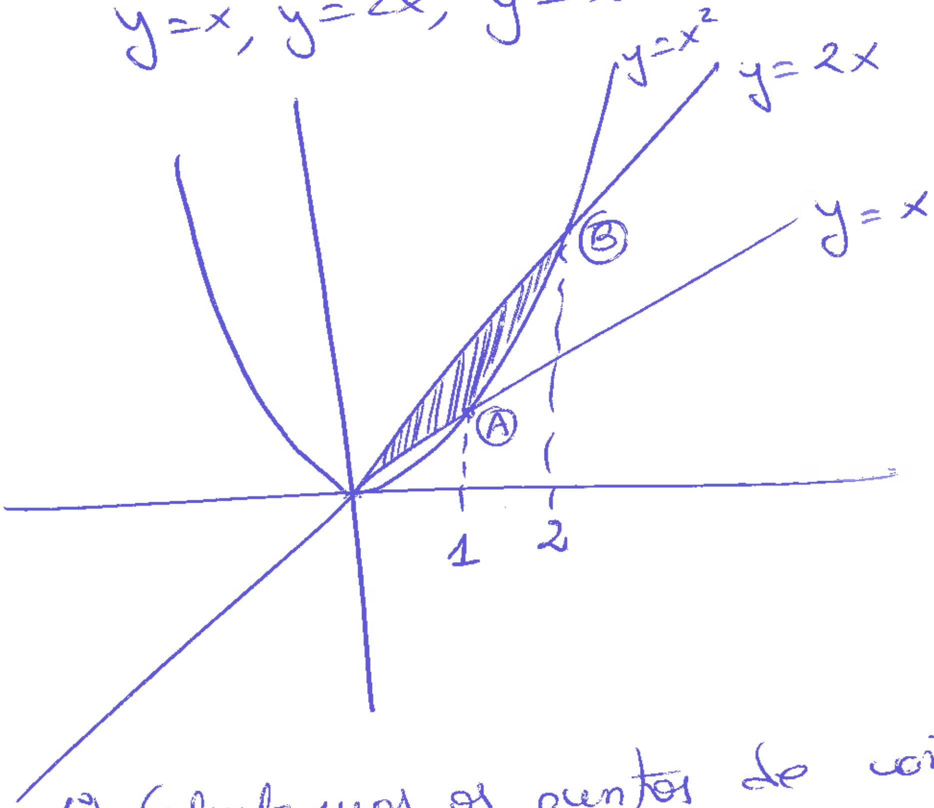
BOLETÍN ABAU INTEGRAL DEFINIDA

MODELO 1. c)

$y=x,\ y=2x,\ y=x^2$



1º Calculamos os puntos de corte A, B:

A $\left.\begin{matrix} y=x \\ y=x^2 \end{matrix}\right\} \Rightarrow x=x^2 \Rightarrow \begin{matrix} x^2-x=0 \\ x(x-1)=0 \end{matrix} \begin{matrix} \nearrow x=0 \Rightarrow (0,0) \\ \searrow x=1 \Rightarrow (1,1)\ A \end{matrix}$

B: $\left.\begin{matrix} y=2x \\ y=x^2 \end{matrix}\right\} \Rightarrow 2x=x^2 \Rightarrow \begin{matrix} x^2-2x=0 \\ x(x-2)=0 \end{matrix} \begin{matrix} \nearrow x=0 \Rightarrow (0,0) \\ \searrow x=2 \Rightarrow (2,4)\ B \end{matrix}$

A rexión limitada polas funcións dadas é a superficie raiada, só nos temos que fixar cales son as funcións que van por encima en cada parte:

$$\grave{A}rea = \int_0^1 (2x - x)\,dx + \int_1^2 (2x - x^2)\,dx =$$

$$= \int_0^1 x\,dx + \int_1^2 (2x - x^2)\,dx = \left.\frac{x^2}{2}\right]_0^1 + \left.\frac{2x^2}{2} - \frac{x^3}{3}\right]_1^2 =$$

$$= \frac{1}{2} + \left. x^2 - x^3/3 \right]_1^2 = \frac{1}{2} + 4 - \frac{8}{3} - \left(1 - 1/3\right) =$$

$$= \frac{1}{2} + 4 - \frac{8}{3} - 1 + \frac{1}{3} = \frac{1}{2} + 3 - \frac{7}{3} = \frac{3+18-14}{6} =$$

$$= \frac{7}{6}\,u^2.$$

MODELO 2.C

$$\int_0^2 \frac{x+3}{\sqrt{x+2}}\,dx = \left[\begin{array}{l} x+2 = t^2 \\ dx = 2t\,dt \\ x+3 = x+2+1 = \\ \quad = t^2+1 \end{array}\right] = \int_0^2 \frac{t^2+1}{\sqrt{t^2}} \cdot 2t\,dt =$$

$$= \int_0^2 \frac{(t^2+1)\,2t}{t}\,dt = 2\int_0^2 (t^2+1)\,dt = 2\left(\frac{t^3}{3} + t\right]_0^2 =$$

$$= 2\left(\frac{(\sqrt{x+2})^3}{3} + \sqrt{x+2}\right]_0^2 = 2\left[\left(\frac{(\sqrt{4})^3}{3} + \sqrt{4}\right) - \left(\frac{(\sqrt{2})^3}{3} + \sqrt{2}\right)\right]$$

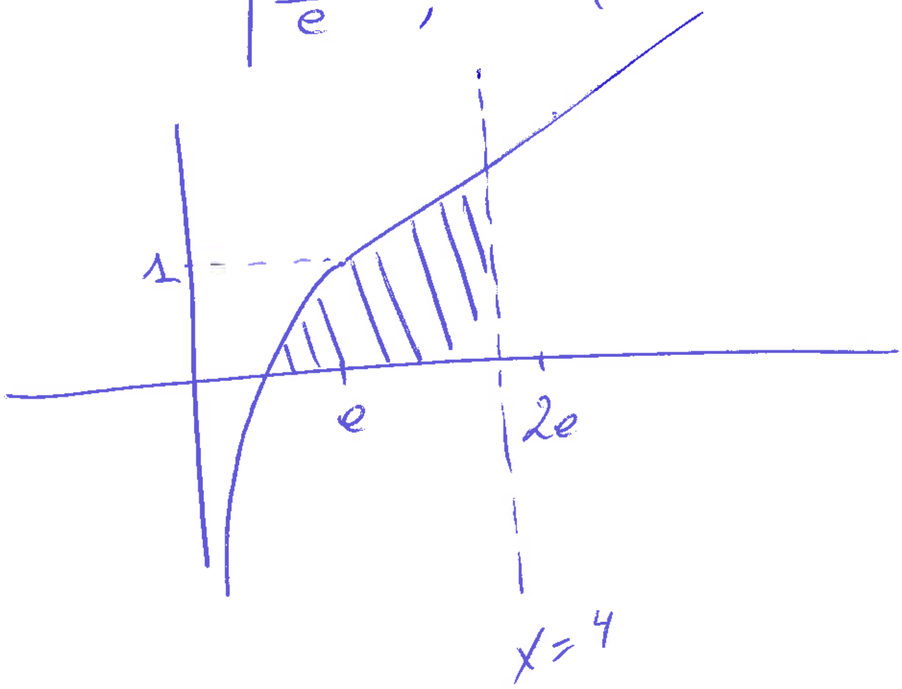
$$= 2\left[\frac{8}{3} + 2 - \left(\frac{2\sqrt{2}}{3} + \sqrt{2}\right)\right] =$$

$$= 2\left[\frac{8}{3} + 2 - \frac{2\sqrt{2}}{3} - \sqrt{2}\right] = \frac{16}{3} + 4 - \frac{4\sqrt{2}}{3} - 2\sqrt{2}$$

$$= \frac{16 + 12 - 4\sqrt{2} - 6\sqrt{2}}{3} = \frac{28 - 10\sqrt{2}}{3}$$

2019 Ex. 3 c)

$$f(x) = \begin{cases} \ln x, & x \in (0, e] \\ \frac{x}{e}, & x \in (e, +\infty) \end{cases}$$



$y = \ln x$

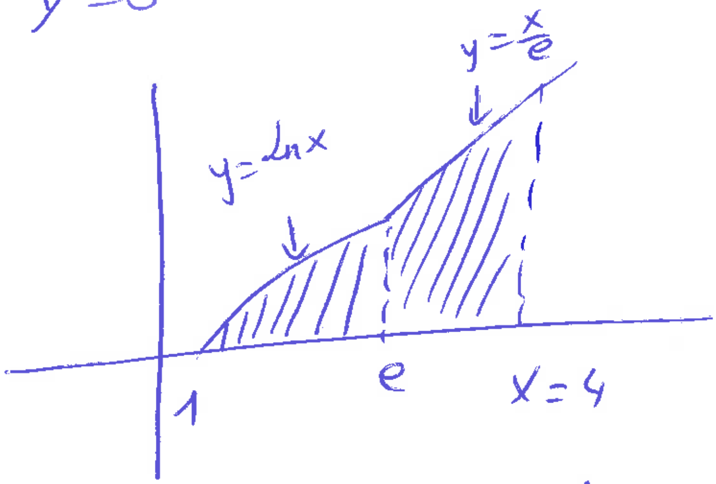
x	y
e	1

$y = \frac{x}{e}$

x	y
e	1
2e	$\frac{2e}{e} = 2$

Calculemos o punto de corte co eixe OX:

$$\left.\begin{array}{l} y = \ln x \\ y = 0 \end{array}\right\} \Rightarrow \quad 0 = \ln x \Rightarrow e^0 = x$$
$$1 = x$$



$$\text{Área} = \int_1^e \ln x \, dx + \int_e^4 \frac{x}{e} \, dx = x \ln x - x \Big]_1^e + \frac{1}{e} \cdot \frac{x^2}{2} \Big]_e^4 =$$

$$\int \underbrace{\ln x}_{u} \underbrace{dx}_{dv} = \left[\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x} dx \\ dv = dx \Rightarrow v = x \end{array} \right] =$$

$$= x \ln x - \int x \frac{1}{x} dx = x \ln x - x$$

$$= e \ln e - e - (1 \ln 1 - 1) + \frac{1}{e} \left[\frac{16}{2} - \frac{e^2}{2} \right] =$$

$$= e - e - (0 - 1) + \frac{1}{e} \left[8 - \frac{e^2}{2} \right] =$$

$$= 1 + \frac{1}{e} \left(8 - \frac{e^2}{2} \right) = 1 + \frac{8}{e} - \frac{e^2}{2e} = 1 + \frac{8}{e} - \frac{e}{2} u^2$$

$$= \frac{2e + 16 - e^2}{2e} u^2$$

2019. 4c)

$$\int \underbrace{x^2}_{u} \underbrace{e^{-x} dx}_{dv} = \left[\begin{array}{l} u = x^2 \Rightarrow du = 2x\,dx \\ e^{-x} dx = dv \Rightarrow v = \int e^{-x} dx = -e^{-x} \end{array} \right]$$

$$= -x^2 e^{-x} - \int (-e^{-x})\, 2x\, dx =$$

$$= -x^2 e^{-x} + 2\int x e^{-x} dx = -x^2 \cdot e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) =$$

