $a^2 = 1 - b^2$

$1 - b^2 - 5b^2 + b = 0$

$-6b^2 + b + 1 = 0 \iff 6b^2 - b - 1 = 0$

$$b = \frac{1 \pm \sqrt{1 - 4 \cdot 6 \cdot (-1)}}{12} = \frac{1 \pm 5}{12} = \begin{cases} \frac{1}{2} \Rightarrow a^2 = 1 - \frac{1}{4} = \frac{3}{4} \\ \text{ó} \\ -\frac{4}{12} = -\frac{1}{3} \Rightarrow a^2 = 1 - \frac{1}{9} = \frac{8}{9} \end{cases}$$

$$\Rightarrow \begin{cases} a = \pm \frac{\sqrt{3}}{2} \;;\; b = \frac{1}{2} \\ \text{ó} \\ a = \pm \frac{\sqrt{8}}{3} = \pm \frac{2\sqrt{2}}{3} \;;\; b = -\frac{1}{3} \end{cases}$$

$a = \cos x$, $b = \operatorname{sen} x$

Posibles soluciones

1ª) $a = \frac{\sqrt{3}}{2}$ y $b = \frac{1}{2}$

1er cuadrante

$x = 30° +$ nº entero de vueltas

comprobación:

$\cos(2 \cdot 30°) + \operatorname{sen} 30° =$

$\cos 60° + \operatorname{sen} 30° = \frac{1}{2} + \frac{1}{2} = 1$

por otro lado: $4 \operatorname{sen}^2 30° = 4\left(\frac{1}{2}\right)^2 = 4 \cdot \frac{1}{4} = 1$

Sí // VALE

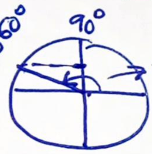
$\boxed{S_1 = \{30° + \text{nº entero de vueltas}\}}$

$= \boxed{\{30° + k \cdot 360° / k \in \mathbb{Z}\}} = \{\frac{\pi}{6} + 2k\pi / k \in \mathbb{Z}\} =$

$30° = \frac{\pi}{6}$

$= \{\frac{\pi + 12k\pi}{6} / k \in \mathbb{Z}\} = \boxed{\{\frac{12k+1}{6}\pi / k \in \mathbb{Z}\}}$

2ª) $a = -\frac{\sqrt{3}}{2}$; $b = \frac{1}{2}$



$x = 150° +$ nº entero vueltas

2º cuadrante

Comprobación: $\cos(2 \cdot 150°) + \operatorname{sen} 150° = \cos 300° + \operatorname{sen} 150°$

$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$

por otro lado $4 \operatorname{sen}^2 150° = 4 \cdot \left(\frac{1}{2}\right)^2 = \boxed{1}$ Sí // VALE

$\boxed{S_2 = \{150° + \text{nº entero de vueltas}\}} = \boxed{\{150° + k \cdot 360° / k \in \mathbb{Z}\}} =$

$= \{\frac{5}{6}\pi + 2k\pi / k \in \mathbb{Z}\} = \{\frac{5\pi + 12k\pi}{6} / k \in \mathbb{Z}\} =$

$= \boxed{\{\frac{5+12k}{6}\pi / k \in \mathbb{Z}\}}$

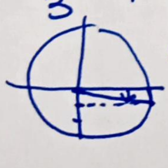
$180 — \pi$
$150 — x$

$x = \frac{150}{180}\pi = \frac{5}{6}\pi$

$\cos x = \frac{2\sqrt{2}}{3}$ y $\operatorname{sen} x = -\frac{1}{3}$

3ª) $a = \frac{2\sqrt{2}}{3}$ y $b = -\frac{1}{3}$, con calculadora:

$x \simeq -19{,}47 \simeq -19°28'16.4''$



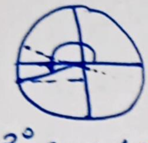
4º cuadrante

hacemos la comprobación

$\cos(2 \cdot (-19{,}47°)) + \operatorname{sen}(-19{,}47°) = 0'7 - 0'3 \neq 1$

NO VALE

4ª) $a = -\frac{2\sqrt{2}}{3}$; $b = -\frac{1}{3}$



3° cuadrante

$x = 180° + 19{,}47° = 199{,}47°$

$\cos(2 \cdot 199{,}47°) + \operatorname{sen}(199{,}47°)$

$0'7 - 0'3 \neq 1$ NO VALE

$\Rightarrow$ Las soluciones son $\boxed{S_1 \cup S_2}$

$\boxed{\text{Ej 8}}$

$$\begin{cases} \operatorname{sen}(x-y) = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x-y = 45° & 1° \text{ cuadrante} \\ \text{ó} \\ x-y = 90° + 45° = 135° & 2° \text{ cuad.} \end{cases} \\ \cos(x+y) = -\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x+y = 135° & 2° \text{ cuad.} \\ \text{ó} \\ x+y = 180° + 45° = 225° & 4° \text{ cuad} \end{cases} \end{cases}$$

