

FICHA DE EJERCICIOS IV - EJERCICIOS DE RADICALES -

1. Extraer factores:

$$a) \sqrt{2 \cdot 3^2 \cdot 5^5} = 3 \cdot 5^2 \sqrt{2 \cdot 5}$$

$$b) \sqrt[4]{2^7 \cdot 3^{14} \cdot 5^4} = 2 \cdot 3^3 \cdot 5 \sqrt[4]{2^3 \cdot 3^2}$$

2. Introducir factores:

$$a) 2\sqrt{3} = \sqrt{2^2 \cdot 3}$$

$$b) 2^2 \cdot 3^3 \sqrt[4]{6} = \sqrt[4]{(2^2)^4 \cdot (3^3)^4 \cdot 6} = \sqrt[4]{2^8 \cdot 3^{12} \cdot 6}$$

3. Poner a común índice:

$$\sqrt{2}$$

$$\sqrt[3]{2^2 \cdot 3^2}$$

$$\sqrt[4]{2^2 \cdot 3^3}$$

$$\sqrt[12]{(2)^6}$$

$$\sqrt[12]{(2^2)^4 \cdot (3^2)^4}$$

$$\sqrt[12]{(2^2)^3 \cdot (3^3)^3}$$

$$\sqrt[12]{2^6}$$

$$\sqrt[12]{2^8 \cdot 3^8}$$

$$\sqrt[12]{2^6 \cdot 3^9}$$

4. Realiza las sumas:

$$a) 2\sqrt{2} - 4\sqrt{2} + \sqrt{2} = -\sqrt{2}$$

$$b) 3\sqrt[4]{5} - 2\sqrt[4]{5} - \sqrt[4]{5} = 0\sqrt[4]{5} = 0$$

$$c) \sqrt{12} - 3\sqrt{3} + 2\sqrt{75} = \sqrt{2^2 \cdot 3} - 3\sqrt{3} + 2\sqrt{3 \cdot 5^2} = 2\sqrt{3} - 3\sqrt{3} + 10\sqrt{3} = 9\sqrt{3}$$

$$d) \sqrt[4]{4} + \sqrt[6]{8} - \sqrt[12]{64} = \sqrt[4]{2^2} + \sqrt[6]{2^3} - \sqrt[12]{2^6} = \sqrt{2} + \sqrt{2} - \sqrt{2} = \sqrt{2}$$

$$e) 2\sqrt{12} - 3\sqrt{75} + \sqrt{27} = 2\sqrt{2^2 \cdot 3} - 3\sqrt{3 \cdot 5^2} + \sqrt{3^3} = 4\sqrt{3} - 15\sqrt{3} + 3\sqrt{3} = -8\sqrt{3}$$

$$f) \sqrt{24} - 5\sqrt{6} + \sqrt{486} = \sqrt{2^3 \cdot 3} - 5\sqrt{6} + \sqrt{2 \cdot 3^5} = 2\sqrt{6} - 5\sqrt{6} + 9\sqrt{6} = 6\sqrt{6}$$

$$g) \frac{2\sqrt{5} + \sqrt{45} + \sqrt{180} - \sqrt{80}}{2\sqrt{5} + 3\sqrt{5} + 6\sqrt{5} - 4\sqrt{5}} = \frac{2\sqrt{5} + \sqrt{3^2 \cdot 5} + \sqrt{2^2 \cdot 3^2 \cdot 5} - \sqrt{2^4 \cdot 5}}{7\sqrt{5}}$$

$$h) \frac{\sqrt[3]{54} - \sqrt[3]{16} + \sqrt[3]{250}}{6\sqrt[3]{2}} = \frac{\sqrt[3]{2 \cdot 3^3} - \sqrt[3]{2^4} + \sqrt[3]{2 \cdot 5^3}}{6\sqrt[3]{2}} = \frac{3\sqrt[3]{2} - 2\sqrt[3]{2} + 5\sqrt[3]{2}}{6\sqrt[3]{2}}$$

$$i) \sqrt{2} + \frac{1}{\sqrt{2}} = \sqrt{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2} + \frac{\sqrt{2}}{2} = \sqrt{2} + \frac{1}{2}\sqrt{2} = \frac{3}{2}\sqrt{2}$$

$$j) \frac{\sqrt[3]{16} + \sqrt[3]{250} + \sqrt[6]{4} - \frac{1}{\sqrt[3]{4}}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2^4} + \sqrt[3]{2 \cdot 5^3} + \sqrt[6]{2^2} - \frac{1}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}}{\sqrt[3]{2}} = \frac{2\sqrt[3]{2} + 5\sqrt[3]{2} + \sqrt[3]{2} - \frac{1}{2}\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{15}{2}\sqrt[3]{2}$$