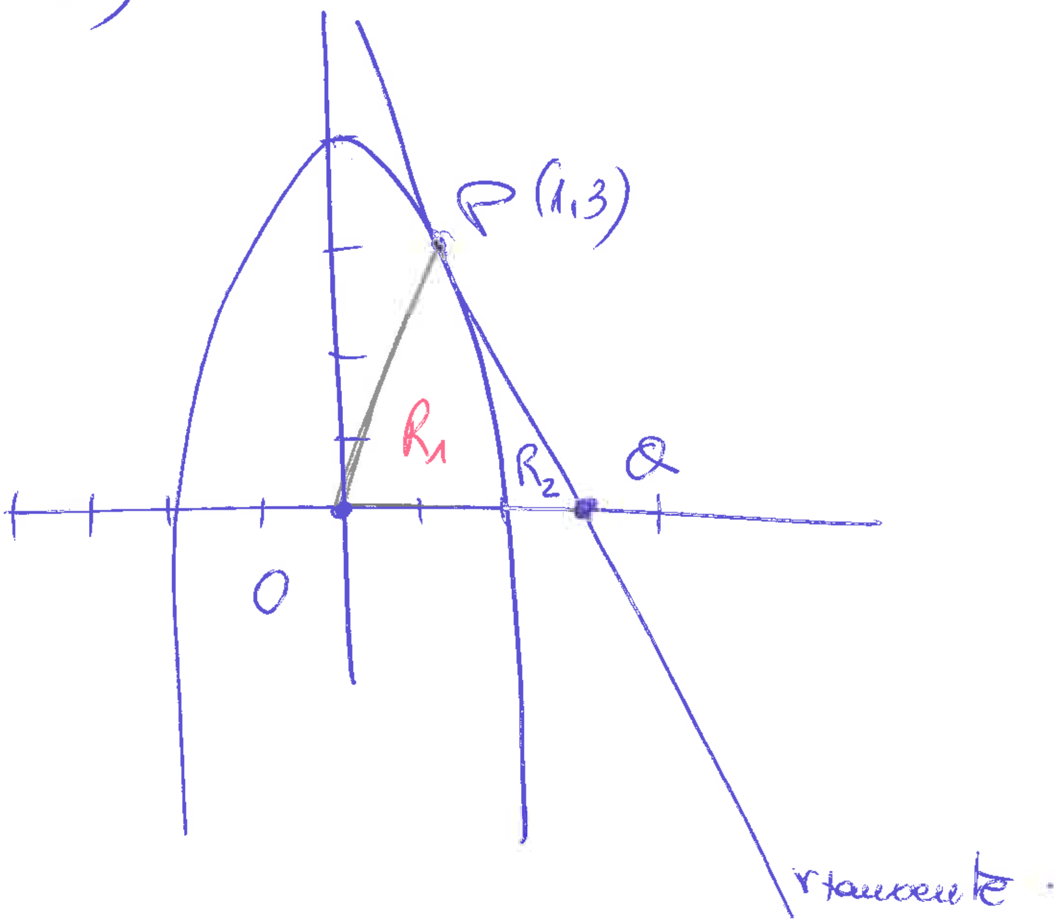
$$\int \underbrace{x}_{u} \underbrace{e^{-x} dx}_{dv} = \left[\begin{array}{l} u = x \Rightarrow du = dx \\ dv = e^{-x} dx \Rightarrow v = -e^{-x} \end{array} \right]$$

$$= -x e^{-x} - \int -e^{-x} dx = -x e^{-x} + \int e^{-x} dx$$

$$= -x e^{-x} - e^{-x}$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2 e^{-x} + C_1$$

6) b)



Recta tangente $\begin{cases} P(1,3) \\ m = f'(1) \end{cases}$

$f(x) = 4 - x^2$

$f'(x) = -2x \Rightarrow f'(1) = -2$

$\Rightarrow$ Recta $\begin{cases} (1,3) \\ m = -2 \end{cases}$ $y - 3 = -2(x-1)$

$\Downarrow$

$y - 3 = -2x + 2$

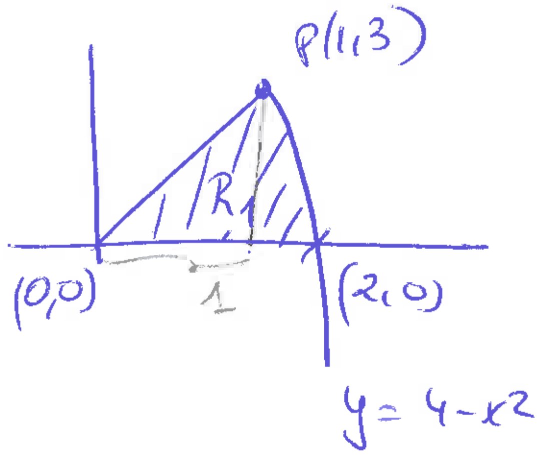
$y + 2x - 5 = 0$

$\boxed{y = 5 - 2x}$

O terceiro vértice é o punto Q, corte co eixe OX da recta tangente: $\begin{cases} y = 5 - 2x \\ y = 0 \end{cases} \Rightarrow x = 5/2 \Rightarrow Q(5/2, 0)$

Area = $R_1 + R_2$

$R_1 \Rightarrow$



$\Rightarrow$

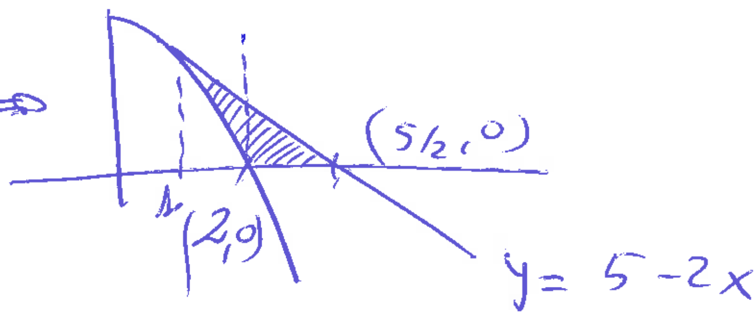
$$\Rightarrow R_1 = \frac{1 \cdot 3}{2} + \int_1^2 4 - x^2 = \frac{3}{2} + \left. 4x - \frac{x^3}{3} \right]_1^2 =$$

$$= \frac{3}{2} + 8 - \frac{8}{3} - \left(4 - \frac{1}{3}\right) = \frac{3}{2} + 8 - \frac{8}{3} - 4 + \frac{1}{3}$$

$$= \frac{3}{2} + \frac{1}{3} + 4 - \frac{8}{3} = \frac{3}{2} - \frac{7}{3} + 4 = \frac{9 - 14 + 24}{6}$$

$$= \frac{19}{6}\ u^2$$

$R_2 \Rightarrow$



$$R_2 = \int_1^2 5 - 2x - (4 - x^2)\,dx + \frac{(5/2 - 2)}{2} = \int_1^2 (5 - 2x - 4 + x^2)\,dx + \frac{1}{4} =$$

$$\int_1^2 (1-2x+x^2)\,dx = x - \frac{2x^2}{2} + \frac{x^3}{3}\Big]_1^2 =$$

$$= 2 - 4 + 8/3 - (1 - 1 + 1/3)$$

$$= -2 + 8/3 - 1/3 = -2 + \frac{7}{3} = 1/3\, u^2$$

$$R_2 = \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}\, u^2.$$

$$\text{Área total} = \frac{16}{6} + \frac{7}{12} = \frac{2 \cdot 16 + 7}{12} = \frac{32+7}{12} = \frac{39}{12}\, u^2$$

Outra forma de calcular R_2:

$$R_2 = \frac{\frac{5}{2} \cdot 3}{2} - \frac{19}{6} = \frac{7}{12}\, u^2.$$

2018. 7b

$y = x^2 - 4x, \quad y = x - 4.$

$y = x^2 - 4x$ Representamos a parábola.

1°) $x_v = \frac{4}{2} = 2$

$y_v = 4 - 8 = -4$ $\Big\} \Rightarrow (2,-4)$

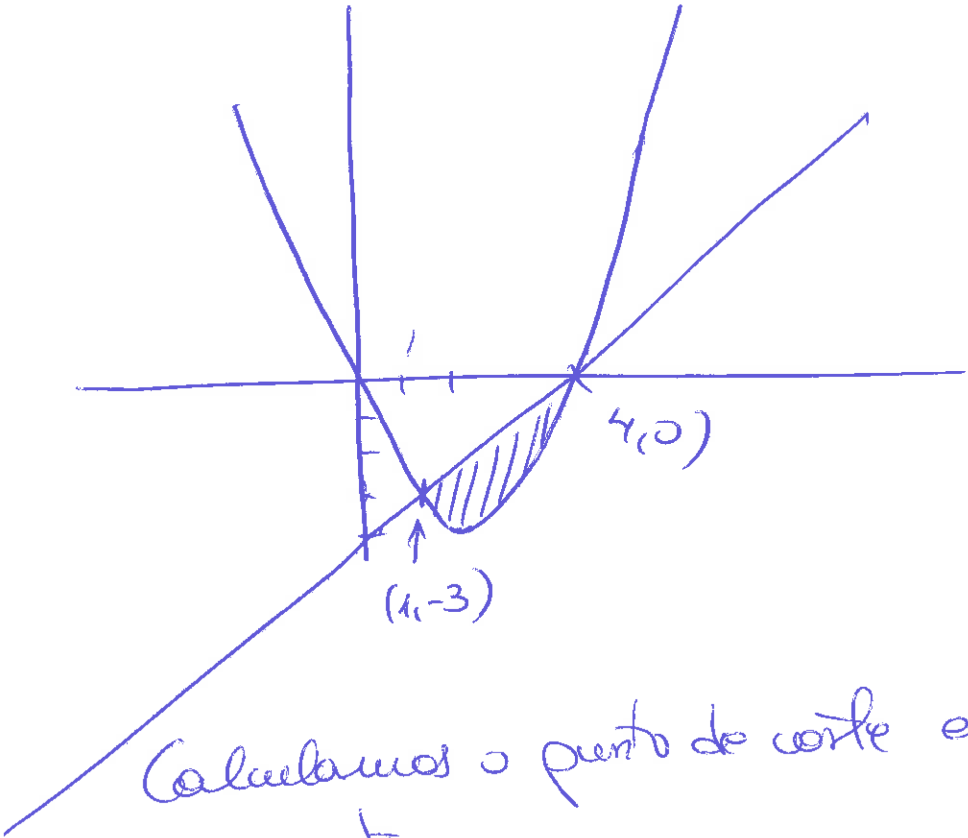
2°) $x = 0 \Rightarrow y = 0 \Rightarrow (0,0)$

$y = 0 \Rightarrow 0 = x^2 - 4x$

$0 = x(x-4) \Rightarrow x = 0 \quad (0,0)$

$x = 4 \Rightarrow (4,0)$

3°) $a = 1 > 0 \Rightarrow \cup$ convexa.