(Muy largo)

Haciendo las posibles combinaciones de las soluciones:

$\left.\begin{array}{l} x-y=45° \\ x+y=135° \end{array}\right\}$ $\left.\begin{array}{l} x-y=45° \\ x+y=225° \end{array}\right\}$ $\left.\begin{array}{l} x-y=135° \\ x+y=135° \end{array}\right\}$ $\left.\begin{array}{l} x-y=135° \\ x+y=225° \end{array}\right\}$

quedan 4 sistemas

1°) $\left.\begin{array}{l} x-y=45° \\ x+y=135° \end{array}\right\}$ $x = 45° + y$

$45° + y + y = 135° \Rightarrow 2y = 90° \Rightarrow y = 45°$

$\Rightarrow x = 90°$

Comprobación $x = 90°$, $y = 45°$

$\left.\begin{array}{l} \operatorname{sen}(x-y) = \operatorname{sen}(90° - 45°) = \operatorname{sen}(45°) = \frac{\sqrt{2}}{2} \text{ sí,} \\ \cos(x+y) = \cos(90° + 45°) = \cos 135° = -\frac{\sqrt{2}}{2} \text{ sí} \end{array}\right\}$ VALE

$$\boxed{S_1 = \left\{ \begin{array}{l} x = 90° + \text{n° entero vueltas} \\ y = 45° + \text{n° entero vueltas} \end{array} \right\}} =$$

$$= \left\{ \begin{array}{l} x = \frac{\pi}{2} + 2k\pi \\ \text{e} \\ y = \frac{\pi}{4} + 2k\pi \end{array} \middle/ k \in \mathbb{Z} \right\} = \boxed{\left\{ \begin{array}{l} x = \frac{1+4k}{2}\pi \\ \text{e} \\ y = \frac{1+8k}{4}\pi \end{array} \middle/ k \in \mathbb{Z} \right\}}$$

2°) $\left.\begin{array}{l} x-y=45° \\ x+y=225° \end{array}\right\}$ $x = 45° + y$

$45° + y + y = 225°$; $2y = 180°$; $\boxed{y = 90°} \Rightarrow \boxed{x = 135°}$

Comprobación

$\operatorname{sen}(135 - 90) = \operatorname{sen} 45 = \frac{\sqrt{2}}{2}$ sí, VALE

$\cos(135 + 90) = \cos 225 = -\frac{\sqrt{2}}{2}$ sí, VALE

$$S_2 = \boxed{\left\{ \begin{array}{l} x = 135^\circ + \text{nº entero de vueltas} \\ \quad e \\ y = 90^\circ + \text{nº entero de vueltas} \end{array} \right\}} =$$

$$= \boxed{\left\{ \begin{array}{l} x = 135^\circ + k \cdot 360^\circ \\ \quad e \\ y = 90^\circ + k\, 360^\circ \end{array} \middle/ k \in \mathbb{Z} \right\}} = \left\{ \begin{array}{l} x = \frac{3\pi}{4} + 2k\pi \\ \quad e \\ y = \frac{\pi}{2} + 2k\pi \end{array} \middle/ k \in \mathbb{Z} \right\} =$$

$$135^\circ = 90^\circ + 45^\circ = \frac{\pi}{2} + \frac{\pi}{4} = \frac{2\pi + \pi}{4} = \frac{3\pi}{4}$$

$$= \boxed{\left\{ \begin{array}{l} x = \frac{3+8k}{4}\pi \\ y = \frac{1+4k}{2}\pi \end{array} \middle/ k \in \mathbb{Z} \right\}}$$

3º $\left.\begin{array}{l} x - y = 135^\circ \\ x + y = 135^\circ \end{array}\right\}$ $x = 135^\circ + y$