5. Realizar los productos:

$$a) \sqrt{2} \cdot \sqrt{6} = \sqrt{12} = 2\sqrt{3}$$

$$b) \sqrt{3} \cdot \sqrt[3]{9} \cdot \sqrt[4]{27} = \sqrt[12]{3^6} \cdot \sqrt[12]{(3^2)^4} \cdot \sqrt[12]{(3^3)^3} = \sqrt[12]{3^6} \cdot \sqrt[12]{3^8} \cdot \sqrt[12]{3^9} = \sqrt[12]{3^{23}} = 3^{12}\sqrt[12]{3^{11}}$$

$$c) \sqrt{12} \cdot \sqrt[3]{36} = \sqrt{2^2 \cdot 3} \cdot \sqrt[3]{2^2 \cdot 3^2} = \sqrt[6]{(2^2 \cdot 3)^3} \cdot \sqrt[6]{(2^2 \cdot 3^2)^2} = \sqrt[6]{2^6 \cdot 3^3} \cdot \sqrt[6]{2^4 \cdot 3^4} = 2\sqrt{3} \cdot \sqrt[3]{2^2 \cdot 3^2}$$

6. Efectúa las divisiones de radicales:

$$a) \frac{\sqrt[6]{128}}{\sqrt[6]{16}} = \sqrt[6]{128} : \sqrt[6]{16} = \sqrt[6]{2^7} : \sqrt[6]{2^4} = \sqrt[6]{2^3} = \sqrt[3]{2}$$

$$b) \frac{\sqrt[3]{4}}{\sqrt{2}} = \sqrt[3]{4} : \sqrt{2} = \sqrt[6]{(2^2)^2} : \sqrt[6]{2^3} = \sqrt[6]{2}$$

$$c) \frac{\sqrt{256}}{\sqrt[3]{16}} = \sqrt{2^8} : \sqrt[3]{2^4} = \sqrt[6]{(2^8)^3} : \sqrt[6]{(2^4)^2} = \sqrt[6]{2^{24}} : \sqrt[6]{2^8} = \sqrt[6]{2^{16}} = 4\sqrt[3]{2^4} = 4\sqrt[3]{2^2}$$

7. Realiza las operaciones:

$$a) (\sqrt{7} - \sqrt{2})^2 = (\sqrt{7})^2 - 2 \cdot \sqrt{7} \cdot \sqrt{2} + (\sqrt{2})^2 = 7 - 2\sqrt{14} + 2 = 9 - 2\sqrt{14}$$

$$b) (2 - \sqrt{3})^2 = (2)^2 - 2 \cdot 2 \cdot \sqrt{3} + (\sqrt{3})^2 = 4 - 4\sqrt{3} + 3 = 7 - 4\sqrt{3}$$

$$c) (\sqrt{5} + 2) \cdot (\sqrt{5} - 2) = (\sqrt{5})^2 - (2)^2 = 5 - 4 = 1$$

$$d) (2\sqrt{5} + 3\sqrt{2}) \cdot (2\sqrt{5} - 3\sqrt{2}) = (2\sqrt{5})^2 - (3\sqrt{2})^2 = 4 \cdot 5 - 9 \cdot 2 = 20 - 18 = 2$$