Representamos:

$y = x - 4$

x	y	
0	-4	(0,-4)
4	0	(4,0)

Calculamos o punto de corte entre a parábola e a recta:

$$\left.\begin{array}{l} y = x - 4 \\ y = x^2 - 4x \end{array}\right\} \Rightarrow x - 4 = x^2 - 4x$$

$$x^2 - 5x + 4 = 0$$

$$x = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \begin{array}{l} \nearrow 4 \\ \searrow 1 \end{array}$$

Se $x = 4 \Rightarrow y = 0 \Rightarrow (4,0)$

$\sqrt{9}$

Se $x = 1 \Rightarrow y = 1 - 4 = -3 \Rightarrow (1, -3)$

$$\text{Área} = \int_1^4 \left[x - 4 - (x^2 - 4x)\right] dx = \int_1^4 (x - 4 - x^2 + 4x)\, dx =$$

$$= \int_1^4 (-x^2 + 5x - 4)\, dx = -\frac{x^3}{3} + \frac{5x^2}{2} - 4x \Big]_1^4 =$$

$$= -\frac{64}{3} + \frac{5 \cdot 16}{2} - 16 - \left(-\tfrac{1}{3} + \tfrac{5}{2} - 4\right)$$

$$= -\frac{64}{3} + 5 \cdot 8 - 16 + \tfrac{1}{3} - \tfrac{5}{2} + 4$$

$$= -\frac{64}{3} + 40 - 16 + \tfrac{1}{3} - \tfrac{5}{2} + 4$$

$$= -\frac{64}{3} + \frac{1}{3} - \tfrac{5}{2} + 44 - 16$$

$$= -\frac{63}{3} - \tfrac{5}{2} + 28 = \frac{-126 - 15 + 6 \cdot 28}{6} =$$

$$= \frac{-141 + 168}{6} = \frac{27}{6} = \frac{9}{2}\, u^2$$

2018. 8c)

$$\int_0^3 x\sqrt{x+1}\,dx = \left[\begin{array}{l} x+1=t^2 \\ dx = 2t\,dt \end{array}\right] =$$

$$= \int_0^3 (t^2-1)\cdot t\cdot 2t\,dt = 2\int_0^3 t^2(t^2-1)\,dt$$

$$= 2\int_0^3 (t^4-t^2)\,dt = 2\left(\frac{t^5}{5}-\frac{t^3}{3}\right]_0^3 =$$

$$= 2\cdot\left(\frac{(\sqrt{x+1})^5}{5}-\frac{(\sqrt{x+1})^3}{3}\right]_0^3 =$$

$$= 2\left[\left(\frac{(\sqrt{4})^5}{5}-\frac{(\sqrt{4})^3}{3}\right)-\left(\frac{(\sqrt{1})^5}{5}-\frac{(\sqrt{1})^3}{3}\right)\right]$$

$$= 2\left[\frac{2^5}{5}-\frac{2^3}{3}-\frac{1}{5}+\frac{1}{3}\right] =$$

$$= 2\left[\frac{32}{5}-\frac{8}{3}-\frac{1}{5}+\frac{1}{3}\right] = 2\left[\frac{31}{5}-\frac{7}{3}\right] =$$

$$= 2\left[\frac{93-35}{15}\right] = 2\left[\frac{58}{15}\right]$$

2018. 9 b)

$y = x^2 - 2x$

$y = x$

Representamos $y = x^2 - 2x$

$x_v = \frac{2}{2} = 1$ } $\Rightarrow$ vértice $(1,-1)$

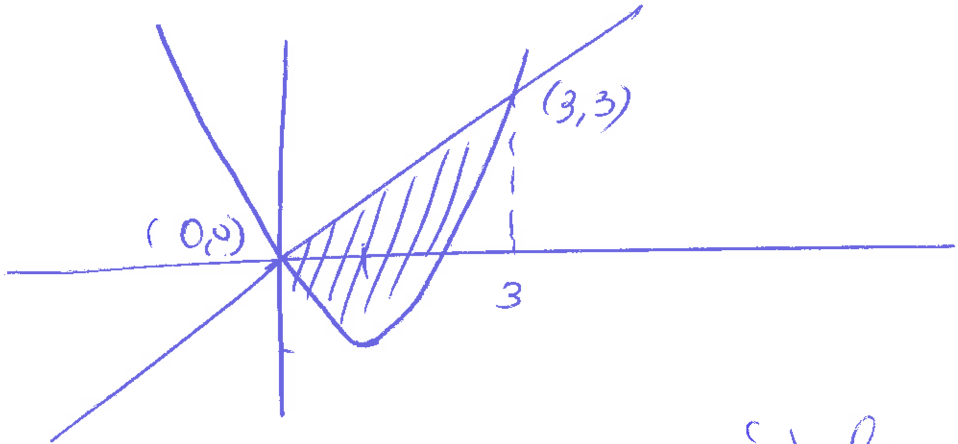
$y_v = 1 - 2 = -1$

$a = 1 > 0 \Rightarrow$ convexa.

$x = 0 \Rightarrow y = 0 \Rightarrow (0,0)$

$y = 0 \Rightarrow 0 = x^2 - 2x$

$0 = x(x-2) \Rightarrow \begin{matrix} x = 0 \\ x = 2 \end{matrix} \Rightarrow \begin{matrix} (0,0) \\ (2,0) \end{matrix}$



Punto de corte entre a parábola e a recta:

$\left.\begin{matrix} y = x^2 - 2x \\ y = x \end{matrix}\right\} \Rightarrow x^2 - 2x = x$

$x^2 - 2x - x = 0$

$x^2 - 3x = 0$

$x(x-3) = 0 \Rightarrow \begin{matrix} x = 0 \Rightarrow (0,0) \\ x = 3 \Rightarrow (3,3) \end{matrix}$

$$\text{Área} = \int_0^3 \left[x - (x^2 - 2x)\right] dx = \int_0^3 (x - x^2 + 2x)\, dx = \int_0^3 (-x^2 + 3x)\, dx$$

$$= -\frac{x^3}{3} + \frac{3x^2}{2}\Big]_0^3 = -\frac{27}{3} + \frac{27}{2} = \frac{-54+81}{6} = \frac{27}{6} = \frac{9}{2}\ u^2$$

$$10)\ c)\ \int_1^e \sqrt{x}\,\underbrace{\ln x}_{u}\,dx = \begin{matrix} u = \ln x \\ dv = \sqrt{x}\,dx \end{matrix} \Rightarrow \begin{matrix} du = \frac{1}{x}\,dx \\ v = \int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} \\ = \frac{2}{3}\sqrt{x^3} \end{matrix}$$

$$= \frac{2}{3}\sqrt{x^3}\cdot \ln x - \int \frac{2}{3}\sqrt{x^3}\cdot \frac{1}{x}\,dx =$$

$$= \frac{2}{3}\sqrt{x^3}\,\ln x - \frac{2}{3}\int x^{3/2-1}\,dx =$$

$$= \frac{2}{3}x\sqrt{x}\,\ln x - \frac{2}{3}\int x^{1/2}\,dx$$

$$= \frac{2}{3}x\sqrt{x}\,\ln x - \frac{2}{3}\,\frac{x^{3/2}}{3/2} = \frac{2}{3}x\sqrt{x}\,\ln x - \frac{4}{9}\sqrt{x^3}\Big]_1^e = \frac{2e\sqrt{e}}{9} + \frac{4}{9}$$

$$11)\ c)\ \int_0^1 x\,\underbrace{\ln(1+x)}_{u}\,dx = \left[\begin{matrix} u = \ln(1+x) \\ x\,dx = dv \end{matrix} \Rightarrow \begin{matrix} du = \frac{1}{1+x}\,dx \\ v = \int x\,dx = \frac{x^2}{2} \end{matrix}\right]$$

$$= \ln(1+x)\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{1+x}\,dx = \ln(1+x)\cdot\frac{x^2}{2} - \frac{1}{2}\int\frac{x^2}{1+x}\,dx$$

$$\int \frac{x^2}{1+x}\,dx = \int (x-1)\,dx + \int \frac{1}{1+x}\,dx = \frac{x^2}{2} - x + \ln(1+x)$$

$$\begin{array}{r|l} x^2 & 1+x \\ \hline -x^2 - x & x-1 \\ \hline -x & \\ +x+1 & \\ \hline 1 & \end{array}$$

$$\frac{x^2}{1+x} = x - 1 + \frac{1}{1+x}$$

Ento:

$$\int_0^1 x\ln(1+x)\,dx = \left.\frac{x^2}{2}\ln(1+x) - \frac{1}{2}\left(\frac{x^2}{2} - x + \ln(1+x)\right)\right]_0^1$$

$$= \left[\frac{1}{2}\ln 1 - \frac{1}{2}\left(\frac{1}{2} - 1 + \ln 1\right)\right] - \left(0 - \frac{1}{2}\left(0 - 0 + \ln 1\right)\right)$$

$$= -\frac{1}{2}\left(\frac{1}{2} - 1\right) - \left(-\frac{1}{2}\,0\right) = -\frac{1}{2}\left(-\frac{1}{2}\right) = \frac{1}{4}.$$

12) b) $P(x) = ax^3 + bx^2 + cx + d$

$(0,5)$ punto inflexión $\Rightarrow P(0) = 5$

$P''(0) = 0$

$(1,1)$ extremo relativo $\Rightarrow P(1) = 1$

$P'(1) = 0$

$\int_0^1 P(x)\,dx$?