$135^\circ + y + y = 135^\circ$; $2y = 0$; $\boxed{y = 0^\circ}$

$\boxed{x = 135^\circ}$

Comprobación

$\left.\begin{array}{l} \operatorname{sen}(x-y) = \operatorname{sen}(135^\circ - 0^\circ) = \operatorname{sen} 135^\circ = \frac{\sqrt{2}}{2} \\ \cos(x+y) = \cos(135^\circ + 0^\circ) = \cos 135^\circ = -\frac{\sqrt{2}}{2} \end{array}\right\}$ Sí, VALE

$$S_3 = \boxed{\left\{ \begin{array}{l} x = 135^\circ + \text{nº entero de vueltas} \\ \quad e \\ y = 0^\circ + \text{nº entero de vueltas} \end{array} \right\}} =$$

$$= \boxed{\left\{ \begin{array}{l} x = 135^\circ + k \cdot 360^\circ \\ y = k \cdot 360^\circ \end{array} \middle/ k \in \mathbb{Z} \right\}} = \left\{ \begin{array}{l} x = \frac{3\pi}{4} + 2k\pi \\ y = 2k\pi \end{array} \middle/ k \in \mathbb{Z} \right\} =$$

$$= \boxed{\left\{ \begin{array}{l} x = \frac{3+8k}{4}\pi \\ \quad e \\ y = 2k\pi \end{array} \middle/ k \in \mathbb{Z} \right\}}$$

4º $\left.\begin{array}{l} x - y = 135^\circ \\ x + y = 225^\circ \end{array}\right\}$ $x = 135^\circ + y$

$135^\circ + y + y = 225^\circ$; $2y = 225^\circ - 135^\circ$

$2y = 90^\circ$; $\boxed{y = 45^\circ}$ $\boxed{x = 180^\circ}$

Comprobación

$\operatorname{sen}(x-y) = \operatorname{sen}(180^\circ - 45^\circ) = \operatorname{sen}(135^\circ) = \frac{\sqrt{2}}{2}$ Sí, VALE

$\cos(x+y) = \cos(180^\circ + 45^\circ) = \cos(225^\circ) = -\frac{\sqrt{2}}{2}$ Sí, VALE

$$S_4 = \boxed{\left\{ \begin{array}{l} x = 180^\circ + \text{nº entero vueltas} \\ y = 45^\circ + \text{nº entero vueltas} \end{array} \right\}} = \boxed{\left\{ \begin{array}{l} x = 180^\circ + k\,360^\circ \\ \quad e \\ y = 45^\circ + k\,360^\circ \end{array} \middle/ k \in \mathbb{Z} \right\}} =$$

$$S_4 = \left\{ \begin{array}{l} x = \pi + 2k\pi \\ \quad e \\ y = \frac{\pi}{4} + 2k\pi \end{array} \middle/ k \in \mathbb{Z} \right\} =$$

$$= \boxed{\left\{ \begin{array}{l} x = (2k+1)\pi \\ \quad e \\ y = \frac{1+8k}{4}\pi \end{array} \middle/ k \in \mathbb{Z} \right\}}$$

Soluciones = $\boxed{S_1 \cup S_2 \cup S_3 \cup S_4}$

—o—

Ej 9

$$\left. \begin{array}{l} \operatorname{sen} x \cos y = -\frac{\sqrt{3}}{4} \\ \cos x \operatorname{sen} y = \frac{\sqrt{3}}{4} \end{array} \right\}$$ usando fórmulas de ángulos suma y diferencia

sumamos 1ª Ec + 2ª Ec $\Rightarrow \boxed{\operatorname{sen} x \cos y + \cos x \operatorname{sen} y = 0}$

1ª Ec − 2ª Ec $\Rightarrow \operatorname{sen} x \cos y - \cos x \operatorname{sen} y = -\frac{2\sqrt{3}}{4}$

$$\boxed{\operatorname{sen} x \cos y - \cos x \operatorname{sen} y = -\frac{\sqrt{3}}{2}}$$

Como sabemos que: $\left\{ \begin{array}{l} \operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta \\ \operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cos\beta - \cos\alpha\operatorname{sen}\beta \end{array} \right.$

Nos queda: $\alpha = x$, $\beta = y$

$$\left. \begin{array}{l} \operatorname{sen}(x+y) = 0 \\ \operatorname{sen}(x-y) = -\frac{\sqrt{3}}{2} \end{array} \right\}$$