8. Racionalizar los radicales:

$$a) \frac{2}{3\sqrt{2}} = \frac{2 \cdot \sqrt{2}}{3\sqrt{2} \cdot \sqrt{2}} = \frac{2 \cdot \sqrt{2}}{3(\sqrt{2})^2} = \frac{2 \cdot \sqrt{2}}{3 \cdot 2} = \frac{\sqrt{2}}{3}$$

$$b) \frac{2}{3\sqrt[5]{4}} = \frac{2}{3\sqrt[5]{2^2}} = \frac{2 \cdot \sqrt[5]{2^3}}{3\sqrt[5]{2^2} \cdot \sqrt[5]{2^3}} = \frac{2 \sqrt[5]{8}}{3\sqrt[5]{2^5}} = \frac{2 \sqrt[5]{8}}{3 \cdot 2} = \frac{\sqrt[5]{8}}{3}$$

$$c) \frac{2}{\sqrt{2} - \sqrt{3}} = \frac{2 \cdot (\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3}) \cdot (\sqrt{2} + \sqrt{3})} = \frac{2\sqrt{2} + 2\sqrt{3}}{(\sqrt{2})^2 - (\sqrt{3})^2} = \frac{2\sqrt{2} + 2\sqrt{3}}{2 - 3} = \frac{2\sqrt{2} + 2\sqrt{3}}{-1} = -2\sqrt{2} - 2\sqrt{3}$$

$$d) \frac{2}{4 - 2\sqrt{2}} = \frac{2 \cdot (4 + 2\sqrt{2})}{(4 - 2\sqrt{2}) \cdot (4 + 2\sqrt{2})} = \frac{2 \cdot (4 + 2\sqrt{2})}{4^2 - (2\sqrt{2})^2} = \frac{2 \cdot (4 + 2\sqrt{2})}{16 - 4 \cdot 2} = \frac{2 \cdot (4 + 2\sqrt{2})}{8} = \frac{4 + 2\sqrt{2}}{4}$$

$$\frac{2\sqrt{2}}{5-2\sqrt{6}} = \frac{2\sqrt{2} \cdot (5+2\sqrt{6})}{(5-2\sqrt{6}) \cdot (5+2\sqrt{6})} = \frac{10\sqrt{2} + 4\sqrt{12}}{5^2 - (2\sqrt{6})^2} =$$

$$e) \quad = \frac{10\sqrt{2} + 4\sqrt{2^2 \cdot 3}}{25 - 4 \cdot 6} = \frac{10\sqrt{2} + 8\sqrt{3}}{25 - 24} = 10\sqrt{2} + 8\sqrt{3}$$

$$f) \quad \frac{5}{2\sqrt{2}} = \frac{5 \cdot \sqrt{2}}{2 \cdot \sqrt{2} \cdot \sqrt{2}} = \frac{5 \cdot \sqrt{2}}{2 \cdot \sqrt{2^2}} = \frac{5 \cdot \sqrt{2}}{2 \cdot 2} = \frac{5 \cdot \sqrt{2}}{4}$$

$$g) \quad \frac{1}{\sqrt[3]{3}} = \frac{\sqrt[3]{3^2}}{\sqrt[3]{3} \cdot \sqrt[3]{3^2}} = \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^3}} = \frac{\sqrt[3]{9}}{3}$$

$$h) \quad \frac{2}{3+\sqrt{3}} = \frac{2 \cdot (3-\sqrt{3})}{(3+\sqrt{3}) \cdot (3-\sqrt{3})} = \frac{6-2\sqrt{3}}{3^2 - (\sqrt{3})^2} =$$

$$\frac{6-2\sqrt{3}}{9-3} = \frac{6-2\sqrt{3}}{6} = \frac{3-\sqrt{3}}{3}$$

$$i) \quad \frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{2} \cdot (\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2}) \cdot (\sqrt{3}+\sqrt{2})} =$$

$$= \frac{\sqrt{6} + \sqrt{2^2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{2 + \sqrt{6}}{3-2} = 2 + \sqrt{6}$$

$$i) \quad \frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} = \frac{(3\sqrt{2} - 2\sqrt{3}) \cdot (3\sqrt{2} - 2\sqrt{3})}{(3\sqrt{2} + 2\sqrt{3}) \cdot (3\sqrt{2} - 2\sqrt{3})} =$$

$$= \frac{(3\sqrt{2} - 2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} = \frac{(3\sqrt{2})^2 - 2 \cdot 3 \cdot \sqrt{2} \cdot 2\sqrt{3} + (2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} =$$

$$= \frac{9 \cdot 2 - 12\sqrt{6} + 4 \cdot 3}{9 \cdot 2 - 4 \cdot 3} = \frac{18 - 12\sqrt{6} + 12}{18 - 12} =$$

$$= \frac{30 - 12\sqrt{6}}{6} = 5 - 2\sqrt{6}$$