$P(0) = 5 \Rightarrow \boxed{d = 5} \Rightarrow P(x) = ax^3 + bx^2 + cx + 5$

$P(1) = 1 \Rightarrow a + b + c + 5 = 1$

$P'(x) = 3ax^2 + 2bx + c \Rightarrow P'(1) = 0 \Rightarrow$

$\Rightarrow \boxed{3a + 2b + c = 0}$

$P''(x) = 6ax + 2b \Rightarrow P''(0) = 0 \Rightarrow 2b = 0$

$\boxed{b = 0}$

$$\left.\begin{array}{l} a + b + c + 5 = 1 \Rightarrow a + c = -4 \\ 3a + 2b + c = 0 \Rightarrow 3a + c = 0 \end{array}\right\} \Rightarrow \begin{array}{r} a + c = -4 \\ -3a - c = 0 \\ \hline -2a = -4 \\ \boxed{a = 2} \end{array}$$

$a + c = -4$

$2 + c = -4$

$\boxed{c = -6} \Rightarrow P(x) = 2x^3 - 6x + 5$

$$\int_0^1 (2x^3-6x+5)\,dx = \frac{2x^4}{4} - \frac{6x^2}{2} + 5x \Big]_0^1 =$$

$$= \frac{2}{4} - \frac{6}{2} + 5 = \frac{1}{2} - 3 + 5 = \frac{1}{2} + 2 = 5/2$$

13) b) $f'(x) = 1 + \ln x$

$(1,4) \Rightarrow f(1) = 4$

$$f'(x) = 1 + \ln x \Rightarrow \int f'(x)\,dx = \int (1+\ln x)\,dx \Rightarrow$$

$$\Rightarrow f(x) = \int 1\,dx + \int \ln x\,dx = x + x\ln x - x + K$$

$$\left.\begin{array}{l} f(x) = x\ln x + K \\ f(1) = 4 \end{array}\right\} \Rightarrow 1\ln 1 + K = 4 \Rightarrow K = 4$$

$$\Rightarrow \boxed{f(x) = x\ln x + 4}$$

14) c) $\int_{-1}^{0} f(x)\,dx$

$$f(x) = \frac{x}{1+|x|} = \begin{cases} \frac{x}{1-x}, & x \leq 0 \\ \frac{x}{1+x}, & x > 0 \end{cases}$$

$$\int_{-1}^{0} f(x)\,dx = \int_{-1}^{0} \frac{x}{1-x}\,dx = \int -1\,dx + \int \frac{1}{1-x}\,dx =$$

$$\begin{array}{l|l} x & 1-x \\ \hline -x+1 & -1 \\ \hline 1 & \end{array}$$

$$\frac{x}{1-x} = -1 + \frac{1}{1-x}$$

$$= -x - \ln|1-x| \Big]_{-1}^{0} = -0 - \ln 1 - (+1 - \ln|2|)$$

$$= -1 + \ln 2$$

16) $f'(x) = \dfrac{1-\ln x}{x^2}$

$(1,0) \Rightarrow f(1)=0$

$$\int f'(x)\,dx = \int \frac{1-\ln x}{x^2}\,dx \Rightarrow f(x) = \int \frac{1-\ln x}{x^2}\,dx =$$

$$= \int \frac{1}{x^2}\,dx - \int \frac{\ln x}{x^2}\,dx = \frac{x^{-1}}{-1} - \int \frac{\ln x}{x^2}\,dx =$$

$$= -\frac{1}{x} - \int \frac{\ln x}{x^2}\,dx =$$

$$\int \frac{\ln x}{x^2}\,dx = \int \frac{1}{x^2}\underbrace{\ln x}_{u}\,dx = \left[\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x}\,dx \\ \frac{1}{x^2}\,dx = dv \Rightarrow v = \frac{-1}{x} \end{array}\right]$$

$$= -\frac{1}{x}\ln x - \int -\frac{1}{x}\,\frac{1}{x}\,dx = -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx =$$

$$= \frac{-\ln x}{x} + \frac{x^{-1}}{-1} = \frac{-\ln x}{x} - \frac{1}{x} + K.$$

$$\left.\begin{array}{l} f(x) = -\frac{1}{x} + \frac{\ln x}{x} + \frac{1}{x} + K \\ f(1)=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} f(x) = +\frac{\ln x}{x} + K \\ f(1)=0 \end{array}\right\} \Rightarrow K=0$$

$$\boxed{f(x) = +\frac{\ln x}{x}}$$

$$f'(x) = \frac{1 - \ln x}{x^2}$$

$$f''(x) = \frac{-\frac{1}{x}(x^2) - (1-\ln x)\cdot 2x}{x^4} = \frac{-x - 2x + 2x\ln x}{x^4}$$

$$f''(x) = \frac{-3x + 2x\ln x}{x^4} = \frac{-3 + 2\ln x}{x^3}$$

$$f''(x) = 0 \iff -3 + 2\ln x = 0$$

$$2\ln x = 3$$

$$\ln x = 3/2 \Rightarrow e^{3/2} = x \text{ posible punto de inflexión}$$

$$\underset{0}{|} \quad \cap \quad \underset{e^{3/2} = \sqrt{e^3} = e\sqrt{e}}{|} \quad \cup$$

$$f''(1) = \frac{-3 + 2\ln 1}{1^3} = \frac{-3}{1} < 0 \quad \cap \text{ cóncava}$$

$$f''(5) = \frac{-3 + 2\ln 5}{5^3} = \frac{+}{+} > 0 \quad \cup \text{ convexa}$$

Cóncava $(0, e^{3/2})$

Convexa $(e^{3/2}, +\infty)$

18) b) $f(x)=\begin{cases} x+2, & x<1 \\ 3(x-2)^2, & x\geq 1 \end{cases}$

$x<1,\ f(x)=x+2$

x	y
0	2
1	3

$x\geq 1,\ f(x)=3(x^2-4x+4)$

$f(x)=3x^2-12x+12$

$x_v=\frac{12}{6}=2$

$y_v=3\cdot 4-24+12=0$

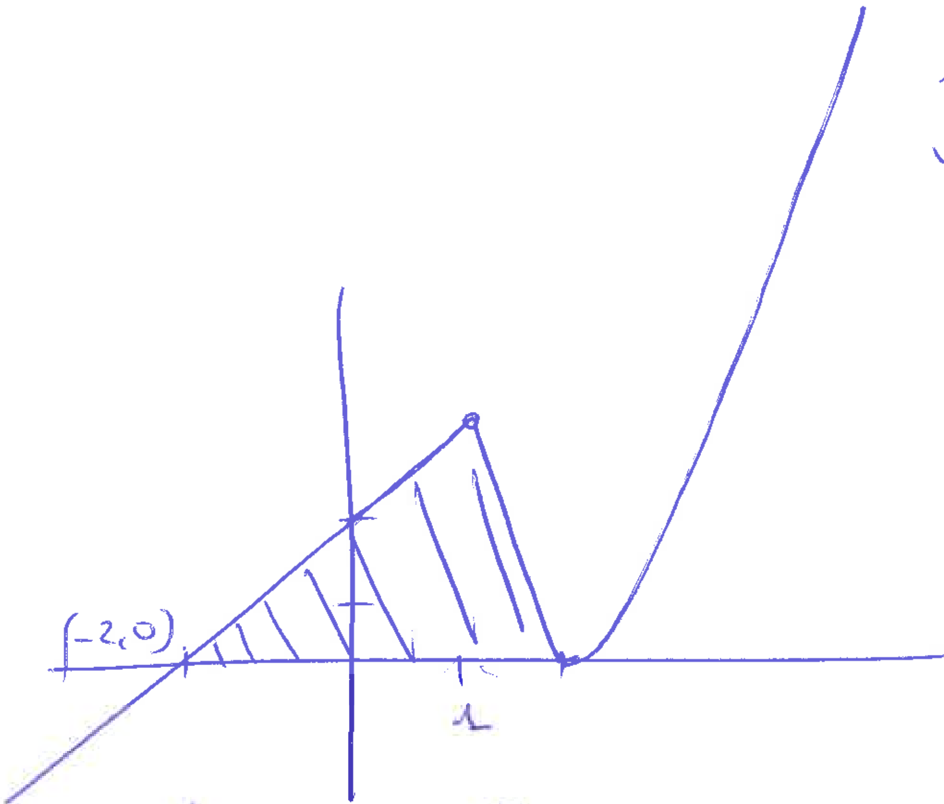
vértice $(2,0)$

$x=0 \Rightarrow y=12 \quad (0,12)$

$y=0 \Rightarrow 0=3x^2-12x+12$

$x=\frac{12\pm\sqrt{144-144}}{6}=2$

$x=1 \Rightarrow y=3-12+12=3$



$$\text{Área}=\int_{-2}^{1}(x+2)\,dx+\int_{1}^{2}(3x^2-12x+12)\,dx=\left.\frac{x^2}{2}+2x\right]_{-2}^{1}+\left.\frac{3x^3}{3}-\frac{12x^2}{2}+12x\right]_{1}^{2}$$

$$=\frac{1}{2}+2-\left(\frac{4}{2}-4\right)+\left[\left(8-6\cdot 4+12\cdot 2\right)-\left(1-6+12\right)\right]$$

$$=\frac{1}{2}+\cancel{2}-\cancel{2}+4+\left[\left(8-24+24\right)-(7)\right]=\frac{1}{2}+4+8-7=\frac{1}{2}+5$$

$$=\frac{11}{2}\,u^2$$

RO

1º) b)

$$f'(x) = \begin{cases} 2x-1, & x<0 \\ e^{2x}-2, & x \geq 0 \end{cases}$$

$f(-1) = 1$

Se $x<0 \Rightarrow f'(x) = 2x-1 \Rightarrow \int f'(x)dx = \int (2x-1)dx$

$\Rightarrow f(x) = \frac{2x^2}{2} - x + k$

$\left. \begin{array}{l} f(x) = x^2 - x + k \\ f(-1) = 1 \end{array} \right\} \Rightarrow$

$\Rightarrow 1+1+k = 1$

$k = -1 \Rightarrow \boxed{f(x) = x^2 - x - 1, \text{ se } x \leq 0}$

Se $x>0$, $f'(x) = e^{2x} - 2 \Rightarrow \int f'(x)dx = \int (e^{2x}-2)\,dx \Rightarrow$

$\Rightarrow f(x) = \frac{1}{2} e^{2x} - 2x + k_2$

$f(x) = \frac{e^{2x}}{2} - 2x + k_2$

A función é derivable en $x=0 \Rightarrow$ A función é continua en $x=0 \Rightarrow \exists \lim_{x \to 0} f(x) = \lim_{x \to 0} f(x) = -1$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} \frac{e^{2x}}{2} - 2x + K_2 = -1$$

$$\Downarrow$$

$$\frac{e^0}{2} - 2\cdot 0 + K_2 = -1$$

$$\frac{1}{2} + K_2 = -1$$

$$K_2 = -1 - 1/2 = -3/2$$

Entón se $x > 0 \Rightarrow f(x) = \frac{e^{2x}}{2} - 2x - 3/2$

$$f(x) = \begin{cases} x^2 - x - 1, & x < 0 \\ \frac{e^{2x}}{2} - 2x - 3/2, & x \geq 0 \end{cases}$$

Recta tangente en $x = -2$ $\begin{cases} (-2, f(-2)) \\ m = f'(-2) \end{cases}$

$f'(-2) = 2\cdot(-2) - 1 = -5$

$f(-2) = 4 + 2 - 1 = 5$ $\Rightarrow$ Recta $\begin{cases} (-2, 5) \\ m = -5 \end{cases}$

$$y - 5 = -5(x+2)$$

$$y - 5 = -5x - 10 \Rightarrow \boxed{y = -5x - 5}$$

Recta tanxente en $x=\frac{\ln 2}{2}$ $\begin{cases} \left(\frac{\ln 2}{2}, f\left(\frac{\ln 2}{2}\right)\right) \\ m=f'\left(\frac{\ln 2}{2}\right) \end{cases}$

$$f\left(\frac{\ln 2}{2}\right)=\frac{e^{2\cdot\frac{\ln 2}{2}}}{2}-2\cdot\frac{\ln 2}{2}-\frac{3}{2}=\frac{2}{2}-\ln 2-\frac{3}{2}$$

$$f(x)=\frac{e^{2x}}{2}-2x-\frac{3}{2} \Rightarrow f\left(\frac{\ln 2}{2}\right)=1-\ln 2-\frac{3}{2}=-\frac{1}{2}-\ln 2$$

$$f'\left(\frac{\ln 2}{2}\right)=e^{2\frac{\ln 2}{2}}-2=2-2=0$$

$$y-\left(-\frac{1}{2}-\ln 2\right)=0\left(x-\frac{\ln 2}{2}\right)$$

$$\boxed{y=-\frac{1}{2}-\ln 2}$$

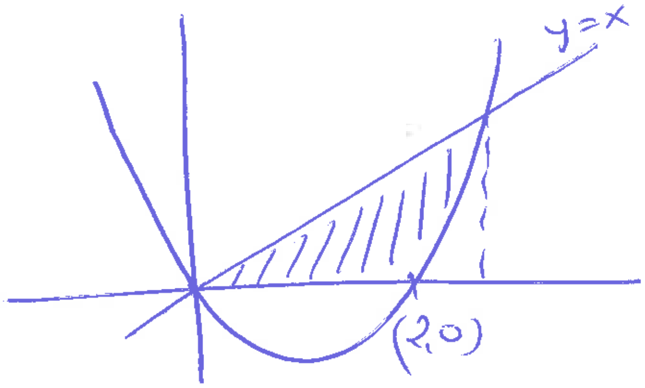
---

201 $y=x(x-2) \Rightarrow y=x^2-2x$

vértice $(1,-1)$

Puntos de corte cos eixes $(0,0),(2,0)$

$a=1 \cup$ convexa.