Para que esto ocurra: $x+y = 0°$ ó $x+y = 180°$

$x-y = -60°$ ó $x-y = 240°$

Los posibles sistemas son:

$$\left. \begin{array}{l} x+y = 0° \\ x-y = 60° \end{array} \right\} ; \quad \left. \begin{array}{l} x+y = 0° \\ x-y = 240° \end{array} \right\} \quad \left. \begin{array}{l} x+y = 180° \\ x-y = -60° \end{array} \right\} \quad \left. \begin{array}{l} x+y = 180° \\ x-y = 240° \end{array} \right\}$$

Hay q. resolver los 4 sistemas

1º) $\left. \begin{array}{l} x+y = 0° \\ x-y = 60° \end{array} \right\}$ $x = 60° + y$

$60° + y + y = 0°$; $2y = -60°$; $\boxed{y = -30°}$

$\boxed{x = 30°}$

Se puede comprobar q. $\operatorname{sen} 30° \cos(-30°) = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \neq -\frac{\sqrt{3}}{4}$ NO VALE

$\cos 30 \cdot \operatorname{sen}(-30°) = \frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{4} \neq \frac{\sqrt{3}}{4}$

NO VALE

2ª) $\left.\begin{array}{l} x+y=0^\circ \\ x-y=240^\circ \end{array}\right\}$ $x=-y$

$-y-y=240^\circ$; $-2y=240^\circ$;

$\boxed{y=-120^\circ}$ $\boxed{x=120^\circ}$

Comprobación $\operatorname{sen}120^\circ \cdot \cos(-120^\circ) = \frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{4}$ Sí VALE

$\cos 120^\circ \cdot \operatorname{sen}(-120^\circ) = -\frac{1}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right) = +\frac{\sqrt{3}}{4}$ Sí VALE

$\boxed{S_2 = \left\{ \begin{array}{l} x = 120^\circ + \text{n° entero de vueltas} \\ \text{e} \\ y = 120^\circ + \text{n° entero de vueltas} \end{array} \right\}} =$

$= \boxed{\left\{ \begin{array}{l} x = 120^\circ + k\,360^\circ \\ \text{e} \\ y = 120^\circ + k\cdot 360^\circ \end{array} \right\}} = \left\{ \begin{array}{l} x = \frac{2}{3}\pi + 2k\pi \\ \text{e} \\ y = \frac{2}{3}\pi + 2k\pi \end{array} \right\} = \boxed{\left\{ \begin{array}{l} x = \frac{2+6k}{3}\pi \\ y = \frac{2+6k}{3}\pi \end{array} \middle/ k\in\mathbb{Z} \right\}}$

$\left.\begin{array}{l} 180 - \pi \\ 120 - x \end{array}\right\}$ $x = \frac{120}{180}\pi = \frac{2}{3}\pi$

3ª) $\left.\begin{array}{l} x+y=180^\circ \\ x-y=-60^\circ \end{array}\right\}$ $x=180^\circ-y$

$180^\circ - y - y = -60^\circ$; $-2y = -60^\circ - 180^\circ$

$-2y = -240^\circ$; $\boxed{y=120^\circ}$ $\boxed{x=60^\circ}$

Comprobación

$\operatorname{sen}60^\circ \cdot \cos 120^\circ = \frac{\sqrt{3}}{2}\cdot\left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{4}$ Sí, vale

$\cos 60^\circ \cdot \operatorname{sen}120^\circ = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$ Sí, vale

$\boxed{S_3 = \left\{ \begin{array}{l} x = 60^\circ + \text{n° entero de vueltas} \\ \text{e} \\ y = 120^\circ + \text{n° entero de vueltas} \end{array} \right\}} =$

$= \boxed{\left\{ \begin{array}{l} x = 60^\circ + k\,360^\circ \\ \text{e} \\ y = 120^\circ + k\,360^\circ \end{array} \middle/ k\in\mathbb{Z} \right\}} = \left\{ \begin{array}{l} x = \frac{\pi}{3} + 2k\pi \\ \text{e} \\ y = \frac{2\pi}{3} + 2k\pi \end{array} \middle/ k\in\mathbb{Z} \right\} =$

$= \boxed{\left\{ \begin{array}{l} x = \frac{6k+1}{3}\pi \\ \text{e} \\ y = \frac{6k+2}{3}\pi \end{array} \middle/ k\in\mathbb{Z} \right\}}$

