

3. Potencias de fracciones. Exponente entero.

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$$\textcircled{a} 5^{-1} = \frac{1}{5} \quad \textcircled{b} 2^{-3} = \frac{1}{2^3} = \frac{1}{8} \quad \textcircled{c} (-6)^0 = 1$$

$$\textcircled{d} \left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 4 \quad \textcircled{e} \left(\frac{2}{3}\right)^{-1} = \left(\frac{3}{2}\right)^1 = \frac{3}{2}$$

$$\textcircled{f} \frac{1}{4^{-2}} = \frac{4^2}{1} = 16 \quad \textcircled{g} \left(\frac{1}{10}\right)^{-1} = \left(\frac{10}{1}\right)^1 = 10$$

$$\textcircled{h} \left[\left(\frac{5}{2}\right)^2\right]^{-1} = \left(\frac{5}{2}\right)^{-2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

Ejercicios 1

$$\textcircled{a} 10^{-3} = \frac{1}{10^3} = \frac{1}{1000} \quad \textcircled{b} 7^{-1} = \frac{1}{7} \quad \textcircled{c} (-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$$

$$\textcircled{d} (-5)^{-3} = \frac{1}{(-5)^3} = \frac{1}{-125} \quad \textcircled{e} \frac{1}{2^{-2}} = \frac{2^2}{1} = 4$$

$$\textcircled{f} \frac{1}{3^{-4}} = \frac{3^4}{1} = 81 \quad \textcircled{g} \left(\frac{7}{3}\right)^{-2} = \left(\frac{3}{7}\right)^2 = \frac{9}{49}$$

$$\textcircled{h} \left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5 = 32 \quad \textcircled{i} \left(-\frac{1}{4}\right)^{-1} = \left(-\frac{4}{1}\right)^1 = -4$$

$$\textcircled{j} \left(-\frac{1}{10}\right)^{-4} = \left(-\frac{10}{1}\right)^4 = +10.000$$

$$\textcircled{k} \left(-\frac{3}{5}\right)^{-3} = \left(-\frac{5}{3}\right)^3 = -\frac{125}{27}$$

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$$\begin{aligned} & \left(\frac{1}{3}\right)^{-1} \cdot \left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^{-3} \cdot (3-2)^5 \cdot 3^7 = \\ & = \underset{\text{"}}{3^1} \cdot \underset{\text{"}}{3^{-2}} \cdot \underset{\text{"}}{3^3} \cdot 3^{-10} \cdot 3^7 = \boxed{3^{-1}} \\ & \left(\frac{3}{1}\right)^1 \left(\frac{3}{1}\right)^{-2} \left(\frac{3}{1}\right)^3 \end{aligned}$$

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(a) $4^{-2} = (2^2)^{-2} = 2^{-4}$

(b) $\left(\frac{1}{8}\right)^{-2} = \left(\frac{2}{1}\right)^2 = (2^3)^2 = 2^6$

(c) $\frac{4^{-1} \cdot 8^{-1}}{(2^{-3})^{-3}} = \frac{(2^2)^{-1} \cdot (2^3)^{-1}}{2^{+9}} = \frac{2^{-2} \cdot 2^{-3}}{2^9} = \frac{2^{-5}}{2^9} =$
 $= 2^{-5-9} = \boxed{2^{-14}}$

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(a) $\frac{(-2)^3 \cdot 4^2}{32} = \frac{-2^3 \cdot (2^2)^2}{2^5} = \frac{-2^3 \cdot 2^4}{2^5} = \frac{-2^7}{2^5} = \boxed{-2^2}$

(b) $\frac{125}{25^2 \cdot (-5)^2} = \frac{5^3}{(5^2)^2 \cdot 5^2} = \frac{5^3}{5^4 \cdot 5^2} = \frac{5^3}{5^6} = \boxed{5^{-3}}$

(c) $\frac{3^2 \cdot 9^4}{(35)^2} = \frac{3^2 \cdot (3^2)^4}{3^{10}} = \frac{3^2 \cdot 3^8}{3^{10}} = \frac{3^{10}}{3^{10}} = \boxed{3^0}$