Puntos de corte entre as funcións $\left.\begin{array}{l} y=x^2-2x \\ y=x \end{array}\right\} \Rightarrow x^2-2x=x$

$x^2-3x=0$

$x(x-3)=0$

$x=0$

$x=3$.

$$\text{Área}=\int_0^2 x\,dx+\int_2^3\left[x-(x^2-2x)\right]dx=\left.\frac{x^2}{2}\right]_0^2+\left.-\frac{x^3}{3}+\frac{3x^2}{2}\right]_2^3=$$

$$= \frac{4}{2} - 0 + \frac{-27}{3} + \frac{3 \cdot 9}{2} - \left(\frac{-8}{3} + \frac{12}{2} \right) =$$

$$= \frac{4}{2} - \frac{27}{3} + \frac{27}{2} + \frac{8}{3} - 6 = \frac{19}{6} u^2$$

22) a) $F(x) = \int_0^x \frac{t^2+6}{2+e^t} dt$

$x = 0$

Recta tangente $\begin{cases} (0, F(0)) \\ m = F'(0) \end{cases}$

$$F(0) = \int_0^0 \frac{t^2+6}{2+e^t} dt = 0$$

$$F'(x) = \frac{x^2+6}{2+e^x} \Rightarrow F'(0) = \frac{6}{2+e^0} = \frac{6}{2+1} = \frac{6}{3} = 2$$

Recta tangente $\begin{cases} (0, 0) \\ m = 2 \end{cases}$ $\quad y - 0 = 2(x-0)$

$$\boxed{y = 2x}$$

22] b) $\int_0^1 x \ln(1+x)\,dx$ 111(c)

24] b) $f(x) = ax^3 + bx + c$

$y = 2x + 1$ recta tanxente en $x = 0$

$\int_0^1 f(x)\,dx = 1$

Se $y = 2x+1$ é a recta tanxente en $x=0 \Rightarrow$

$\Rightarrow f(0) = 1 \Rightarrow \boxed{c = 1}$

$f'(0) = 2 \Rightarrow f'(x) = 3ax^2 + b \Rightarrow$

$\Rightarrow f'(0) = 2 \Rightarrow \boxed{b = 2}$

$f(x) = ax^3 + 2x + 1$

$$\int_0^1 (ax^3 + 2x + 1)\,dx = 1$$

$$\left. a\frac{x^4}{4} + \frac{2x^2}{2} + x \right]_0^1 = \frac{a}{4} + 1 + 1 = 1$$

$$\frac{a}{4} = 1 - 2$$

$$\frac{a}{4} = -1 \Rightarrow \boxed{a = -4}$$

29] $f(0)=0$

$f'(x)=(2-x)\,e^{3x} \Rightarrow \int f'(x)dx=\int(2-x)e^{3x}dx$

$\Rightarrow f(x)=\int \underbrace{(2-x)}_{u}\ \underbrace{e^{3x}dx}_{dv} = \left[\begin{array}{l} u=2-x \Rightarrow du=-dx \\ dv=e^{3x}dx \Rightarrow v=\frac{1}{3}e^{3x}\end{array}\right]$

$= \frac{1}{3}e^{3x}(2-x)-\int\frac{1}{3}e^{3x}(-dx) =$

$= \frac{1}{3}e^{3x}(2-x)+\frac{1}{3}\int e^{3x}dx =$

$= \frac{1}{3}e^{3x}(2-x)+\frac{1}{9}e^{3x}+K$

$f(x)=\frac{1}{3}e^{3x}(2-x)+\frac{1}{9}e^{3x}+K$ $\Big\} \Rightarrow$

$f(0)=0$

$\Rightarrow \frac{1}{3}e^{0}\cdot 2+\frac{1}{9}e^{0}+K=0$

$\frac{2}{3}+\frac{1}{9}+K=0$

$K=-\frac{1}{9}-\frac{2}{3}=\frac{-1-6}{9}=-\frac{7}{9}$

$\boxed{f(x)=\frac{1}{3}e^{3x}(2-x)+\frac{1}{9}e^{3x}-\frac{7}{9}} \Rightarrow f''(x)=(5-3x)e^{3x}$

Convexa en $(-\infty, 5/3)$

Cóncava en $(5/3, +\infty)$

28) $f(x) = 4x - x^2$

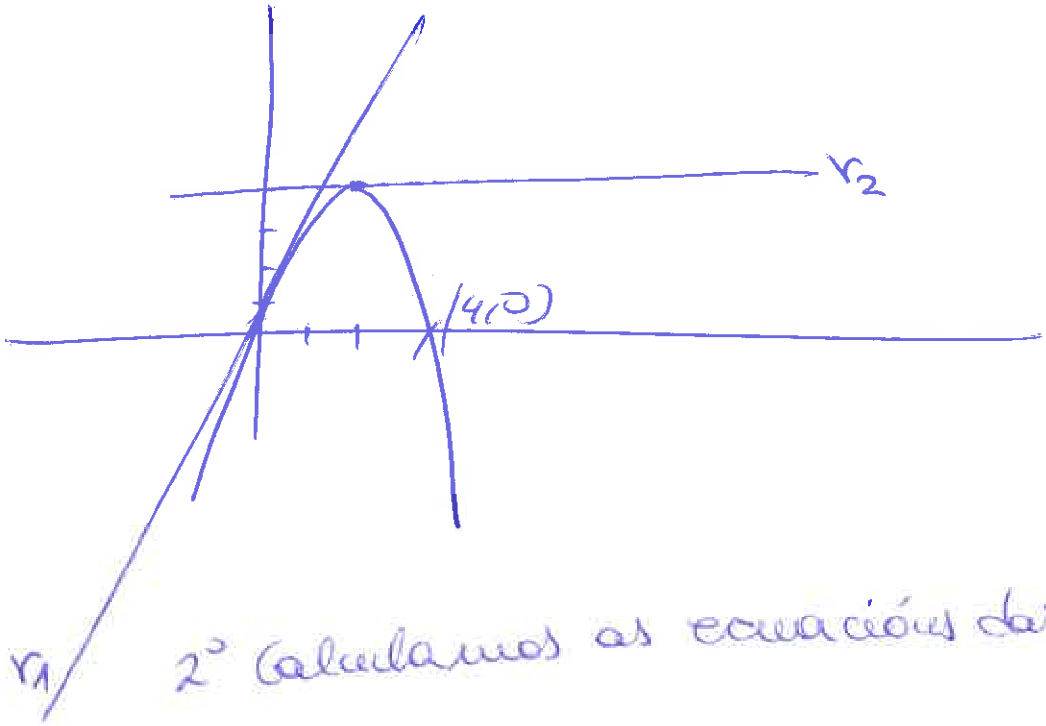
$f(x) = -x^2 + 4x$ 1º Representamos a parábola:

$$\left.\begin{array}{l} x_v = \frac{-4}{-2} = 2 \\ y_v = -4 + 8 = 4 \end{array}\right\} \Rightarrow \text{Vértice } (2,4)$$

$x = 0 \Rightarrow y = 0 \quad (0,0)$

$y = 0 \Rightarrow 0 = 4x - x^2$

$0 = x(4 - x) \Rightarrow \begin{array}{l} x = 0 \\ x = 4 \end{array} \Rightarrow \begin{array}{l} (0,0) \\ (4,0) \end{array}$



2º Calculamos as ecuacións das rectas tanxente

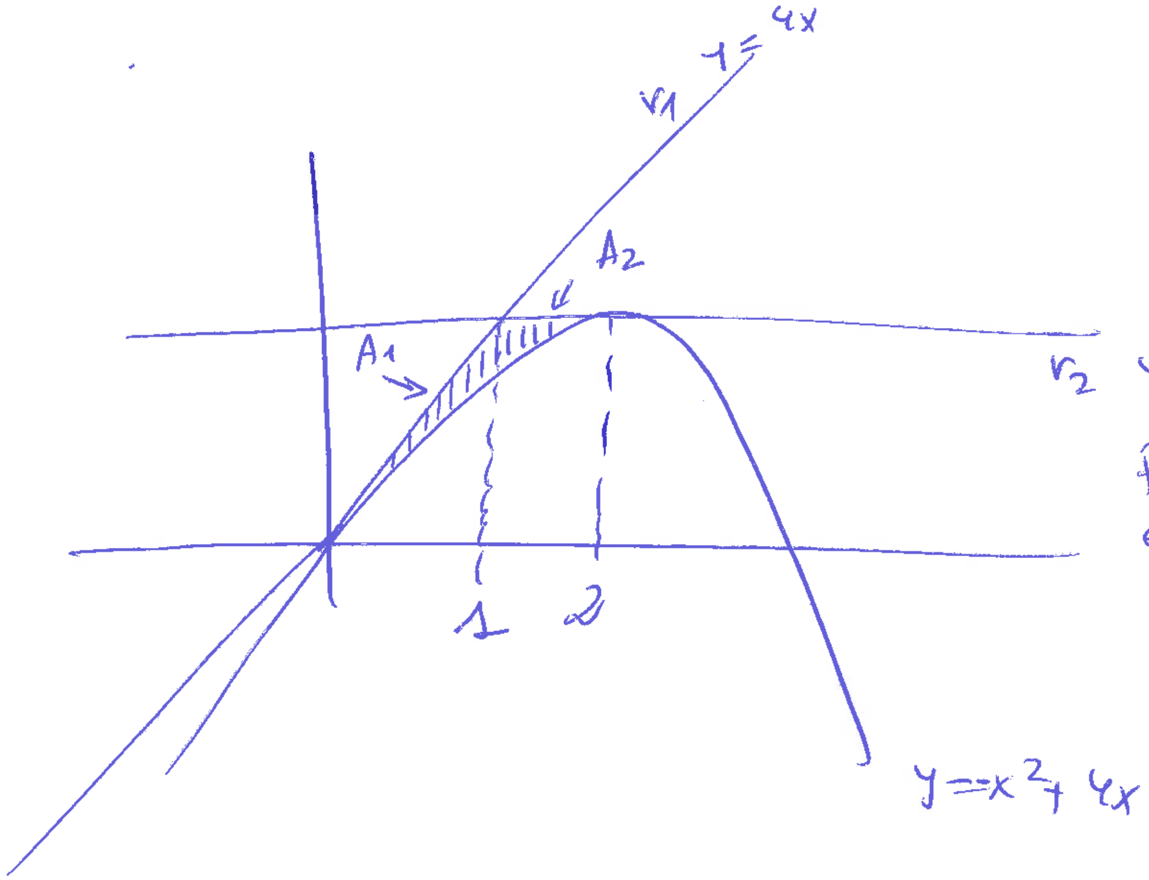
$$r_1 \left\{ \begin{array}{l} (0, f(0)) \\ m = f'(0) \end{array} \right. \Rightarrow r_1 \left\{ \begin{array}{l} (0,0) \\ m = 4 \end{array} \right. \quad \boxed{y = 4x}\ r_1$$

