Soluciones Ficha Límites L'Hôpital

Básicos

1) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} 2x}{3x} = \dfrac{0}{0}$ L'Hôpital

$= \lim\limits_{x\to 0} \dfrac{2\cos 2x}{3} = \boxed{\dfrac{2}{3}}$

2) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} x}{x} \underset{\uparrow}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{1} = \boxed{1}$

$\frac{0}{0}$ L'Hôpital

3) $\lim\limits_{x\to 0^+} \dfrac{\ln x}{\ln x^2} = \lim\limits_{x\to 0^+} \dfrac{\ln x}{2\ln x} = \lim\limits_{x\to 0^+} \dfrac{1}{2} = \boxed{\dfrac{1}{2}}$

4) $\lim\limits_{x\to 0} \dfrac{2x^3}{x-\operatorname{sen} x} \underset{\uparrow}{=} \lim\limits_{x\to 0} \dfrac{6x^2}{1-\cos x} \underset{\uparrow}{=} \lim\limits_{x\to 0} \dfrac{12x}{\operatorname{sen} x} = \dfrac{0}{0}$

$\frac{0}{0}$ L'Hôpital (1ª vez) $\quad$ $\frac{0}{0}$ L'Hôpital (2ª vez) $\quad$ L'Hôpital (3ª vez)

$= \lim\limits_{x\to 0} \dfrac{12}{\cos x} = \boxed{12}$

5) $\lim\limits_{x\to 1} \dfrac{e^x - e}{x^2-1} \underset{\uparrow}{=} \lim\limits_{x\to 1} \dfrac{e^x}{2x} = \boxed{\dfrac{e}{2}}$

$\frac{0}{0}$ L'Hôpital

6) $\lim\limits_{x\to 0} \dfrac{x\cos x - \operatorname{sen} x}{x^2} \underset{\uparrow}{=} \lim\limits_{x\to 0} \dfrac{\cos x + x(-\operatorname{sen} x) - \cos x}{2x} =$

$\frac{0}{0}$ L'Hôpital

$= \lim\limits_{x\to 0} \dfrac{-x\operatorname{sen} x}{2x} \underset{\uparrow}{=} \lim\limits_{x\to 0} -\dfrac{\operatorname{sen} x}{2} = \boxed{0}$

como $x \neq 0$

7) $\lim\limits_{x\to 0} \dfrac{1-2x-e^x+\operatorname{sen} 3x}{x^2} \underset{\uparrow}{=} \lim\limits_{x\to 0} \dfrac{-2-e^x+3\cos 3x}{2x} \underset{\uparrow}{=}$

$\frac{0}{0}$ L'Hôpital $\quad$ $\frac{0}{0}$ L'Hôpital 2ª vez

$= \lim\limits_{x\to 0} \dfrac{-e^x - 9\operatorname{sen}(3x)}{2} = \boxed{-\dfrac{1}{2}}$

8) $\lim\limits_{x\to \infty} \dfrac{2x^2-1}{e^{2x}} \underset{\uparrow}{=} \lim\limits_{x\to \infty} \dfrac{4x}{2e^{2x}} = \lim\limits_{x\to \infty} \dfrac{2x}{e^{2x}} \underset{\uparrow}{=} \lim\limits_{x\to \infty} \dfrac{2}{2e^{2x}} =$

$\frac{\infty}{\infty}$ L'Hôpital $\quad$ $\frac{\infty}{\infty}$ L'Hôpital (2ª vez)

$= \dfrac{1}{\infty} = \boxed{0}$

9) $\lim_{x\to 0} \frac{x e^{x^2}}{\cos x - 1} \underset{\frac{0}{0}\text{ L'Hospital}}{=} \lim_{x\to 0} \frac{e^{x^2} + x e^{x^2} 2x}{-\operatorname{sen} x} =$

$= \lim_{x\to 0} \frac{e^{x^2}(1+2x^2)}{-\operatorname{sen} x} = \frac{1}{0}$ indet.