4ª y última) $\left.\begin{array}{l} x+y=180^\circ \\ x-y=240^\circ \end{array}\right\}$ $x=180^\circ-y$

$180^\circ - y - y = 240^\circ$; $-2y = 240^\circ - 180^\circ$

$-2y = 60^\circ$; $\boxed{y=-30^\circ}$ $x = 180^\circ - (-30^\circ) = \boxed{210^\circ}$

$4^{\underline{a}}$ ~) $x = 210^\circ$, $y = -30^\circ$

Comprobación

$\operatorname{sen} 210^\circ \cdot \cos(-30^\circ) = -\frac{1}{2} \cdot \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{4}$ sí //

$\cos 210^\circ \cdot \operatorname{sen}(-30^\circ) = -\frac{\sqrt{3}}{2} \cdot \left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{4}$ sí //

$$\boxed{S_4 = \left\{ \begin{array}{l} x = 210^\circ + \text{n}^\circ \text{ entero de vueltas} \\ \quad \text{e} \\ y = -30^\circ + \text{n}^\circ \text{ entero de vueltas} \end{array} \right\}}$$

$$= \boxed{\left\{ \begin{array}{l} x = 210^\circ + k \cdot 360^\circ \\ \quad \text{e} \\ y = -30^\circ + k \cdot 360^\circ \end{array} \Big/ k \in \mathbb{Z} \right\}} = \left\{ \begin{array}{l} x = \frac{7}{6}\pi + 2k\pi \\ \quad \text{e} \\ y = -\frac{\pi}{6} + 2k\pi \end{array} \Big/ k \in \mathbb{Z} \right\} =$$

$$\left. \begin{array}{l} 180 \text{ — } \pi \\ 210 \text{ — } x \end{array} \right\} \; x = \frac{210}{180}\pi = \frac{7}{6}\pi$$

$$= \boxed{\left\{ \begin{array}{l} x = \frac{12k+7}{6}\pi \\ \quad \text{e} \\ y = \frac{12k-1}{6}\pi \end{array} \Big/ k \in \mathbb{Z} \right\}}$$

$\Rightarrow$ $\boxed{\text{Soluciones} = S_2 \cup S_3 \cup S_4}$

(las obtenidas para S_1 NO VALEN, ya lo comprobamos)

—— o ——

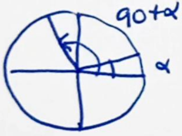
Ej 10

$$\left. \begin{array}{l} \operatorname{sen} x + \cos y = \sqrt{3} \\ x - y = \frac{\pi}{2} \end{array} \right\}$$

En grados

$$\left. \begin{array}{l} \operatorname{sen} x + \cos y = \sqrt{3} \\ x - y = 90^\circ \end{array} \right\}$$

$x = 90^\circ + y \Rightarrow \operatorname{sen}(90^\circ + y) + \cos y = \sqrt{3}$



$\operatorname{sen}(90 + \alpha) = \cos(\alpha)$ Utilizando esto en la ecuación, queda:

$\cos y + \cos y = \sqrt{3}$

$2\cos y = \sqrt{3} \Leftrightarrow \cos y = \frac{\sqrt{3}}{2}$ hay 2 posibilidades:

$y = 30^\circ$ ó $y = -30^\circ$

para $\boxed{y = 30^\circ} \Rightarrow x = 90^\circ + 30^\circ = \boxed{120^\circ}$

para $\boxed{y = -30^\circ} \Rightarrow x = 90^\circ + (-30^\circ) = \boxed{60^\circ}$

Comprobación

1ª) $\left.\begin{array}{l} x=120^\circ \\ y=30^\circ \end{array}\right\}$ ¿ $\operatorname{sen} x+\cos y=\sqrt{3}$?