$f(0) = 0$

$f'(x) = 4 - 2x \Rightarrow f'(0) = 4$

$$r_2 \begin{cases} (2, f(2)) = (2, 4) \\ m = f'(2) = 4 - 4 = 0 \end{cases} \Rightarrow \quad y - 4 = 0(x-2)$$

$$\boxed{y = 4} \; r_2$$



Punto de corte entre $\left.\begin{array}{l} y = 4 \\ y = 4x \end{array}\right\} \Rightarrow 4x = 4$, $x = 1$

$$A = A_1 + A_2 = \int_0^1 \left[4x - (-x^2 + 4x)\right] dx + \int_1^2 \left[4 - (-x^2 + 4x)\right] dx$$

$$= \int_0^1 (4x + x^2 - 4x)\, dx + \int_1^2 (4 + x^2 - 4x)\, dx =$$

$$= \int_0^1 x^2\, dx + \int_1^2 (x^2 - 4x + 4)\, dx = \left.\frac{x^3}{3}\right]_0^1 + \left.\frac{x^3}{3} - \frac{4x^2}{2} + 4x\right]_1^2 =$$

$$= \frac{1}{3} + \left(\frac{8}{3} - \cancel{\frac{16}{2}} + \cancel{8}\right) - \left(\frac{1}{3} - \frac{4}{2} + 4\right) = \frac{1}{3} + \frac{8}{3} - \frac{1}{3} - 2$$

$$= \frac{8}{3} - 2 = \frac{2}{3}\, u^2$$

30) b)

$$\int (x \operatorname{sen} x)\, dx = \;?$$

$F(\pi) = 0$

$$\int \underbrace{x}_{u} \underbrace{\operatorname{sen} x\, dx}_{dv} = \left[\begin{array}{l} u = x \Rightarrow du = dx \\ dv = \operatorname{sen} x\, dx \Rightarrow v = -\cos x \end{array} \right]$$

$$= -x\cos x + \int \cos x\, dx = -x\cos x + \operatorname{sen} x + K$$

$F(x) = -x\cos x + \operatorname{sen} x + K \Rightarrow -\pi\cos\pi + \operatorname{sen}\pi + K = 0$

$F(\pi) = 0$ $\qquad \pi + 0 + K = 0$

$K = -\pi$

$$\boxed{F(x) = -x\cos x + \operatorname{sen} x - \pi}$$

32 b)

$$\int_0^1 \frac{e^x}{e^{2x} + 3e^x + 2}\, dx = \left[\begin{array}{l} e^x = t \\ e^x dx = dt \end{array} \right] = \int_1^e \frac{1}{t^2+3t+2}\, dt =$$

$$\frac{1}{t^2+3t+2} = \frac{A}{t+2} + \frac{B}{t+1} \Rightarrow 1 = A(t+1) + B(t+2)$$

$t^2+3t+2 = 0$ $\qquad t = -1 \Rightarrow 1 = B$

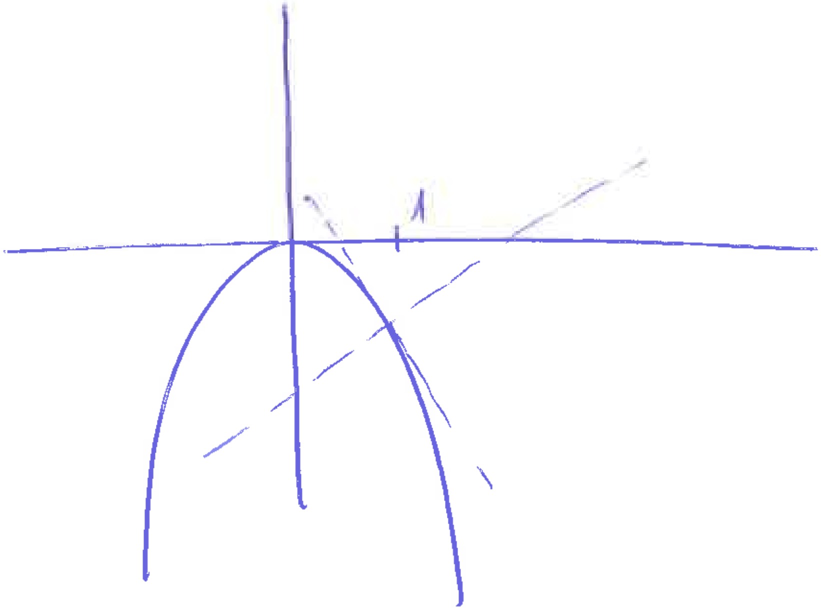
$t = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} \begin{cases} -2 \\ -1 \end{cases}$ $\qquad t = -2 \Rightarrow 1 = -A \Rightarrow A = -1$

$$= \int \frac{-1}{t+2}\, dt + \int \frac{1}{t+1}\, dt = -\ln|t+2| + \ln|t+1| =$$

$$= -\ln|e^x+2| + \ln|e^x+1| \Big]_0^1 =$$

$$= -\ln(e+2) + \ln(e+1) - (-\ln 3 + \ln 2)$$

34) $f(x) = -x^2 \Rightarrow v(0,0),\ a=-1\ \cap$ cóncava, $(0,0)$



Recta normal $\begin{cases} (1, f(1)) \\ m = -\dfrac{1}{f'(1)} \end{cases}$

$f(1) = -1$

$f'(x) = -2x \Rightarrow f'(1) = -2$

$m = -\dfrac{1}{f'(1)} = \dfrac{-1}{-2} = \dfrac{1}{2}$

Recta normal $\begin{cases} (1, -1) \\ m = \frac{1}{2} \end{cases} \Rightarrow y + 1 = \frac{1}{2}(x-1)$

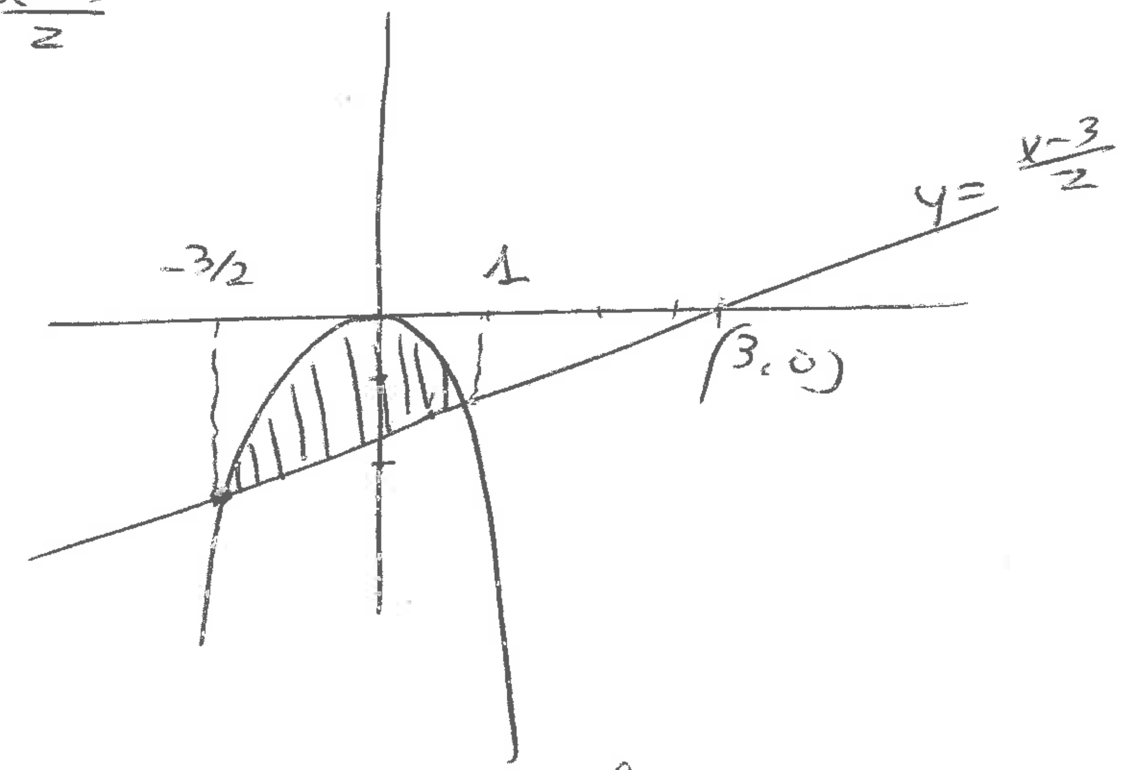
$y = \frac{1}{2}x - \frac{1}{2} - 1$

$y = \frac{1}{2}x - \frac{3}{2}$

$y = \dfrac{x-3}{2}$

$y = \frac{x-3}{2}$

x	y
0	$-3/2$
3	0



Calculemos os puntos de corte:

$$\left.\begin{array}{l} y = -x^2 \\ y = \frac{x-3}{2} \end{array}\right\} \Rightarrow -x^2 = \frac{x-3}{2}$$

$$-2x^2 = x - 3$$

$$-2x^2 - x + 3 = 0$$

$$x = \frac{1 \pm \sqrt{1+24}}{-4} = \frac{1\pm 5}{-4} \begin{cases} -4/4 = 1 \\ \frac{6}{-4} = -3/2 \end{cases}$$

$$\text{Área} = \int_{-3/2}^{1} \left[-x^2 - \left(\frac{x-3}{2}\right)\right] dx = -\frac{x^3}{3} - \frac{1}{2}\frac{x^2}{2} + \frac{3}{2}x \Big]_{-3/2}^{1}$$

$$= -1/3 - \frac{1}{4} + \frac{3}{2} - \left(-\frac{1}{3}\left(\frac{-27}{8}\right) - \frac{1}{4}\cdot\frac{9}{4} + \frac{3}{2}\left(-3/2\right)\right) =$$

$$= \frac{-1}{3} - \frac{1}{4} + \frac{3}{2} - \frac{9}{8} + \frac{9}{16} + \frac{9}{4} = \frac{125}{48} u^2$$

$$= -\frac{9}{8} + \frac{45}{16} + \frac{1}{3} + \frac{1}{4} - \frac{3}{2}$$

36) $f''(x) = 4e^{2x} - 2x$

$f(0) = 1$

$x - y + 3 = 0 \Rightarrow y = x + 3$

A tangente à gráfica no ponto $(0,1)$ é $\parallel$ $y = x+3$

$m = 1$

$f'(0) = 1$

$f(0) = 1$

$$f'(x) = \int (4e^{2x} - 2x)\, dx = 2e^{2x} - \frac{2x^2}{2} + k_1$$

$f'(x) = 2e^{2x} - x^2 + k_1 \Rightarrow f'(0) = 1 \Rightarrow 2e^0 - 0 + k_1 = 1$