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$$(a) \frac{(-6^2)^3}{(-6^3)^2} = \frac{-6^6}{+6^6} = \boxed{-6^0} = \boxed{-1}$$

$$(b) \frac{(-1)^4}{3^4} \cdot 9^2 = \frac{1}{3^4} \cdot (3^2)^2 = \frac{3^4}{3^4} = \boxed{3^0} = \boxed{1}$$

$$(c) \frac{15^3}{3^2 \cdot 5^4} = \frac{(3 \cdot 5)^3}{3^2 \cdot 5^4} = \frac{3^3 \cdot 5^3}{3^2 \cdot 5^4} = \boxed{\frac{3}{5}}$$

$$(d) \frac{(6^4)^3}{2^{10} \cdot (3^6)^2} = \frac{6^{12}}{2^{10} \cdot 3^{12}} = \frac{(2 \cdot 3)^{12}}{2^{10} \cdot 3^{12}} = \frac{2^{12} \cdot 3^{12}}{2^{10} \cdot 3^{12}} = \frac{2^2}{1} = \boxed{4}$$

$$(e) \frac{(-2)^5 \cdot (-3)^5}{6^4} = \frac{-6^5}{6^4} = \boxed{-6}$$

$$(f) \frac{(-1)^5 \cdot 10^4}{5^4 \cdot 2^3} = \frac{-(2 \cdot 5)^4}{5^4 \cdot 2^3} = \frac{-2^4 \cdot 5^4}{5^4 \cdot 2^3} = \boxed{-2}$$

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$$a) \frac{-3^2}{(-3)^3} = \frac{-3^2}{-3^3} = +3^{-1} = \left(\frac{1}{3}\right)$$

$$b) \left(\frac{2}{3}\right)^2 \cdot \left(\frac{-3}{2}\right)^4 = \left(\frac{2}{3}\right)^2 \cdot \left(\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)^{-2} \cdot \left(\frac{3}{2}\right)^4 = \\ = \left[\left(\frac{3}{2}\right)^2\right] = \left[\frac{9}{4}\right]$$

$$c) \left(\frac{1}{2}\right)^3 : \left(\frac{1}{4}\right)^2 = \frac{1^3}{2^3} : \frac{1^2}{4^2} = \frac{4^2}{2^3} = \frac{(2^2)^2}{2^3} = \frac{2^4}{2^3} = (2)$$

$$d) \left(\frac{2}{5}\right)^2 : \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^{-1} = \left(\frac{5}{2}\right)$$

$$e) \frac{2^3 \cdot (-3)^2 \cdot 4^2}{6^3 \cdot 9^2} = \frac{2^3 \cdot 3^2 \cdot 4^2}{6^3 \cdot 9^2} = \frac{2^3 \cdot 3^2 \cdot (2^2)^2}{(2 \cdot 3)^3 \cdot (3^2)^2} = \frac{2^3 \cdot 3^2 \cdot 2^4}{2^3 \cdot 3^3 \cdot 3^4} \\ = \frac{2^7 \cdot 3^2}{2^3 \cdot 3^7} = \left(\frac{2^4}{3^5}\right) = (2^4 \cdot 3^{-5}) = \left[\frac{16}{243}\right]$$

$$f) \left[\left(\frac{1}{2}\right)^{-3}\right]^2 = \left[\left(\frac{1}{2}\right)^{-6}\right] = \left(\frac{2}{1}\right)^6 = [64]$$

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$$(a) \frac{(-7)^4}{7^{-2}} = \frac{7^4}{7^{-2}} = 7^{4-(-2)} = (7^6)$$

$$(b) \frac{1}{5^2 \cdot 5^4} = \frac{1}{5^{-2}} = (5^2)$$

$$(c) \frac{6^4 \cdot 2^{-2}}{2^2 \cdot 3^{-1}} = \frac{6^4 \cdot 3}{2^2 \cdot 2^2} = \frac{2^4 \cdot 3^4 \cdot 3}{2^4} = (3^5)$$

$$(d) \left(\frac{1}{2^3}\right)^{-2} \cdot \left(\frac{1}{2^{-3}}\right)^2 = \left(\frac{2^3}{1}\right)^2 \cdot \left(\frac{2^{-3}}{1}\right)^{-2} = 2^6 \cdot 2^6 = (2^{12})$$

$$(e) \left(\frac{1}{5^3}\right)^2 \cdot (5^3)^{-2} = \left(\frac{5^3}{1}\right)^{-2} \cdot 5^{-6} = 5^{-6} \cdot 5^{-6} = (5^{-12})$$

$$(f) \frac{3^{-4}}{9^{-3}} = \frac{9^3}{3^4} = \frac{(3^2)^3}{3^4} = \frac{3^6}{3^4} = (3^2)$$

$$(g) \frac{5^2 \cdot 10^{-2}}{2^2} = \frac{5^2}{2^2 \cdot 10^2} = \frac{5^2}{2^2 \cdot 2^2 \cdot 5^2} = \frac{5^2}{2^4 \cdot 5^2} = \frac{1}{2^4} = (2^{-4})$$

$$(h) \frac{12^2 \cdot 5^{-5}}{15^2 \cdot 8^{-1}} = \frac{(3 \cdot 2^2)^2 \cdot 8}{3^2 \cdot 5^2 \cdot 5^5} = \frac{3^2 \cdot 2^4 \cdot 2^3}{3^2 \cdot 5^7} = \frac{2^7 \cdot \cancel{3^2}}{5^7 \cdot \cancel{3^2}}$$