$\operatorname{sen} 120^\circ+\cos 30^\circ=\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\sqrt{3}$

SÍ, VALE

$$\boxed{S_1=\left\{\begin{array}{l} x=120^\circ+\text{nº entero de vueltas} \\ \text{e} \\ y=30^\circ+\text{nº entero de vueltas} \end{array}\right\}}=$$

$$=\boxed{\left\{\begin{array}{l} x=120^\circ+k\,360^\circ \\ \text{e} \\ y=30^\circ+k\,360^\circ \end{array} \Big/ k\in\mathbb{Z}\right\}}=\left\{\begin{array}{l} x=\frac{2\pi}{3}+2k\pi \\ \text{e} \\ y=\frac{\pi}{6}+2k\pi \end{array} \Big/ k\in\mathbb{Z}\right\}=$$

$\left.\begin{array}{l} 180 \text{ — } \pi \\ 120 \text{ — } x \end{array}\right\}\ x=\frac{120}{180}\pi=\frac{2}{3}\pi$

$\left.\begin{array}{l} 180 \text{ — } \pi \\ 30 \text{ — } x \end{array}\right\}\ x=\frac{30}{180}\pi=\frac{\pi}{6}$

$$=\boxed{\left\{\begin{array}{l} x=\frac{6k+2}{3}\pi \\ \text{e} \\ y=\frac{12k+1}{6}\pi \end{array} \Big/ k\in\mathbb{Z}\right\}}$$

2ª) $\left.\begin{array}{l} x=60^\circ \\ y=-30^\circ \end{array}\right\}$

comprobación

$\operatorname{sen} 60+\cos(-30^\circ)=\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\sqrt{3}$ SÍ, VALE

$$\boxed{S_2=\left\{\begin{array}{l} x=60^\circ+\text{nº entero de vueltas} \\ \text{e} \\ y=-30^\circ+\text{nº entero de vueltas} \end{array}\right\}}=$$

$$=\boxed{\left\{\begin{array}{l} x=60+k\,360^\circ \\ \text{e} \\ y=-30^\circ+k\,360^\circ \end{array} \Big/ k\in\mathbb{Z}\right\}}=\left\{\begin{array}{l} x=\frac{\pi}{3}+2k\pi \\ \text{e} \\ y=-\frac{\pi}{6}+2k\pi \end{array} \Big/ k\in\mathbb{Z}\right\}=$$

$$=\boxed{\left\{\begin{array}{l} x=\frac{6k+1}{3}\pi \\ \text{e} \\ y=\frac{12k-1}{6}\pi \end{array} \Big/ k\in\mathbb{Z}\right\}}$$

$$\boxed{\text{Soluciones}=S_1\cup S_2}$$

Ej 11 $\operatorname{sen} 2x = \operatorname{sen} x$

razones ángulo doble: $\operatorname{sen} 2\alpha = 2 \operatorname{sen}\alpha \cos\alpha$

$\Rightarrow \quad 2 \operatorname{sen}x \cos x = \operatorname{sen}x \iff 2\operatorname{sen}x\cos x - \operatorname{sen}x = 0$

$\iff \quad \operatorname{sen}x\,(2\cos x - 1) = 0 \quad \begin{cases} \operatorname{sen}x = 0 \\ \text{ó} \\ 2\cos x - 1 = 0 \end{cases}$

- si $\operatorname{sen}x = 0 \Rightarrow$ x puede ser $\begin{cases} 0^\circ \\ \text{ó} \\ 180^\circ \end{cases}$

- si $2\cos x - 1 = 0 \iff 2\cos x = 1 \iff \cos x = \frac{1}{2} \quad \begin{cases} x = 60^\circ \\ \text{ó} \\ x = -60^\circ \end{cases}$

Comprobación

1º) $x = 0^\circ \Rightarrow \operatorname{sen} 2\cdot 0 = \operatorname{sen} 0$ sí, VALE

$\boxed{S_1 = \{0^\circ + \text{nº entero de vueltas}\}} = \boxed{\{0 + k360^\circ / k\in\mathbb{Z}\}}$

$= \boxed{\{2k\pi / k \in \mathbb{Z}\}}$

2º) $x = 180^\circ \Rightarrow \operatorname{sen} 2\cdot 180^\circ = \operatorname{sen} 360^\circ = 0 = \operatorname{sen} 180^\circ$

sí, VALE

$S_2 = \boxed{\{180^\circ + \text{nº entero de vueltas}\}} =$

$\boxed{\{180^\circ + k\cdot 360^\circ / k\in\mathbb{Z}\}} = \{\pi + 2k\pi / k\in\mathbb{Z}\} =$

$= \boxed{\{(2k+1)\pi / k \in \mathbb{Z}\}}$

También se puede ver que $S_1 \cup S_2 = \boxed{\{k\pi / k\in\mathbb{Z}\}}$

3º) $x = 60^\circ \Rightarrow$ comprobación $\operatorname{sen}(2\cdot 60^\circ) = \operatorname{sen} 120^\circ = \frac{\sqrt{3}}{2} = \operatorname{sen}60^\circ$