$2 \cdot 1 + k_1 = 1$

$\boxed{k_1 = -1}$

$$f'(x) = 2e^{2x} - x^2 - 1$$

$$f(x) = \int (2e^{2x} - x^2 - 1)\, dx = e^{2x} - \frac{x^3}{3} - x + k_2$$

$f(0) = 1 \Rightarrow e^0 - 0 - 0 + k_2 = 1$

$1 + k_2 = 1$

$k_2 = 0 \Rightarrow \boxed{f(x) = e^{2x} - \frac{x^3}{3} - x}$

$$\int_0^{\pi/2} \underbrace{x}_{u}\, \underbrace{\text{sen}\,(2x+\pi)\,dx}_{dv} = \qquad u = x \Rightarrow du = dx$$

$$dv = \text{sen}\,(2x+\pi)\,dx \Rightarrow v = \frac{-\cos(2x+\pi)}{2}$$

$$= \frac{-\cos(2x+\pi)}{2}\cdot x - \int \frac{-\cos(2x+\pi)}{2}\,dx$$

$$= -\frac{x\cos(2x+\pi)}{2} + \frac{1}{2}\int \cos(2x+\pi)\,dx$$

$$= -\frac{x\cos(2x+\pi)}{2} + \frac{1}{4}\,\text{sen}\,(2x+\pi)\Big]_0^{\pi/2}$$

$$= \frac{1}{2}\left(-\frac{\pi}{2}\cos 2\pi\right) + \frac{1}{4}\,\text{sen}\,2\pi - \left(0 + \frac{1}{4}\,\text{sen}\,\pi\right)$$

$$= -\frac{\pi}{4} + 0 - \frac{1}{4}\cdot 0 = -\frac{\pi}{4} - \frac{1}{4}\cdot 0 = -\frac{\pi}{4}$$

38) $$\int_0^1 \frac{2}{3+3e^x}\,dx = \int \frac{2}{3(1+e^x)}\,dx = \frac{2}{3}\int \frac{1}{1+e^x}\,dx$$

$$\int \frac{1}{1+e^x}\,dx = \left[\begin{array}{l} e^x = t \\ e^x dx = dt \\ dx = \dfrac{dt}{t} \end{array}\right] = \int \frac{1}{1+t}\cdot\frac{dt}{t} =$$

$$\int \frac{1}{t^2+t} = \int \frac{1}{t}\,dt + \int \frac{1}{t+1}\,dt =$$

$$\frac{1}{t^2+t} = \frac{1}{t(1+t)} = \frac{A}{t} + \frac{B}{t+1}$$

$$1 = A(t+1) + Bt$$

$$t=0 \Rightarrow 1 = A$$

$$t=-1 \Rightarrow 1 = -B \Rightarrow B = -1$$

$$= \ln|t| - \ln|t+1| = \ln|e^x| - \ln|e^x+1| + K$$

$$\int_0^1 \frac{2}{3+3e^x}\,dx = \frac{2}{3}\int_0^1 \frac{1}{1+e^x}\,dx = \frac{2}{3}\Big[\ln|e^x| - \ln|e^x+1|\Big]_0^1$$

$$= \frac{2}{3}\Big(\big(\ln e - \ln|e+1|\big) - \big|\ln e^0 - \ln(e^0+1)\big| \Big)$$

$$= \frac{2}{3}\Big(1 - \ln|e+1| + \ln 2 \Big) = \frac{2}{3}\Big(1 - \ln|e+1| + \ln 2\Big)$$

38) b) $\lim\limits_{x\to 0} \dfrac{F(x)}{x} = \left[\dfrac{F(0)}{0} = \dfrac{0}{0}\right] \underset{L'H}{=}$

$= \lim\limits_{x\to 0} \dfrac{F'(x)}{1} = \lim\limits_{x\to 0} \dfrac{2}{3+3e^{x}} = \dfrac{2}{3+3\cdot e^{0}} =$

$= \dfrac{2}{3+3} = \dfrac{2}{6} = 1/3$

40) $f(x) = x^3 - 4x^2 + 4x$

a) $f'(x) = 3x^2 - 8x + 4$

$f'(x) = 0 \Leftrightarrow 3x^2 - 8x + 4 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 4\cdot 4\cdot 3}}{6} = \dfrac{8 \pm \sqrt{64 - 16\cdot 3}}{6}$

$x = \dfrac{8 \pm \sqrt{64-48}}{6} = \dfrac{8 \pm \sqrt{16}}{6}$

$x = \dfrac{8 \pm 4}{6}$ $\begin{cases} \dfrac{12}{6} = 2 \\ \dfrac{4}{6} = \dfrac{2}{3} \end{cases}$

$\nearrow \quad 2/3 \quad \searrow \quad 2 \quad \nearrow$

$f'(0) = 4 > 0$

$f'(1) = \cancel{3\cdot 1} - 8 + 4 < 0$

$f'(3) = 27 - 24 + 4 > 0$

Crecente en $(-\infty, 2/3) \cup (2, +\infty)$ $(2/3, f(2/3))$ máximo. $(2/3, 1'19)$

Decrecente $(2/3, 2)$

$(2, f(2))$ mínimo, $f(2) = 8 - 16 + 8 = 0 \Rightarrow (2, 0)$

Concavidade e convexidade:

$f''(x) = 6x - 8 \Rightarrow f''(x) = 0 \Leftrightarrow x = 8/6 = 4/3$

$$\begin{array}{c|c} \cap & \cup \\ \hline \multicolumn{2}{c}{4/3} \end{array}$$

$f''(0) = -8 < 0 \Rightarrow \cap$

$f''(2) = 12 - 8 > 0 \cup$

Cóncava en $(-\infty, 4/3)$

Convexa en $(4/3, +\infty)$

$(4/3, f(4/3))$ punto de inflexión. $(4/3, 0'59)$

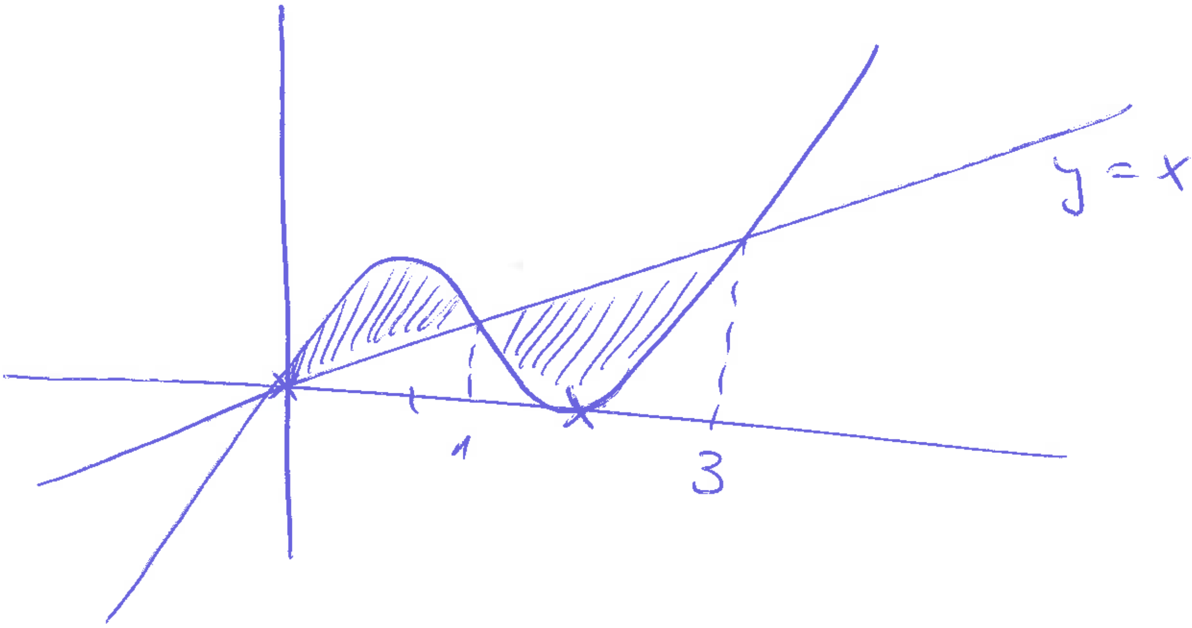
b) Puntos de corte cos eixes:

$x = 0 \Rightarrow 0 = x^3 - 4x^2 + 4x$

$0 = x(x^2 - 4x + 4)$

$x^2 - 4x + 4 = 0 \Rightarrow x = 2 \quad (2, 0)$

$x = \frac{4 \pm \sqrt{16 - 16}}{2} = 2 \qquad x = 0 \Rightarrow (0, 0)$



Puntos de corte entre as funcións:

$$\left.\begin{array}{l} y = x \\ y = x^3 - 4x^2 + 4x \end{array}\right\} \Rightarrow \begin{array}{l} x^3 - 4x^2 + 4x = x \\ x^3 - 4x^2 + 3x = 0 \end{array}$$

$1 \quad -4$

$$x^3 - 4x^2 + 3x = x\,(x^2 - 4x + 3)$$

$$x^2 - 4x + 3 = 0$$

$$x = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} \begin{array}{l} \nearrow 3 \\ \searrow 1 \end{array}$$

$$\text{Área} = \int_0^1 \left[(x^3 - 4x^2 + 4x) - x\right] dx + \int_1^3 \left[x - (x^3 - 4x^2 + 4x)\right] dx$$

$$= \int_0^1 (x^3 - 4x^2 + 3x)\,dx + \int_1^3 (-x^3 + 4x^2 - 3x)\,dx$$

$$= \left.\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right]_0^1 + \left.\frac{-x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2}\right]_1^3$$

$$= \frac{1}{4} - \frac{4}{3} + \frac{3}{2} + \left(-\frac{81}{4} + \frac{4 \cdot 27}{3} - 3 \cdot \frac{9}{2}\right) - \left(-\frac{1}{4} + \frac{4}{3} - \frac{3}{2}\right)$$

$$= \frac{1}{4} - \frac{4}{3} + \frac{3}{2} - \frac{81}{4} + 36 - \frac{27}{2} + \frac{1}{4} - \frac{4}{3} + \frac{3}{2}$$

$$= \frac{1}{2} + 3 - \frac{27}{2} - \frac{8}{3} - \frac{81}{4} + 36$$

$$= -\frac{26}{2} - \frac{8}{3} - \frac{81}{4} + 39$$

$$= -13 - \frac{8}{3} - \frac{81}{4} + 39 = 26 - \frac{8}{3} - \frac{81}{4} = \frac{37}{12} u^2.$$