$$= \frac{2^7}{5^7} = \left(\frac{2}{5} \right)^7$$

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$$(a) \frac{2^2 \cdot 3^4}{9 \cdot 12 \cdot 6} = \frac{2^2 \cdot 3^4}{3^2 \cdot 3 \cdot 2^2 \cdot 2 \cdot 3} = \frac{2^2 \cdot \cancel{3^4}}{\cancel{3^4} \cdot 2^3} = \left[\frac{1}{2} \right]$$

$$(b) \frac{4 \cdot 45 \cdot 24}{3^3 \cdot 2^4 \cdot 5} = \frac{2^2 \cdot 3^2 \cdot 5 \cdot 3 \cdot 2^3}{3^3 \cdot 2^4 \cdot 5} = \frac{2^5 \cdot 3^3 \cdot \cancel{5}}{2^4 \cdot 3^3 \cdot \cancel{5}} = [2]$$

$$(c) \frac{8 \cdot 27^{-1}}{12^{-1}} = \frac{8 \cdot 12}{27} = \frac{2^3 \cdot 3 \cdot 2^2}{3^3} = \frac{2^5 \cdot 3}{3^3} = \frac{2^5}{3^2} = \left[\frac{32}{9} \right]$$

PRÁCTICA

Ejercicio 2 Simplifica estas expresiones empleando las propiedades de las potencias y calcula su resultado.

$$a) \frac{81^2 \cdot 32}{16^2 \cdot 27^2} = \frac{(3^4)^2 \cdot 2^5}{(2^4)^2 \cdot (3^3)^2} = \frac{3^8 \cdot 2^5}{2^8 \cdot 3^6} = \frac{3^2}{2^3} = \boxed{\frac{9}{8}}$$

$$b) \frac{25^2 \cdot 5}{625^2} = \frac{(5^2)^2 \cdot 5}{(5^4)^2} = \frac{5^4 \cdot 5}{5^8} = \frac{5^5}{5^8} = \frac{1}{5^3} = \boxed{\frac{1}{125}}$$

$$c) \frac{49^4 \cdot 5^6}{7^7 \cdot 125^2} = \frac{(7^2)^4 \cdot 5^6}{7^7 \cdot (5^3)^2} = \frac{7^8 \cdot 5^6}{7^7 \cdot 5^6} = \boxed{7}$$

$$d) \frac{121^3 \cdot 64}{11^4 \cdot 2^5} = \frac{(11^2)^3 \cdot 2^6}{11^4 \cdot 2^5} = \frac{11^6 \cdot 2^6}{11^4 \cdot 2^5} = 11^2 \cdot 2 = \boxed{242}$$

$$e) \frac{2^{-5} \cdot 4^3}{16} = \frac{(2^2)^3}{2^4 \cdot 2^5} = \frac{2^6}{2^9} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$$

$$f) \frac{2^5 \cdot 3^2 \cdot 4^{-1}}{2^3 \cdot 9^{-1}} = \frac{2^5 \cdot 3^2 \cdot 4}{2^3 \cdot 9} = \frac{2^5 \cdot 3^2 \cdot 2^2}{2^3 \cdot 3^2} = \frac{2^5 \cdot 3^4}{2^5} = 3^4 = \boxed{81}$$

$$\begin{aligned}
 \textcircled{g} \quad & \frac{6^4 \cdot 3^{-2}}{8^{-2} \cdot 2^3 \cdot 2^4} = \frac{6^4 \cdot 3^2}{3^2 \cdot 2^3 \cdot 2^4} = \frac{(2 \cdot 3)^4 \cdot (2^3)^2}{3^2 \cdot 2^3 \cdot 2^4} \\
 & = \frac{2^4 \cdot 3^4 \cdot 2^6}{3^2 \cdot 2^3 \cdot 2^4} = \frac{2^{10} \cdot 3^4}{2^7 \cdot 3^2} = 2^3 \cdot 3^2 = \boxed{72}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{h} \quad & \frac{10^{-1} \cdot 4^2}{12^2 \cdot 15^{-2}} = \frac{15^2 \cdot 4^2}{12^2 \cdot 10^1} = \frac{(3 \cdot 5)^2 \cdot (2^2)^2}{(3 \cdot 2^2)^2 \cdot 2 \cdot 5} = \frac{3^2 \cdot 5^2 \cdot 2^4}{3^2 \cdot 2^4 \cdot 2 \cdot 5} \\
 & = \frac{3^2 \cdot 5^2 \cdot 2^4}{3^2 \cdot 2^5 \cdot 5} = \boxed{\frac{5}{2}}
 \end{aligned}$$