Sí, VALE

$\operatorname{sen}60^\circ = \frac{\sqrt{3}}{2}$

$\boxed{S_3 = \{60^\circ + \text{nº entero de vueltas}\}} =$

$= \boxed{\{60^\circ + k360^\circ / k\in\mathbb{Z}\}} = \{\frac{\pi}{3} + 2k\pi / k\in\mathbb{Z}\} =$

$= \boxed{\{\frac{6k+1}{3}\pi / k \in\mathbb{Z}\}}$

4º) $x = -60^\circ$; comprobación

$\operatorname{sen}(2\cdot(-60^\circ)) = \operatorname{sen}(-120^\circ) = -\frac{\sqrt{3}}{2}$
$\operatorname{sen}(-60^\circ) = -\frac{\sqrt{3}}{2}$ } sí, VALE

$\boxed{S_4 = \{-60^\circ + \text{nº entero de vueltas}\}} =$

$= \boxed{\{-60^\circ + k\cdot 360^\circ / k \in \mathbb{Z}\}} = \{-\frac{\pi}{3} + 2k\pi / k \in \mathbb{Z}\} =$

$= \boxed{\{\frac{6k-1}{3}\pi / k \in \mathbb{Z}\}}$

Soluciones : $\boxed{S_1 \cup S_2 \cup S_3 \cup S_4}$

—•—

Ej 12 $\cos x - \operatorname{sen} x = 0$

añadimos $\cos^2 x + \operatorname{sen}^2 x = 1$ y cambio variable $\begin{cases} a = \cos x \\ b = \operatorname{sen} x \end{cases}$; me queda el sistema:

$\left.\begin{matrix} b - a = 0 \\ a^2 + b^2 = 1 \end{matrix}\right\}$ $b = a$
$a^2 + a^2 = 1$; $2a^2 = 1$; $a^2 = \frac{1}{2}$; $a = \pm\frac{1}{\sqrt{2}}$

$a = \pm\frac{\sqrt{2}}{2}$

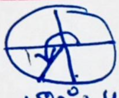
Soluciones 1º) $a = \frac{\sqrt{2}}{2}$ y $b = \frac{\sqrt{2}}{2}$ } $x = 45^\circ$ se ve fácilmente la comprobación $\operatorname{sen} 45^\circ = \cos 45^\circ$

$a = \cos x$ y $b = \operatorname{sen} x$

1er cuadrante

$\boxed{S_1 = \{45^\circ + \text{nº entero de vueltas}\}} = \boxed{\{45^\circ + k\, 360^\circ / k \in \mathbb{Z}\}} =$

$= \{\frac{\pi}{4} + 2k\pi / k \in \mathbb{Z}\} = \boxed{\{\frac{8k+1}{4}\pi / k \in \mathbb{Z}\}}$

2º) $a = -\frac{\sqrt{2}}{2}$ y $b = -\frac{\sqrt{2}}{2}$ 3º cuadrante 

$180^\circ + 45^\circ = 225^\circ$

se comprueba fácilmente q. $\operatorname{sen} 225^\circ = \cos 225^\circ$

$S_2 = \boxed{\{225^\circ + \text{nº entero de vueltas}\}} = \boxed{\{225^\circ + k\cdot 360^\circ / k \in \mathbb{Z}\}}$

$S_2 = \left\{ \frac{5}{4}\pi + 2k\pi \mid k \in \mathbb{Z} \right\} = \boxed{\left\{ \frac{8k+5}{4}\pi \mid k \in \mathbb{Z} \right\}}$

π — $180°$
x — $225°$

$x = \frac{225}{180}\pi = \frac{15\pi}{12} = \frac{5}{4}\pi$

$\boxed{\text{Soluciones} = S_1 \cup S_2}$

Ej 13

$$\left.\begin{array}{l} \operatorname{sen} x \cos y = \frac{1}{4} \\ \cos x \operatorname{sen} y = \frac{3}{4} \end{array}\right\}$$

Vamos a usar las razones trigonométricas del ángulo suma y diferencia

$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha \cos\beta + \cos\alpha \operatorname{sen}\beta$
$\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha \cos\beta - \cos\alpha \operatorname{sen}\beta$

* si hacemos 1ª Ecuación + 2ª Ec. nos queda:

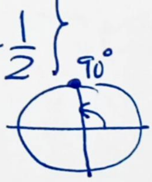
$$\underbrace{\operatorname{sen} x \cos y + \cos x \operatorname{sen} y}_{\operatorname{sen}(x+y)} = \frac{1}{4} + \frac{3}{4} \Rightarrow \boxed{\operatorname{sen}(x+y) = 1}$$

* Si hacemos 1ª Ec − 2ª Ec. nos queda:

$$\underbrace{\operatorname{sen} x \cos y - \cos x \operatorname{sen} y}_{\operatorname{sen}(x-y)} = \underbrace{\frac{1}{4} - \frac{3}{4}}_{-\frac{1}{2}} \Rightarrow \boxed{\operatorname{sen}(x-y) = -\frac{1}{2}}$$

El nuevo sistema queda:

$$\left.\begin{array}{l} \operatorname{sen}(x+y) = 1 \\ \operatorname{sen}(x-y) = -\frac{1}{2} \end{array}\right\}$$



Para $\operatorname{sen}(x+y) = 1 \Rightarrow \boxed{x+y = 90°}$

Para $\operatorname{sen}(x-y) = -\frac{1}{2}$ $\left\langle \begin{array}{l} x-y = -30° \\ \text{ó} \\ x-y = 210° \end{array}\right.$

Salen 2 posibles sistemas:

$$\left.\begin{array}{l} x+y = 90° \\ x-y = -30° \end{array}\right\} \quad \text{ó} \quad \left.\begin{array}{l} x+y = 90° \\ x-y = 210° \end{array}\right\}$$

4° cuadrante — 3° cuad.

1º) $\left.\begin{array}{l} x+y=90^\circ \\ x-y=210^\circ \end{array}\right\}$ $x=210^\circ+y$

$210^\circ+y+y=90^\circ$; $2y=90^\circ-210^\circ$

$2y=-120^\circ$; $\boxed{y=-60^\circ}$ $\Rightarrow$ $\boxed{x=150^\circ}$

Comprobación ¿ $\left\{\begin{array}{l} \operatorname{sen}150^\circ \cos(-60^\circ)=\frac{1}{4} \\ \cos 150^\circ \operatorname{sen}(-60^\circ)=\frac{3}{4} \end{array}\right.$?

$\operatorname{sen}150^\circ \cos(-60^\circ)=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$ Sí//

$\cos 150^\circ \operatorname{sen}(-60^\circ)=-\frac{\sqrt{3}}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right)=\frac{3}{4}$ Sí// VALE

$\boxed{S_1=\left\{\begin{array}{l} x=150^\circ+\text{nº entero de vueltas} \\ \text{e} \\ y=-60^\circ+\text{nº entero vueltas} \end{array}\right\}}=$

$=\boxed{\left\{\begin{array}{l} x=150^\circ+k360^\circ \\ \text{e} \\ y=-60^\circ+k360^\circ \end{array} \middle/ k\in\mathbb{Z}\right\}}=\left\{\begin{array}{l} x=\frac{5\pi}{6}+2k\pi \\ \text{e} \\ y=-\frac{\pi}{3}+2k\pi \end{array} \middle/ k\in\mathbb{Z}\right\}=$

$\left.\begin{array}{l} 180 — \pi \\ 150 — x \end{array}\right\}$ $x=\frac{150}{180}\pi=\frac{10}{12}\pi=\frac{5}{6}\pi$

$=\boxed{\left\{\begin{array}{l} x=\frac{12k+5}{6}\pi \\ y=\frac{6k-1}{3}\pi \end{array} \middle/ k\in\mathbb{Z}\right\}}$

2º) $\left.\begin{array}{l} x+y=90^\circ \\ x-y=-30^\circ \end{array}\right\}$ $x=90^\circ-y$

$90^\circ-y-y=-30^\circ$; $-2y=-30^\circ-90^\circ$; $-2y=-60^\circ$

$\boxed{y=30^\circ}$ $\Rightarrow$ $\boxed{x=60^\circ}$

Comprobación $\operatorname{sen}60^\circ\cos 30^\circ=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}=\frac{3}{4}\neq\frac{1}{4}$

NO VALE

$\cos 60^\circ\operatorname{sen}30^\circ=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}\neq\frac{3}{4}$

Las Soluciones son $\boxed{S_1}$