42. b) $\displaystyle\int \frac{x^3+3}{x^2-x}\,dx = \int (x+1)\,dx + \int \frac{x+3}{x^2-x}\,dx$

$$\begin{array}{l|l} x^3 + 3 & x^2 - x \\ \hline -x^3 + x^2 & x + 1 \\ \hline x^2 + 3 & \\ -x^2 + x & \\ \hline x + 3 & \end{array}$$

$$\frac{x^3+3}{x^2-x} = x + 1 + \frac{x+3}{x^2-x}$$

$$\int \frac{x+3}{x^2-x}\,dx$$

$$\frac{x+3}{x^2-x} = \frac{A}{x} + \frac{B}{x-1}$$

$$x+3 = A(x-1) + Bx$$

$$x=0 \Rightarrow 3 = -A \Rightarrow A = -3$$

$$x=1 \Rightarrow 4 = B \Rightarrow B = 4$$

$$\int \frac{x+3}{x^2-x}\,dx = \int \frac{-3}{x}\,dx + \int \frac{4}{x-1}\,dx =$$

$$= -3\ln|x| + 4\ln|x-1|$$

$$\int \frac{x^3+3}{x^2-x}\,dx = \frac{x^2}{2} + x - 3\ln|x| + 4\ln|x-1| + K.$$

431 b) $y = f(x)$ continua en $[1,4]$

$$\int_1^2 f(x)\,dx = 2, \quad \int_1^4 f(x)\,dx = -4, \quad \int_2^4 5f(x)\,dx?$$

$$\int_2^4 5f(x)\,dx = 5\int_2^4 f(x)\,dx =$$

$$\int_1^4 f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^4 f(x)\,dx$$

$$-4 = 2 + \int_2^4 f(x)\,dx \Rightarrow \int_2^4 f(x)\,dx = -4-2 = -6$$

$$\int_2^4 5f(x)\,dx = 5\int_2^4 f(x)\,dx = 5\cdot(-6) = -30$$

44) $f(x) = -x^2+9x$, $y=20$, $x-y+15=0$

$f(x) = -x^2+9x$

1º) $x_v = \frac{-9}{-2} = \frac{9}{2}$

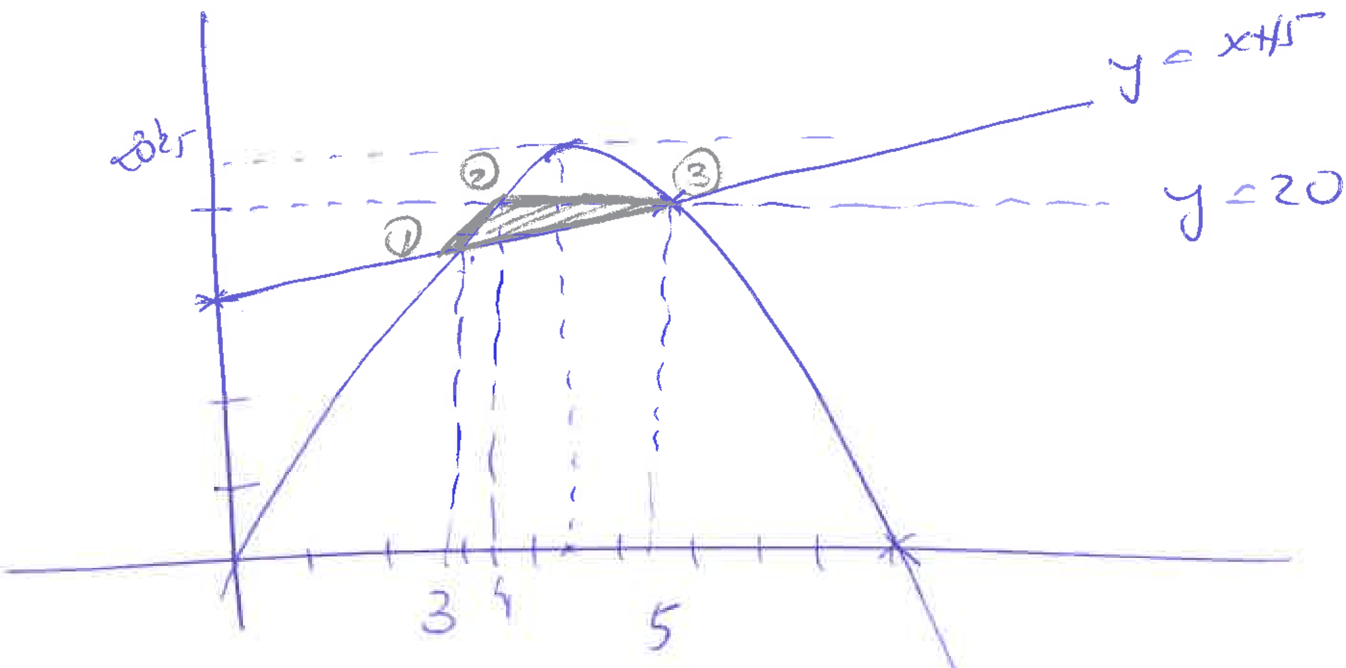
$y_v = -\frac{81}{4} + \frac{81}{2} = \frac{-81+162}{4} = \frac{81}{4} \Rightarrow v\left(\frac{9}{2}, \frac{81}{4}\right) = (4'5, 20'$

2º) $a=-1<0$ ∩ cóncava

3º) $x=0 \Rightarrow y=0 \Rightarrow (0,0)$

$y=0 \Rightarrow 0=-x^2+9x$

$0 = x(-x+9) \Rightarrow \begin{matrix} x=0 \\ x=9 \end{matrix} \Rightarrow \begin{matrix} (0,0) \\ (9,0) \end{matrix}$



① Punto de corte $\left.\begin{array}{l} y = -x^2 + 9x \\ y = x + 15 \end{array}\right\} \Rightarrow x + 15 = -x^2 + 9x$

$x^2 - 8x + 15 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 60}}{2} = \dfrac{8 \pm 2}{2} \begin{array}{l} \nearrow 5 \\ \searrow 3 \end{array}$

$\Rightarrow$ ① ; $(3, f(3)) = (3, 18)$

② Punto de corte $\left.\begin{array}{l} y = -x^2 + 9x \\ y = 20 \end{array}\right\} \Rightarrow -x^2 + 9x = 20$

$-x^2 + 9x - 20 = 0 \Rightarrow$

$\Rightarrow x^2 - 9x + 20 = 0 \Rightarrow x = \dfrac{+9 \pm \sqrt{81 - 80}}{2} = \dfrac{+9 \pm 1}{2} \begin{array}{l} \nearrow +5 \\ \searrow +4 \end{array}$

$\Rightarrow$ ② $(4, 20)$

③ $(5, 20)$

$$A = \int_3^4 \left[(-x^2+9x) - (x+15)\right] dx + \int_4^5 \left[20 - (x+15)\right] dx$$

$$= \int_3^4 (-x^2+8x-15)\, dx + \int_4^5 (-x+5)\, dx =$$

$$= -\frac{x^3}{3} + \frac{8x^2}{2} - \frac{15x}{x} \Big]_3^4 + -\frac{x^2}{2} + 5x \Big]_4^5 =$$

$$= -\frac{x^3}{3} + 4x^2 - 15x \Big]_3^4 + \left(-\frac{x^2}{2} + 5x \right]_4^5 =$$

$$= -\frac{64}{3} + 64 - 60 - \left(-\frac{27}{3} + 36 - 45\right) + \left(-\frac{25}{2} + 25\right) - \left(-\frac{16}{2} + 2\ldots\right.$$

$$= -\frac{64}{3} + \cancel{6}4 - \left(-9 + (9)\right) + \left(-\frac{25}{2} + 25\right) - \left(-8 + 20\right)$$

$$= -\frac{64}{3} + 4 + 18 - \frac{25}{2} + 25 - 12$$

$$= -\frac{64}{3} - \frac{25}{2} + 47 - 12 = -\frac{64}{3} - \frac{25}{2} + 35$$

$$= \frac{-128 - 75 + 210}{6} = \frac{-203 + 210}{6} = \frac{7}{6} u^2$$

46) b) $\displaystyle\int_2^3 \frac{x^3+2}{x^2-1}\,dx = \int x\,dx + \int \frac{x+2}{x^2-1}\,dx$

$$\begin{array}{l|l} x^3+2 & x^2-1 \\ \underline{-x^3+x} & x \\ x+2 & \end{array}$$

$$\frac{x^3+2}{x^2-1} = x + \frac{x+2}{x^2-1}$$

$$\int \frac{x+2}{x^2-1}\,dx = \int \frac{3/2}{x-1}\,dx + \int \frac{-1/2}{x+1}\,dx =$$

$$\left\{\begin{array}{l} \dfrac{x+2}{x^2-1} = \dfrac{A}{x-1} + \dfrac{B}{x+1} \\ x+2 = A(x+1) + B(x-1) \\ x=1 \Rightarrow 3 = 2A \Rightarrow A = 3/2 \\ x=-1 \Rightarrow 1 = -2B \Rightarrow B = -1/2 \end{array}\right.$$

$$= \frac32 \ln|x-1| - \frac12 \ln|x+1| \Rightarrow$$

$$\int_2^3 \frac{x^3+2}{x^2-1}\,dx = \left[\frac{x^2}{2} + \frac32 \ln|x-1| - \frac12 \ln|x+1|\right]_2^3 =$$

$$= \frac92 + \frac32 \ln 2 - \frac12 \ln 4 - \left(\frac42 + \frac32 \ln 1 - \frac12 \ln 3\right)$$

$$= \frac92 + \frac32 \ln 2 - \frac12 \ln 4 - 2 - \frac32 \cdot 0 + \frac12 \ln 3$$

$$= \frac92 + \frac32 \ln 2 - \frac12 \ln 4 - \qquad = \frac92 + \frac{3\ln 2}{2} - \frac{2\ln 2}{2} -$$

$$= \frac92 + \frac{\ln 2^3}{2} - \frac{\ln 2^2}{2} - 2 + \frac{\ln 3}{2} =$$

$$-2 + \frac{\ln 3}{2} = \frac92 + \frac{\ln 2}{2} - 2 + \frac{\ln 3}{2} = \frac52 + \frac{\ln 2}{2} + \frac{\ln 3}{2} =$$

$$= \frac{5 + \ln 2 + \ln 3}{2} = \frac{5 + \ln 6}{2}$$