Miramos los límites laterales:

$$\left.\begin{array}{l} \lim_{x\to 0^-} \frac{e^{x^2}(1+2x^2)}{-\operatorname{sen} x} = +\infty \\ \lim_{x\to 0^+} \frac{e^{x^2}(1+2x^2)}{-\operatorname{sen} x} = -\infty \end{array}\right\} \Rightarrow \nexists \lim$$

10) $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} = 1^\infty$ tipo nº e

$= e^{\lim_{x\to 0} (\cos x - 1)\frac{1}{x^2}} = e^{\lim_{x\to 0} \frac{\cos x - 1}{x^2}} \underset{\frac{0}{0}\text{ L'Hospital}}{=}$

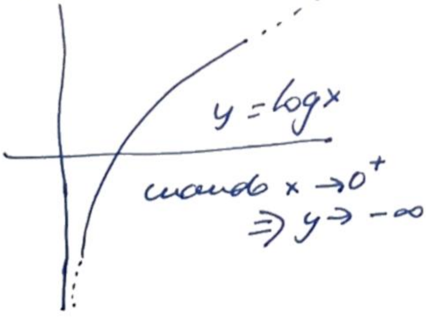
$= e^{\lim_{x\to 0} \frac{-\operatorname{sen} x}{2x}} \underset{\text{L'Hospital otra vez}}{\overset{\frac{0}{0}}{=}} e^{\lim_{x\to 0} \frac{-\cos x}{2}} = e^{-1/2} = \boxed{\frac{1}{\sqrt{e}}}$

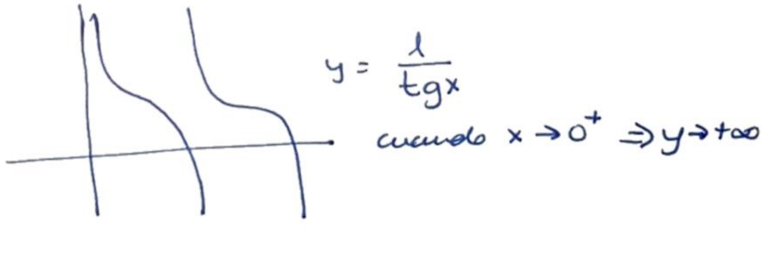
11) $\lim_{x\to 1} x^{\frac{1}{1-x}} = 1^\infty$ tipo nº e

$= e^{\lim_{x\to 1} (x-1)\frac{1}{1-x}} = e^{\lim_{x\to 1} \frac{-(1-x)}{1-x}} = e^{-1} = \boxed{\frac{1}{e}}$

12) $\lim_{x\to 0} \frac{\log x}{\operatorname{cotg} x} = \lim_{x\to 0} \frac{\log x}{\frac{1}{\operatorname{tg} x}} = -\frac{\infty}{\infty}$

para verlo, dibujar las gráficas de $\log x$ y $\operatorname{tg} x$





12) aplicando L'Hopital:

$$= \lim_{x\to 0^+} \frac{(\log x)'}{\left(\frac{1}{tg x}\right)'} = \lim_{x\to 0^+} \frac{\frac{1}{x}\log e}{-\frac{(tg\,x)'}{tg^2 x}} = \lim_{x\to 0^+} \frac{\frac{\log e}{x}}{-\left[\frac{\frac{1}{\cos^2 x}}{\frac{\operatorname{sen}^2 x}{\cos^2 x}}\right]} =$$

$$= \lim_{0^+} \frac{\frac{\log e}{x}}{-\frac{\cos^2 x}{\cos^2 x \operatorname{sen}^2 x}} = \lim_{x\to 0^+} \frac{\frac{\log e}{x}}{-\frac{1}{\operatorname{sen}^2 x}} = \lim_{x\to 0^+} \frac{-\log e \operatorname{sen}^2 x}{x} =$$

$$= \lim_{x\to 0^+} \frac{\log e \; 2 \operatorname{sen} x \cos x}{1} = \boxed{0}$$

$\frac{0}{0}$ L'Hopital 2ª vez

13) $$\lim_{x\to 0} (\operatorname{cotg} x \cdot \operatorname{arc\,sen} x) = \lim_{x\to 0} \frac{1}{tg x} \operatorname{arc\,sen} x = \frac{0}{0}$$

L'Hopital

$$= \lim_{x\to 0} \frac{(\operatorname{arc\,sen} x)'}{(tg x)'} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{\frac{1}{\cos^2 x}} = \lim_{x\to 0} \frac{\cos^2 x}{\sqrt{1-x^2}} = \boxed{1}$$

hasta aquí para el examen del lunes